# Accuracy Compensation Technology of Closed-Loop Feedback of Industrial Robot Joints

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**Abstract:** The existing kinematic parameter calibration method cannot further improve the absolute positioning accuracy of the robot due to the uncertainty of positioning error caused by robot joint backlash. In view of this problem, a closed-loop feedback accuracy compensation method for robot joints was proposed. Firstly, a Chebyshev polynomial error estimation model was established which took geometric error and non-geometric error into account. In addition, the absolute linear grating scale was installed at each joint of the robot and the positioning error of the robot end was mapped to the joint angle. And the joint angle corrected value was obtained. Furthermore, the closed-loop feedback of robot joints was established to realize the online correction of the positioning error. Finally, an experiment on the KUKA KR210 industrial robot was conducted to demonstrate the effectiveness of the method. The result shows that the maximum absolute positioning error of the robot is reduced by 75% from 0.76 mm to 0.19 mm. This method can compensate the robot joint backlash effectively and further improve the absolute positioning accuracy of the robot.

**Key words:** accuracy compensation; closed-loop feedback of robot joint; Chebyshev polynomial; robot joint backlash

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## 0 Introduction

The automatic drilling and riveting system based on industrial robot is gaining favour in aircraft assembly industry due to its advantages of high flexibility, high efficiency, and low cost of manufacturing and maintenance[1-3]. When using robot off-line programming technology to plan drilling and riveting task, the absolute positioning accuracy of robot determines the position accuracy of off-line program. In fact, the absolute positioning accuracy of the industrial robot with series structure is only 1—2 mm, which cannot meet the requirements of the aviation manufacturing field for positioning accuracy (for example, the drilling accuracy of aircraft parts is required to be within 0.5 mm). Therefore, it is of great theoretical significance and practical value to study the robot accuracy compensation technology and improve the absolute positioning accuracy of industrial robots.

The error sources that affect the positioning accuracy of robot can be divided into geometric error and non-geometric error. Geometric errors are error sources that can be represented by geometric quantities, mainly referring to the kinematics parameter errors of the robot. Among them, the joint angle errors have the largest impact on positioning errors, accounting for nearly 90% of all the errors[4-5]. Non-geometric errors are the error sources that are difficult to be represented by geometric quantities. Among them, joint flexibility, friction caused by relative motion, joint transmission error and joint back-

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lash will all affect the motion accuracy of robot joints. Cordes et al.[6] studied the influence of joint backlash on robot milling, and believed that the robot motion speed and joint position had a great influence on the joint backlash. Therefore, the accurate control of the robot joint angle is an effective way to improve the absolute positioning accuracy of the robot.

Scholars have conducted a lot of research on robot precision compensation methods, which are mainly divided into on-line error correction method[7-8] and off-line error calibration method[9-13]. The main idea of the on-line error correction method is to install external sensors on the joints of the robot to detect the actual position deviation of the robot, and the control system will adjust the pose of the robot in real time according to the deviation so as to achieve the accuracy requirements. Qu et al.[7] used the laser tracker to conduct real-time closed-loop feedback control on the robot's terminal posture, so that the absolute positioning accuracy of the robot could reach within 0.2 mm. However, this method has relatively high cost and higher requirements for the openness, lighting and other measuring environments of the industrial site.

The basic principle of the off-line error calibration method is to identify the kinematic parameter error of the robot by measuring the positioning error of several positions in the whole working space of the robot and combining with the kinematic error model, so as to correct the kinematic model of the robot. This method is widely used because it does not require external detection equipment, however, it can only compensate the geometric errors caused by the manufacturing and assembly errors of the robot, and other non-geometric errors such as joint flexibility and joint transmission errors cannot be compensated. In view of the above shortcomings, Hong et al.[10] comprehensively considered the influence of geometric error and flexibility error of the robot, and proposed a variable parameter error model with spatial grid, which effectively solved the problem of uneven spatial distribution of parameter error. Zeng et al.[11-12] proposed the concept of error similarity, and believed that when the input of each joint of the robot was similar, the corresponding positioning error had the similarity. By establishing the mapping relationship between the positioning error and joint rotation angle, the positioning error of the target point is predicted, and the error post-processing compensation strategy is adopted to compensate the positioning error without modifying the robot control parameters. Nguyen et al.[13] proposed the idea of hierarchical compensation, which used extended Kalman filtering algorithm to identify the geometric errors of the robot, and then used the residual error and robot joint angle after parameter calibration as the output and input of artificial neural network (ANN) respectively to realize the compensation of the non-geometric error of the robot. The above off-line calibration methods are highly dependent on the repetitive positioning accuracy of the robot. In fact, the one-way repetitive positioning accuracy of the robot is very high, but the multi-directional repetitive positioning accuracy is relatively poor. Due to the influence of the accuracy change of the robot's multi-direction pose[14], the sampling point error has an uncertain component during the error measurement, so the off-line calibration method cannot further improve the compensation effect. Therefore, it is necessary to improve the robot accuracy compensation method.

In this paper, the robot accuracy compensation technology was studied, starting from the suppression of joint backlash. And on the basis of the modified Denavit-Hartenberg (MDH) model kinematics parameter calibration method, a robot joint closed-loop feedback accuracy compensation method was proposed. Firstly, the spatial error estimation model was established. Then, the positioning error of the robot was corrected on-line by the closed-loop feedback control of the joint. Finally, experiments verified the effectiveness of the proposed method for further improving the absolute positioning accuracy of the robot.

## 1 Calibration Method of Kinematics Parameters

It is very important to choose the appropriate kinematics model for robot kinematics error modeling. Denavit-Hartenberg (DH) model[15] is one of the most commonly used. In order to avoid the phenomenon that the axes of the adjacent joints of the robot have parallelism error which leads to the singularity of the transformation matrix, MDH model[16] is often adopted for robot error modeling and parameter calibration. The kinematics equation of $n$-DOF (Degree of freedom) robot established with MDH model is as follows

$$ {}^{0}_{n}T = {}^{0}_{1}T\,{}^{1}_{2}T \cdots {}^{i}_{i+1}T \cdots {}^{n-1}_{n}T \tag{1}$$

where ${}^{i}_{i+1}T$ is the spatial transformation relation between the robot link coordinate system $\{i+1\}$ and the robot link coordinate system $\{i\}$ and is expressed as follows

$$ {}^{i}_{i+1}T(\theta_i) = T_{r,z}(\theta_i)T_{t,z}(d_i)T_{t,x}(a_i)T_{r,x}(\alpha_i)T_{r,y}(\beta_i) \tag{2}$$

where $\theta_i, d_i, a_i, \alpha_i$, and $\beta_i$ are kinematics parameters of the robot.

