# Equivalent Stiffness of Metal Clip-Like Piezoelectric Spring Structure

*BIAN Kan*[1], *WANG Yue*[2], *HUANG Xuewen*[3], *WU Yipeng*[2*]

1. School of Materials Science and Engineering, Nanjing Institute of Technology, Nanjing 211167, P.R. China;
2. State Key Laboratory of Mechanics and Control of Mechanical Structures, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, P.R. China;
3. Huaneng Shandong Ruyi (Pakistan) Energy (Private) Co., Ltd. Jinan 250014, P.R. China

(Received 10 August 2020; revised 20 November 2020; accepted 27 November 2020)

**Abstract:** Piezoelectric ceramic is hard to be integrated with the normal spring structure. To address the above problem, this paper proposed a new geometry of a clip-like spring which is very similar to binder clip in our daily life. The equivalent stiffness of the designed piezoelectric clip-like spring is thoroughly researched and discussed through the theoretical model, the finite element simulation and the experimental measurement. The results confirm the possibility of designing a compact piezoelectric clip-like spring, and the equivalent stiffness can be tuned through the several key geometric parameters. Finally, theoretical predictions confirmed by experimental results show that the equivalent stiffness of the spring structure is as function of the instantaneous angle of the clip, this stiffness variation caused by the geometric nonlinearity can be ignored in some practical engineering applications, which means it is possible to linearize the clip-like spring and simplify the following dynamic model of the corresponding piezoelectric oscillators.

**Key words:** piezoelectric spring; low stiffness; Mohr's method; finite element simulation

**CLC number:** TB122 **Document code:** A **Article ID:** 1005-1120(2020)06-0962-08

## 0 Introduction

Due to the intrinsic characteristics of piezoelectric materials, there are numerous applications, such as piezoelectric motors[1] and sonars[2], that benefit from their use. However, in the above mature applications, the piezoelectric materials usually operate in a condition of high-frequencies (e.g. > 20 kHz). When the operation frequency is low, for instance, in the structure vibration control field and the vibration energy harvesting field, the frequencies are usually lower than 100 Hz. The value of piezoelectric coupling coefficient in the above cases will drastically decrease. Moreover, low-frequency operation usually corresponds to large deformation of rigid structure, which is a huge challenge to integrate most piezoelectric ceramics[3]. Consequently, though thousands of researchers have proposed various vibration control or energy harvesting techniques based on piezoelectric materials[4-7], it is still hard to find their practical applications around our daily life.

Taking mechanical vibration control or energy harvesting as examples, Ji et al.[8] proposed an unsymmetrical synchronized switch damping technique used for ultra-low-frequency (0.98 Hz) vibration suppression. They selected a flexible cantilevered beam as an experimental target. The amplitude of vibration displacement was reduced by 60% after the control. Tsukamoto et al.[9] presented a flexible 3D meshed-core piezoelectric cantilever for low-frequency (18.7 Hz) vibration energy harvesting. Thanks to the proposed meshed-core structure, the first-order resonant frequency of the cantilevered beam is

*Corresponding author, E-mail address: yipeng.wu@nuaa.edu.cn.

**How to cite this article**: BIAN Kan, WANG Yue, HUANG Xuewen, et al. Equivalent stiffness of metal clip-like piezoelectric spring structure[J]. Transactions of Nanjing University of Aeronautics and Astronautics, 2020, 37(6): 962-969.

http://dx.doi.org/10.16356/j.1005-1120.2020.06.014

15.8% lower than the conventional solid cantilever, but the output electrical power is increased by 68%. Among these cantilevered structures, low frequency leads to large tip deflection of beam and large strain of piezoelectric element. Macro fiber composite (MFC) and polyvinylidene fluoride (PVDF) are the two most popular piezoelectric materials to be selected, which create new problems of high cost or low piezoelectric coupling coefficient. Karami and Inman[10] tried to design a special structure integrated with piezoelectric ceramic and applied in low-frequency vibrations. A zigzag beam and very thin piezoelectric layer were finally presented. The zigzag structure is suitable for micro-electro-mechanical systems (MEMS), but the strain at the fixed end of the beam is usually large, the structure may easily be broken or damaged during the oscillation. Clementino et al.[11] reported on the design and experimental verification of a pitch link system using piezoelectric material for vibration attenuation. The active pitch link was developed by the Carleton University's Rotorcraft Research Group and can be considered as a piezoelectric spring structure. An X-shape supporting structure designed by Ji et al.[8] can also be regarded as a piezoelectric spring structure which was applied in low-frequency vibration energy harvesting or vibration isolation. The nonlinear stiffness characteristic is explored to significantly improve vibration energy harvesting or isolation performances[12-14]. Wu et al.[4] proposed a clip-like spring which is similar to binder clips. The piezoelectric ceramics can be easily integrated with the metal clip due to the planar structure. Moreover, the clip-like spring has the basic attribute of low stiffness comparing with its elastic modulus, which makes it suitable for low-frequency operation field. However, Wu et al.[4] simply considered the clip-like spring as a linear spring and provided the equivalent stiffness value, but neither the detailed stiffness model nor the approximate equivalence method had been mentioned. In fact, the geometric nonlinearity of clip-like springs can be directly observed during their operation state. The effect of the nonlinearity on spring stiffness needs to be accurately analyzed.