The kinematic parameter error of the robot can be regarded as a tiny displacement. This differential transformation caused by the kinematic parameter error is transmitted through the joints of the robot to the tool coordinate system at the end of the robot and results in positioning error at the end of the robot. By means of the parameter calibration method, the kinematics parameters can be as close as possible to the real value so as to realize the error compensation based on the kinematics model. Let the theoretical position of the robot end be $\boldsymbol{P}$ and its position error is $\Delta\boldsymbol{P}$, and the position error of the robot end can be approximately written as a linear combination of errors of each kinematics parameter, which is shown as follows

$$\Delta \boldsymbol{P} = \frac{\partial \boldsymbol{P}}{\partial \theta}\Delta\theta + \frac{\partial \boldsymbol{P}}{\partial d}\Delta d + \frac{\partial \boldsymbol{P}}{\partial a}\Delta a + \frac{\partial \boldsymbol{P}}{\partial \alpha}\Delta\alpha + \frac{\partial \boldsymbol{P}}{\partial \beta}\Delta\beta = \boldsymbol{J}_\theta\Delta\theta + \boldsymbol{J}_d\Delta d + \boldsymbol{J}_a\Delta a + \boldsymbol{J}_\alpha\Delta\alpha + \boldsymbol{J}_\beta\Delta\beta \tag{3}$$

where $\Delta\theta, \Delta d, \Delta a, \Delta\alpha, \Delta\beta$ are the errors of kinematic parameter of each joint, and $\boldsymbol{J}_\theta, \boldsymbol{J}_d, \boldsymbol{J}_a, \boldsymbol{J}_\alpha, \boldsymbol{J}_\beta$ the $3\times n$ matrixes composed of the kinematic parameters of each joint. Assuming that there are three groups of robot positioning error measured values, according to Eq.(3), it can be obtained

$$\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \vdots \\ \Delta P_m \end{bmatrix} = \begin{bmatrix} J_\theta^1 & J_d^1 & J_a^1 & J_\alpha^1 & J_\beta^1 \\ J_\theta^2 & J_d^2 & J_a^2 & J_\alpha^2 & J_\beta^2 \\ & & \vdots & & \\ J_\theta^m & J_d^m & J_a^m & J_\alpha^m & J_\beta^m \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta d \\ \Delta a \\ \Delta\alpha \\ \Delta\beta \end{bmatrix} \tag{4}$$

The above formula can be written as

$$\Delta P = \boldsymbol{J} \cdot \Delta X \tag{5}$$

where $\Delta\boldsymbol{X}$ is the error vector of robot kinematics parameters, and the least square method is used to solve the error vector of robot kinematics parameters, as shown below

$$\Delta \boldsymbol{X} = (\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J})^{-1}\boldsymbol{J}^{\mathrm{T}}\Delta \boldsymbol{P} \tag{6}$$

The error of kinematics parameters calculated is substituted into the kinematics model, and a modified kinematics model is established to estimate the positioning error of the robot.

The above MDH model-based kinematic parameter calibration method has the following two shortcomings:

(1) It is impossible to represent the multi-directional pose accuracy variation of the robot due to the robot joint backlash.

(2) It is impossible to describe the real robot kinematics model which contains non-geometric errors such as gear friction, load change, and joint transmission error.

## 2 Closed-Loop Feedback Accuracy Compensation Method for Robot Joint

In order to further improve the absolute positioning accuracy of the robot, it is necessary to establish a spatial error estimation model that comprehensively considers the geometric errors and non-geometric errors of the robot and realize the spatial error compensation of the robot by means of the closed-loop feedback of the hardware sensor.

### 2.1 Spatial error estimation model and its coefficient identification

#### 2.1.1 Chebyshev polynomial error modeling

In order to build a real robot kinematics model reflecting the characteristics of positioning error distribution, this paper proposes a spatial error estimation model based on Chebyshev higher order polynomial. The actual kinematics model of the robot is considered as the introduction of error transfer matrix based on the theoretical kinematics model, which is used to describe the relative position relationship between the theoretical robot link coordinate system and the actual robot link coordinate system, as shown in Fig.1. Taking the 6-DOF series industrial robot as the research object, the error transfer matrix is introduced in the coordinate system of each link, and the actual kinematics model of the robot can be expressed as

$$F_a(\theta)=T_1(\theta_1)\boldsymbol{E}_1(\theta_1)T_2(\theta_2)\boldsymbol{E}_2(\theta_2)\cdots T_n(\theta_n)\boldsymbol{E}_n(\theta_n) \tag{7}$$

where $\boldsymbol{E}_i(\theta_i)$ is error transfer matrix. In general, when the geometric and non-geometric errors of the robot work together, the spatial transformation relationship between adjacent joints will also have errors, which can be regarded as differential transformation caused by geometric errors and non-geometric errors. Therefore, the error transfer matrix can be expressed as follows, including three rotational error terms and three translational error terms

$$\boldsymbol{E}_i(\theta_i)=\begin{bmatrix} 1 & -\varepsilon_{i,z}(\theta_i) & \varepsilon_{i,y}(\theta_i) & \delta_{i,x}(\theta_i) \\ \varepsilon_{i,z}(\theta_i) & 1 & -\varepsilon_{i,x}(\theta_i) & \delta_{i,y}(\theta_i) \\ -\varepsilon_{i,y}(\theta_i) & \varepsilon_{i,x}(\theta_i) & 1 & \delta_{i,z}(\theta_i) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

where $\varepsilon_{i,x}$, $\varepsilon_{i,y}$, and $\varepsilon_{i,z}$ are the rotational error terms of the link coordinate system $\{i\}$ around the $x$, $y$, and $z$ axes, respectively; and $\delta_{i,x}$, $\delta_{i,y}$, and $\delta_{i,z}$ the translational error terms of the link coordinate system $\{i\}$ along the $x$, $y$, and $z$ axes, respectively.

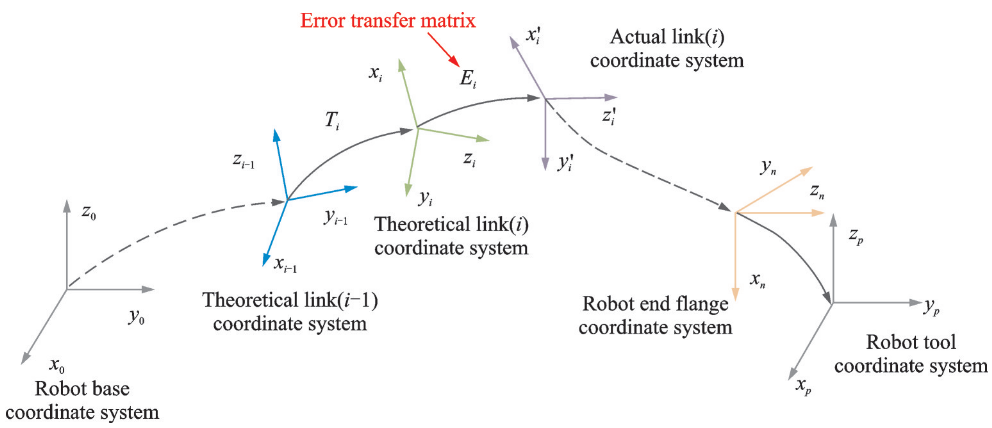


Fig.1 Schematic diagram of spatial error estimation model

In order to accurately characterize the geometric errors and non-geometric errors of the robot, the error transfer matrix is considered as the mapping function of joint angle. Therefore, each differential transformation in the error transfer matrix can be written as a polynomial based on the joint rotation angle. The order of polynomial is an optimization parameter, which not only ensures the accuracy of robot error modeling, but also prevents error overfitting. In general, Taylor series expansion is used to approximate the function. However, the fitting error of Taylor series expansion is small near the expansion point, but the farther it is from the expansion point, the fitting error will increase obviously, especially at the edge of the region. Chebyshev polynomial fitting is often used for function approximation and bounded error in the whole range, which is better than the Taylor series expansion of the same order. It has been widely used in digital signal processing, satellite orbit, radar detection and other fields[17-19]. Ref.[20] used Chebyshev polynomials to establish a 6-DOF kinematics error model to characterize the error distribution characteristics of the 5-axis CNC machine tool, and reduced the absolute positioning error of the machine tool to 0.1 mm. In this paper, Chebyshev polynomials are introduced to model robot positioning errors.

Chebyshev polynomials are a series of orthogonal polynomials defined in a recursive way, with the definition field of $[-1, 1]$, and its $m$-order polynomial is shown as

$$f(x)=\lambda_0 c_0(x)+\lambda_1 c_1(x)+\lambda_2 c_2(x)+\cdots+\lambda_m c_m(x) \tag{9}$$

where $c_0(x)=1$, $c_1(x)=x$, $\cdots$, $c_m(x)=2xc_{m-1}(x)-c_{m-2}(x)$ and $\lambda_0,\lambda_1,\lambda_2,\cdots,\lambda_m$ are the Chebyshev polynomial coefficients. The differential transformations in the error transfer matrix are written in the form of Chebyshev polynomials as follows

$$\begin{gathered}\varepsilon_{i,k}(\tilde{\theta}_i)=\lambda_{0,k}^{(i)}c_0(\tilde{\theta}_i)+\lambda_{1,k}^{(i)}c_1(\tilde{\theta}_i)+\cdots+\lambda_{m,k}^{(i)}c_m(\tilde{\theta}_i)\\ \delta_{i,k}(\tilde{\theta}_i)=\gamma_{0,k}^{(i)}c_0(\tilde{\theta}_i)+\gamma_{1,k}^{(i)}c_1(\tilde{\theta}_i)+\cdots+\gamma_{m,k}^{(i)}c_m(\tilde{\theta}_i)\\ i=1,\cdots,n;k=x,y,z\end{gathered} \tag{10}$$

where $\theta_i=\dfrac{2(\theta_i-\theta_{i,\min})}{\theta_{i,\max}-\theta_{i,\min}}-1$ is the Chebyshev variable within the closed interval $[-1, 1]$, and $\theta_{i,\min}$, $\theta_{i,\max}$ are the minimum and maximum values of the rotation angles of each robot joint, respectively.

The zeroth order term in Eq.(10) is not a function of joint rotation angle, which can be regarded as geometric errors (constant errors) such as kinematics parameter errors of the robot, and the other higher order terms are mapping functions of joint rotation angle, representing non-geometric errors such as gear friction, joint transmission errors and joint flexibility of the robot. It can be seen that Chebyshev polynomials can accurately model geometric errors and non-geometric errors of robots.

The Chebyshev polynomial error estimation model can be obtained by substituting Eq.(10) into Eq.(7). Since the measuring point of the laser tracker is the actual position of the tool coordinate system at the end of the robot relative to the base coordinate system, the positioning error of the corresponding point can be obtained by comparing with the theoretical position. Therefore, the position error, $\Delta \boldsymbol{P}_\mathrm{e}$, at the end of robot can be expressed as

$$\Delta \boldsymbol{P}_\mathrm{e}(\theta)=\boldsymbol{P}_\mathrm{a}-\boldsymbol{P}_\mathrm{t}=\boldsymbol{F}_\mathrm{a}(\lambda,\gamma,\theta)\boldsymbol{L}_\mathrm{t}^\mathrm{f}-\boldsymbol{P}_\mathrm{t} \tag{11}$$

$$\boldsymbol{L}_\mathrm{t}^\mathrm{f}=[L_x,L_y,L_z,1]^\mathrm{T} \tag{12}$$

where $\boldsymbol{P}_\mathrm{a}, \boldsymbol{P}_\mathrm{t}$ are the actual and theoretical positions at the end of robot, and $\boldsymbol{L}_\mathrm{t}^\mathrm{f}$ is the actual position of the tool coordinate system relative to the flange coordinate system. Partial derivatives are obtained for each Chebyshev coefficient in Eq.(11), and the positioning error at the end of robot was approximately written into a linear combination form of each coefficient, namely

$$\Delta \boldsymbol{P}_\mathrm{e}=\frac{\partial \boldsymbol{P}_\mathrm{a}}{\partial \boldsymbol{\lambda}}\Delta\boldsymbol{\lambda}+\frac{\partial \boldsymbol{P}_\mathrm{a}}{\partial \boldsymbol{\gamma}}\Delta\boldsymbol{\gamma}=\boldsymbol{J}(x)\Delta x \tag{13}$$

where $\Delta\boldsymbol{\lambda}=[\lambda_{0,k}^{(1)},\cdots,\lambda_{m,k}^{(1)},\cdots,\lambda_{0,k}^{(6)},\cdots,\lambda_{m,k}^{(6)}]^\mathrm{T}$, $\Delta\boldsymbol{\gamma}=[\gamma_{0,k}^{(1)},\cdots,\gamma_{m,k}^{(1)},\cdots,\gamma_{0,k}^{(6)},\cdots,\gamma_{m,k}^{(6)}]^\mathrm{T}$ are the Chebyshev coefficients to be identified.

#### 2.1.2 Chebyshev coefficient identification

The purpose of coefficient identification is to search for the optimal solution of each coefficient in the kinematics model, and the optimal estimation of Chebyshev coefficient needs to minimize the difference between the estimation error of sampling point and the actual error. Accordingly, the coefficient identification problem is a typical regression problem, and the least square method is one of the most simple and effective methods to solve the regression problem. On this basis, the researchers put forward many iterative improved algorithms of least square method, among which Levenberg-Marquardt (L-M) algorithm[21] is widely used in the field of robot kinematics calibration. By introducing adaptive damping factor to change the step size and direction of correction, L-M algorithm can improve the sensitivity of initial value selection or the inexistence of inverse matrix in Gauss-Newton algorithm in the process of searching for the optimal solution, which realizes the combination of advantages of Gauss-Newton algorithm and gradient descent method. For higher order nonlinear equations, the traditional L-M algorithm does not converge because the Jacobian matrix of the equations is nearly singular. Therefore, an improved L-M algorithm with higher order convergence was adopted for the identification of the coefficients of Chebyshev polynomials[22]. The iterative process is as follows:

(1) Parameter initialization. Set initial iteration number as $k=1$ and convergence accuracy as $\varphi=0.000\,01$, where optimization factor is $\upsilon_1$, iteration parameters are $m$, $P_0$, $P_1$, $P_2$, and condition $0<P_0<P_1<P_2<1$, $\upsilon_1>m>0$ should be satisfied.

(2) According to Eq.(13), calculate the robot Jacobian matrix $\boldsymbol{J}(x_k)$ and position error $\Delta\boldsymbol{P}_{\mathrm{e}}(x_k)$ for the $k$th iteration.

(3) Introduce the adaptive damping factor $\mu_k$, and obtain the $k$th Chebyshev coefficient iteration step $d_{k1}$, that is

$$\begin{cases} d_{k1}=-(\boldsymbol{J}^{\mathrm{T}}(x_k)\boldsymbol{J}(x_k)+\mu_k\boldsymbol{I})^{-1}\boldsymbol{J}^{\mathrm{T}}(x_k)\Delta\boldsymbol{P}_{\mathrm{e}}(x_k) \\ \mu_k=\upsilon_k\left\|\Delta\boldsymbol{P}_{\mathrm{e}}(x_k)\right\|^2 \end{cases} \tag{14}$$

where $\boldsymbol{I}$ is the identity matrix and $x_k$ the Chebyshev coefficient of the $k$th iteration.

(4) According to Eqs.(15) and (16), calculate the updating step $y_k$ and Chebyshev coefficient approximate iteration step $d_{k2}$, and the updating step $z_k$ and Chebyshev coefficient approximate iteration step $d_{k3}$

$$\begin{cases} y_k=x_k+d_{k1} \\ d_{k2}=-(\boldsymbol{J}^{\mathrm{T}}(x_k)\boldsymbol{J}(x_k)+\mu_k\boldsymbol{I})^{-1}\boldsymbol{J}^{\mathrm{T}}(x_k)\Delta\boldsymbol{P}_{\mathrm{e}}(y_k) \end{cases} \tag{15}$$

$$\begin{cases} z_k=y_k+d_{k2} \\ d_{k3}=-(\boldsymbol{J}^{\mathrm{T}}(x_k)\boldsymbol{J}(x_k)+\mu_k\boldsymbol{I})^{-1}\boldsymbol{J}^{\mathrm{T}}(x_k)\Delta\boldsymbol{P}_{\mathrm{e}}(z_k) \end{cases} \tag{16}$$

(5) Define the total step size of iteration as $s_k$

$$s_k=d_{k1}+d_{k2}+d_{k3} \tag{17}$$

(6) Calculate the iterative updating coefficient $r_k$ from the following Eqs.(18) and (19), that is

$$\begin{cases} A_k=\left\|\Delta\boldsymbol{P}_{\mathrm{e}}(x_k)\right\|^2-\left\|\Delta\boldsymbol{P}_{\mathrm{e}}(x_k+s_k)\right\|^2 \\ P_k=\left\|\Delta\boldsymbol{P}_{\mathrm{e}}(x_k)\right\|^2-\left\|\Delta\boldsymbol{P}_{\mathrm{e}}(x_k)+\boldsymbol{J}(x_k)d_{k1}\right\|^2+\left\|\Delta\boldsymbol{P}_{\mathrm{e}}(y_k)\right\|^2- \\ \qquad \left\|\Delta\boldsymbol{P}_{\mathrm{e}}(y_k)+\boldsymbol{J}(x_k)d_{k2}\right\|^2+\left\|\Delta\boldsymbol{P}_{\mathrm{e}}(z_k)\right\|^2-\left\|\Delta\boldsymbol{P}_{\mathrm{e}}(z_k)+\boldsymbol{J}(x_k)d_{k3}\right\|^2 \end{cases} \tag{18}$$

$$r_k=A_k/P_k \tag{19}$$

(7) Update the Chebyshev coefficient $x_{k+1}$ and optimization factor $\upsilon_{k+1}$ of the $k+1$ iterations

$$x_{k+1}=\begin{cases} x_k+s_k & r_k>P_0 \\ x_k & r_k\leqslant P_0 \end{cases} \tag{20}$$

$$\upsilon_{k+1}=\begin{cases} 2\upsilon_k & r_k>P_2 \\ \upsilon_k & P_1\leqslant r_k\leqslant P_2 \\ \max\left\{\dfrac{\upsilon_k}{2},m\right\} & r_k>P_2 \end{cases} \tag{21}$$

(8) Update iteration number $k=k+1$, return to Step (2), when $\left\|\Delta\boldsymbol{P}_{\mathrm{e}}\left(x_{k+1}\right)-\Delta\boldsymbol{P}_{\mathrm{e}}\left(x_k\right)\right\|^2\leqslant\varphi$, the error norm is very close and converges, or the iteration number $k$ is 100, the loop ends.

The Chebyshev coefficient calculated above is substituted into the space error estimation model to obtain the actual kinematics model of the robot, and then the error compensation of the robot is carried out.

## 2.2 Closed-loop feedback of robot joint

On the basis of the Chebyshev polynomial error estimation model in Section 2.1, in order to achieve joint closed-loop feedback control, it is necessary to map the estimated spatial Cartesian errors to the front three joints of the robot and obtain their modified angle values. The positioning error of the robot end can be expressed as the partial differential of the robot end pose to the rotation angle of the front three joints, that is, the joint mapping model is established as follows

$$\Delta\boldsymbol{P}_{\mathrm{e}}=\frac{\partial\boldsymbol{P}_{\mathrm{t}}}{\partial\theta_1}\Delta\theta_1+\frac{\partial\boldsymbol{P}_{\mathrm{t}}}{\partial\theta_2}\Delta\theta_2+\frac{\partial\boldsymbol{P}_{\mathrm{t}}}{\partial\theta_3}\Delta\theta_3=\frac{\partial\boldsymbol{P}_{\mathrm{t}}}{\partial\theta}\Delta\theta \tag{22}$$

The rotation angle error of the front three joints obtained by using the least square method is as follows

$$\Delta\theta=\left(\left(\frac{\partial\boldsymbol{P}_{\mathrm{t}}}{\partial\theta}\right)^{\mathrm{T}}\frac{\partial\boldsymbol{P}_{\mathrm{t}}}{\partial\theta}\right)^{-1}\left(\frac{\partial\boldsymbol{P}_{\mathrm{t}}}{\partial\theta}\right)^{\mathrm{T}}\Delta\boldsymbol{P}_{\mathrm{e}} \tag{23}$$

Therefore, the modified angular values of the front three joints of the robot are as follows

$$\theta_i' = \theta_i - \Delta\theta_i \qquad i = 1,2,3 \tag{24}$$

In order to improve the control accuracy of joint angle, a PID controller is adopted. The control signal in the continuous time domain can be expressed as follows

$$\mu(t) = k_\mathrm{P} e(t) + k_\mathrm{I}\int_0^t e(t)\,\mathrm{d}t + k_\mathrm{D}\frac{\mathrm{d}e(t)}{\mathrm{d}t} \tag{25}$$

where $\mu(t)$ corresponds to the output of the control model and $e(t)$ is the difference between the target value of joint angle and the actual value. $k_\mathrm{P}$, $k_\mathrm{I}$ and $k_\mathrm{D}$ are the proportional, integral and differential gain constants, respectively.

In the closed-loop feedback control of the robot joint, the robot controller sends pulses to the joint motor every 12 ms, which requires discretization of Eq.(25). In the actual control process, the ideal joint closed-loop feedback control curve should approach the target angle quickly without overshoot, but the existence of integral term will make the stability of the control system worse and even damage the robot equipment. A PD controller is used to avoid this. Proportional and differential gain constants in the controller are obtained through parameter setting. The control model of joint closed-loop feedback is shown as follows

$$\mu(n) = k_{\mathrm{P}d} e(n) + k_{\mathrm{D}d}[e(n) - e(n-1)] \tag{26}$$

where $k_{\mathrm{P}d} = k_p$, $k_{\mathrm{D}d} = k_p T_\mathrm{D}/T$, and $T$ is the sampling period.

## 2.3 Accuracy compensation method

The compensation process of the closed-loop feedback precision compensation method for robot joint is shown in Fig.2. The specific steps are as follows:

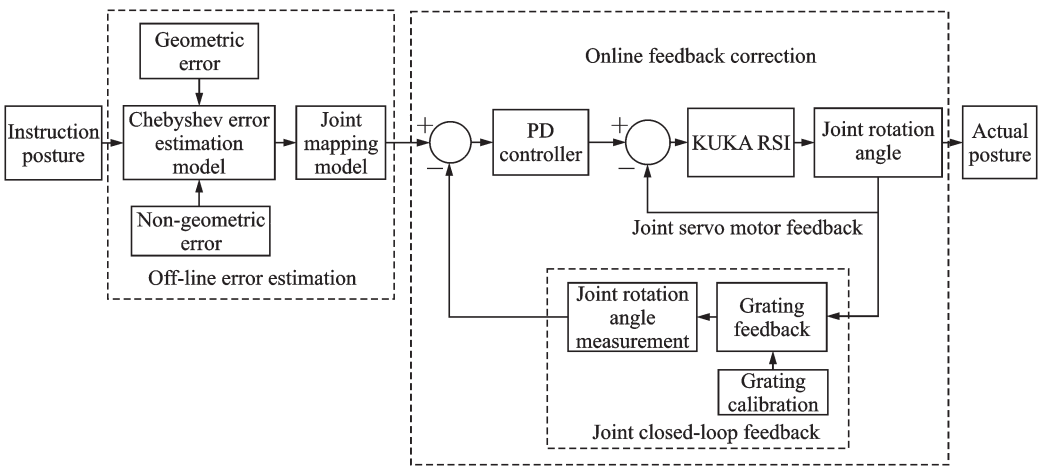


Fig.2 Closed-loop feedback accuracy compensation method for robot joints

(1) Calibrate the absolute linear grating mounted on the front three joints of the robot to determine the corresponding relationship between the grating reading and joint angle.

(2) Determine the motion range of each joint in the robot workspace, and generate a number of sampling points and a number of target points in this range. Input the theoretical position of these sampling points to the robot, and measure the actual position of the end of the robot before compensation with the laser tracker, and then calculate the corresponding positioning error values of the sampling points.

(3) According to the Chebyshev polynomial spatial error modeling method proposed in this paper, establish the spatial error estimation model by using the theoretical position and positioning error values of sampling points obtained in Step (2).

(4) By substituting the target points into the spatial error estimation model in Step (3), calculate the estimated positioning errors of the target points, and then map the estimated spatial position errors to the front three robot joints by the joint mapping model, so as to obtain the modified angle values of

the front three joints of the robot. The spatial error estimation model and joint mapping model constitute the feedforward part of the control loop.

(5) Design a PD controller as the closed-loop feedback controller of the robot joint. Input the modified joint rotation angle to the robot, and combine with the real-time interactive environment of RSI (Robot sensor interface). Control the joint rotation angle by closed-loop feedback to realize the accurate positioning of the end of robot.

At this point, the target point error compensation is completed.

## 3 Experimental Verification and Result Analysis

The experimental verification idea of the closed-loop feedback accuracy compensation method for robot joint is as follows: Firstly, the joint backlash is compensated by closed-loop feedback control to verify whether the multi-direction pose accuracy of the robot is improved. And then the sampling points and target points are planned in the robot workspace to verify the feasibility of the closed-loop feedback precision compensation method.

KUKA KR210 R2700 extra 6-DOF industrial robot was taken as the research object in the experiment, and its repetitive positioning accuracy was ±0.06 mm. The experimental verification platform is shown in Fig.3. The measuring equipment used is an API RADIAN laser tracker with an absolute ranging accuracy of 15 μm within the range of 10 m. The SMR (Spherically mounted retroreflectors) was installed in a fixed position on the side of the end-effector of the robot. All the data were measured in the base coordinate system of the robot. Renishaw RTLA-S series absolute value linear grating scale with a resolution of 50 nm was installed on the finishing surface around the corresponding joint of the robot, and its installation position is shown in Fig.3.

Absolute linear grating adopts the BiSS (A kind of full-duplex synchronous serial bus communication) bus protocol. In order to enable the grating

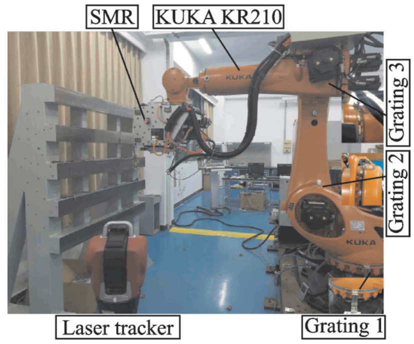


Fig.3 Robot accuracy compensation experiment platform

signal to be collected by the integrated control software of the upper computer, it is necessary to convert the BiSS signal into USB (universal serial bus) signal and finally connect the feedback signal of the robot joint to the industrial personal computer through the hub. RSI real-time interactive environment is a robot communication interface specially opened by KUKA Company. It can conduct real-time data interaction with external systems through Ethernet or I/O bus, and its signal processing cycle is 12 ms. After obtaining the grating feedback signal, the integrated control software of the upper computer will calculate the actual angle of the robot joint by the grating calibration model, and at the same time process the difference between the target angle and the actual angle to be the robot joint angle correction by PD controller. With RSI real-time interactive environment, the robot receives the joint angle correction sent by the upper computer to complete the joint position correction. Fig.4 is the flow chart of the robot joint closed-loop feedback correction, which introduces the principle and actual correction process of closed-loop feedback.

Before the experiment, it is necessary to establish the robot base coordinate system and the tool coordinate system fixed with SMR in the measurement software of the laser tracker. In this paper, the method in Ref.[23] is adopted to establish the coordinate system. The base coordinate system of the robot is taken as the reference, and the pose of the tool coordinate system relative to the base coordinate system is taken as the measurement pose. The laser tracker can be used to measure the positioning

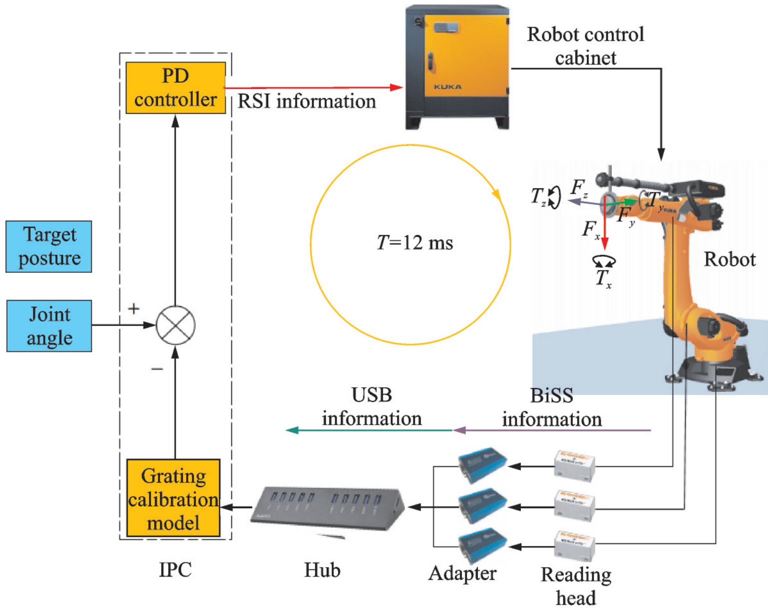


Fig.4 Robot joint closed-loop feedback correction process

error of the robot.

### 3.1 Verification robot multi-directional pose accuracy

According to the accuracy compensation method of robot joint closed-loop feedback proposed in this paper, it is necessary to calibrate the absolute linear gratings on the circular arc surface of the robot joints to determine the relationship between the grating readings and joint rotation angles. Considering that in a RSI signal processing period, excessive joint correction will cause severe vibration of the robot, so this paper divides the joint closed-loop feedback correction process into rough correction process and fine correction process. Each joint was modified by $\pm 0.002°/T$ in the rough correction. When the difference between the joint target angle and the actual angle was detected to be less than 0.007°, the fine correction process was started. PD controller is adopted for fine correction process. After parameter setting, the proportional and differential gain constants are $k_P = 0.05$, $k_D = 0.65$, respectively, and the joint closed-loop feedback can achieve better control accuracy.

As shown in Fig.5, the 600 mm×1 000 mm×600 mm rectangular region selected in the robot workspace was used as the test area. $P_1$, $P_2$ and $P_3$ were the test points on the diagonal of the test area. Repeat 30 times for each test point according to the three mutually perpendicular motion directions shown in Fig.5, and the motion distance is 100 mm. Calculate the multi-direction pose accuracy of the robot base on the ISO 9283[14] standard, and the test results of multi-direction pose accuracy variation of the robot are shown in Fig.6.

As can be seen from Fig.6, the multi-directional pose accuracy of the robot without joint closed-loop feedback correction changes greatly, which is more than 3 times of the robot's repeated positioning accuracy. But after the robot joint closed-loop feedback correction, the accuracy change of multi-direction pose of the robot decreases as a whole, which indicates that the motion error of the front three joints of the robot has a great influence on the accuracy change of the multi-direction pose of the robot end. The closed-loop feedback correction of the joint can effectively reduce its motion error. At the same time, because the motion error of the back three joints of the robot still exists, the error of the accuracy change of the multi-direction pose still remains to some extent. And the results further illustrate that error of sample point itself has uncertainties, which does not guarantee that the error of reaching the same sampling point every time is not much different. Therefore, the off-line calibration

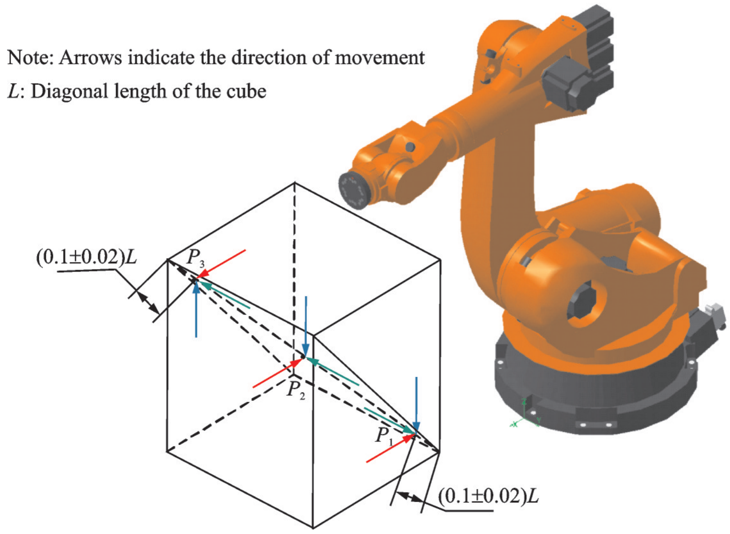


Fig.5　Measurement points selection of multi-direction pose accuracy variation

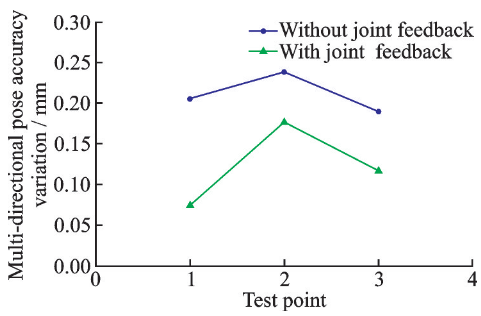


Fig.6　Multi-direction posture accuracy of robot with joint closed-loop feedback correction

methods, such as kinematics parameter calibration, error similarity accuracy compensation method, and spatial interpolation compensation method, cannot further improve the absolute positioning accuracy of the robot.

### 3.2 Verification of precision compensation method of robot joint closed-loop feedback

In the rectangular region shown in Fig.5 and within the interval [−10°, 10°] of robot end posture angles $A$, $B$, and $C$ (The robot end posture angles $A$, $B$, and $C$ are the angles of the robot end around the $x$, $y$, and $z$ axes, respectively), 100 sampling points and 104 target points are randomly generated. Among them, 100 sampling points are used to construct the robot's spatial error estimation model, and 104 target points are used to verify the effectiveness of the joint closed-loop feedback accuracy compensation method. The motion range of each joint of the robot is shown in Table 1.

**Table 1　Motion range of robot joints**　(°)

| Joint | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Max | 25 | −70 | 120 | 15 | 110 | 20 |
| Min | −25 | −110 | 70 | −15 | 60 | −40 |

According to the closed-loop feedback accuracy compensation method for the robot joint described in Section 2.3, spatial error modeling was carried out on 100 sampling points by using the third-order Chebyshev polynomials to identify the optimal value of Chebyshev coefficient and obtained the mapping relationship between each differential transformation error term in error transfer matrix and joint rotation angle. The 104 target points were respectively verified by accuracy compensation experiments using kinematics parameter calibration method and joint closed-loop feedback accuracy compensation method, and the positioning errors of the target points before and after compensations were measured by the laser tracker. The experimental results are shown in Figs.7—10. Figs.7—9 show the positioning error distribution in the three directions of $x$, $y$, and $z$ in the basic coordinate system of the robot. Fig.10 shows the absolute positioning error distribution of the robot. The absolute positioning error of the robot is calculated according to Eq.(27)

$$\Delta = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} \tag{27}$$

The frequency histogram of the absolute positioning error of the target point is shown in Fig.11. Frequency histogram represents the absolute positioning error distribution of the target point using different precision compensation methods. In order to display the experimental results more intuitively, the statistical data of the positioning error of the robot before and after compensations are shown in Table 2.

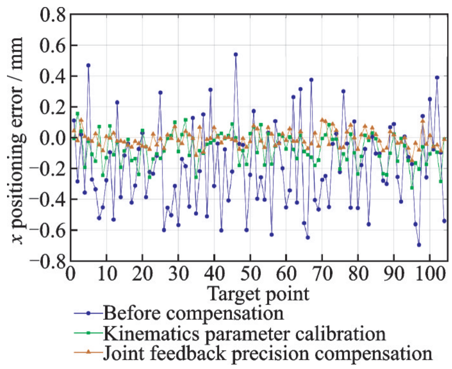


Fig.7 Comparison of positioning errors in $x$ direction before and after accuracy compensations

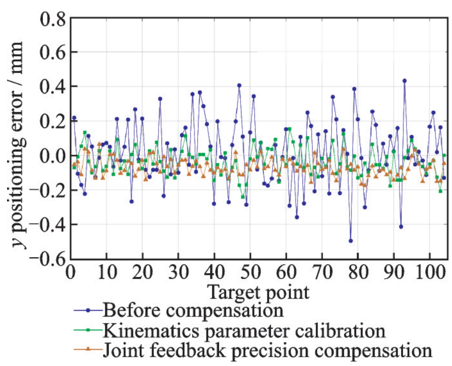


Fig.8 Comparison of positioning errors in $y$ direction before and after accuracy compensations

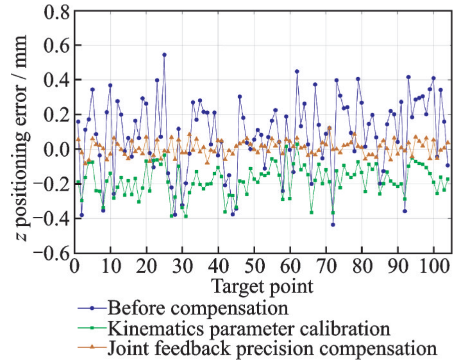


Fig.9 Comparison of positioning errors in $z$ direction before and after accuracy compensations

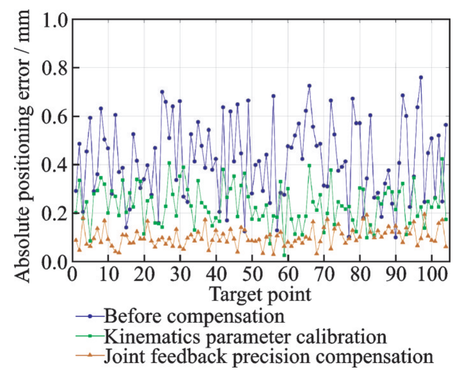


Fig.10 Comparison of absolute positioning errors before and after accuracy compensations

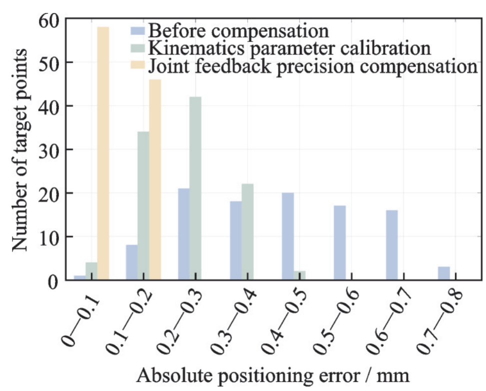


Fig.11 Comparison of target point accuracy compensation results

**Table 2 Experimental data statistics** mm

| Item | Before the compensation | Calibration of kinematics parameters | Joint feedback error compensation |
|---|---|---|---|
| Range of positioning error | [0.10, 0.76] | [0.03, 0.42] | [0.03, 0.19] |
| Mean value | 0.42 | 0.24 | 0.10 |
| Standard deviation | 0.16 | 0.08 | 0.04 |

After the compensation by the kinematics parameter calibration method, the maximum value of the absolute positioning error of the target point decreased from 0.76 mm before compensation to 0.42 mm after compensation, and the mean value of the absolute positioning error decreased from 0.42 mm before compensation to 0.24 mm after compensation. The absolute positioning accuracy of the robot is within 0.5 mm, and the compensation effect is significant.

After the joint closed-loop feedback accuracy compensation, the positioning error of the robot is further reduced. In the basic coordinate system of the robot, the error components in $x$, $y$, and $z$ directions are compensated to within $\pm 0.2$ mm. The fluctuation range of positioning error is greatly reduced, and the mean value of error components in the three directions is close to zero, which shows higher stability than before compensation. The maximum absolute positioning error of the robot that was compensated by the proposed method fell 75% and 55%, compared to that before compensation and that compensation by the kinematics parameter calibration method, respectively. And the absolute positioning error is within 0.2 mm, which satisfy the aviation manufacturing requirements for robot absolute positioning accuracy. At the same time, it is proved that the closed-loop feedback accuracy compensation method of robot joint is feasible and effective.

## 4 Conclusions

(1) When using the kinematics parameter calibration method to compensate the geometric errors of the robot, the influence of the non-geometric errors of the robot is not considered. Therefore, it is necessary to establish a real kinematics model reflecting the error distribution characteristics of the robot.

(2) Aiming at the complexity of the modeling of robot joint backlash, the closed-loop feedback effect of the absolute linear grating mounted on the joint of the robot is used to correct it, so as to improve the multi-direction pose accuracy of the robot and ensure the error sampling accuracy.

(3) The Chebyshev polynomial error estimation model is established. And combined with the joint closed-loop feedback correction, the maximum absolute positioning error of the robot is reduced from 0.76 mm before compensation to 0.19 mm after compensation. The compensation effect is significant, which can meet the requirements of the industrial robot automatic drilling and riveting system for the absolute positioning accuracy of the robot.

(4) Robot is a nonlinear control system under the action of strong coupling of multiple factors. The single PD controller cannot guarantee the accurate control of the joint movement of the robot in any pose, and the control of the robot joint in different poses needs to be further studied.

(5) Only the front three joints are equipped with absolute linear gratings, so it is impossible to eliminate the influence of the backlash of the back three joints of the robot. In the future, if gratings are installed on all joints of the robot, the absolute positioning accuracy of the robot can be further improved.

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**Author contributions** Dr. **ZHANG Lin** contributed to the discussion and background of the study, designed the study proposal and wrote the manuscript. Prof. **TIAN Wei** provided the industrial robot experimental platform for this research. Mr. **ZHENG Faying** established the mathematical models for the study, conducted the experiments and analysis, and interpreted the results. Prof. **LIAO Wenhe** provided financial support for this research. All authors commented on the manuscript draft and approved the submission.

**Competing interests** The authors declare no competing interests.

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# 工业机器人关节闭环反馈精度补偿技术

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**摘要：**现有的运动学参数标定方法无法进一步提升机器人的绝对定位精度，是由于关节运动回差引起机器人定位误差具有不确定性。为此，提出了一种机器人关节闭环反馈精度补偿方法。首先，建立了综合考虑几何误差和非几何误差的切比雪夫多项式误差估计模型。然后，为了降低关节运动回差的影响，在机器人关节处安装绝对式直线光栅，将末端位置误差映射到关节转角上，得到关节转角修正量，通过关节闭环反馈控制实现机器人末端位置误差的在线修正。最后，以工业机器 KUKA KR210 为对象进行试验验证，试验结果表明，机器人的绝对定位误差最大值由补偿前的 0.76 mm 降低到 0.19 mm，最大绝对定位误差降幅达到 75%。该方法能够对关节运动回差进行有效地补偿，从而进一步提高机器人的绝对定位精度。

**关键词：**精度补偿；机器人关节闭环反馈；切比雪夫多项式；机器人关节运动回差