The stiffness model of a clip-like spring is established in this paper, theoretically integrated with piezoelectric ceramics based on Mohr's method. The accuracy of the simplified model is validated by a finite element analysis tool and a universal tensile testing machine. The investigation shows that the clip-like spring can be simply considered as a linear spring from the tension curve. The theoretical performance of the proposed structure is reasonable.

## 1 Piezoelectric Spring and Stiffness Model

The clip-like spring is shown in Fig.1. Two sturdy wire handles are removed and can be replaced by some specially designed pins. Usually, the metal clip is made of tempered spring steel and manufactured by the punch machine, hence, the cost is low because of the high productivity. In addition, the clip-like spring structure has six flat surfaces and can be simply integrated with the plate-like piezoelectric ceramics, which successfully solves the aforementioned difficulty in the Introduction. However, according to the theory of mechanics of materials, the stress at the bottom surfaces is high and uniform. The stress distribution characteristic is benefit to maximize the electromechanical performance of piezoelectric materials. This phenomenon had been theoretically analyzed and a definition of normalized power density was provided. Besides, it is reported that the maximal power density can be obtained only when the piezoelectric ceramics are

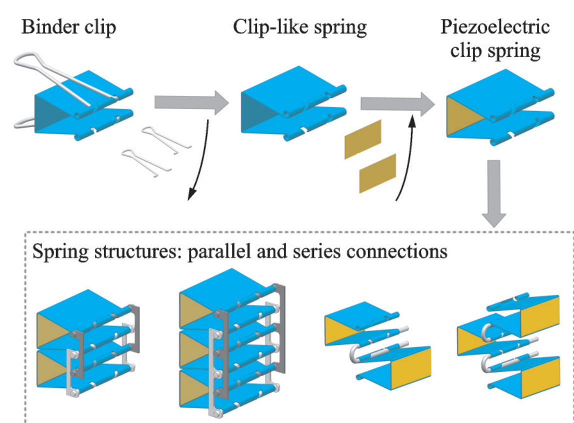



Fig.1 Diagram of clip-like spring integrated with piezoelectric ceramics (Spring structures composed of one or several clips are also shown)

glued to the two bottom surfaces of the clip-like spring[15]. Consequently, the piezoelectric integration strategy of the maximal power density is selected in this paper. Therefore, the oscillation of the spring deformation generates electrical energy due to the direct piezoelectric effect, and the driven voltage added to the piezoelectric element also causes the deformation of the clip-like spring.

The spring structure can be composed of one or several clips, and the clips can also be connected in series or parallel via special designed pins. In addition, different connections correspond to various equivalent stiffness. For one clip, the equivalent stiffness can be calculated based on the Mohr integration. The simplified force diagram of the clip is shown in Fig.2. The force $F$ added to the clip-like spring leads to an expansion deformation $\delta$, and it is assumed that $\delta$ is mainly caused by the arc radius deformation of arc $BC$.

Under the action of the force $F$, an angle between the two lines $AB$ and $DC$ is denoted by $\alpha$, which is also assumed to be an initial angle. As soon as the force $F$ changes to $F+\Delta F$, the expansion

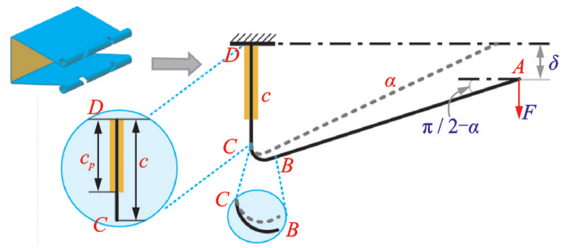



Fig.2 Simplified force diagram of piezoelectric clip-like spring

deformation $\delta$ will then become $\delta+\Delta\delta$, the difference of the deformation $\Delta\delta$ (which is also call deflection) can be considered as affecting by the structure strain. In this case, the stiffness $K$ of the designed piezoelectric clip-like spring can be calculated by Eq.(1).

$$K=\frac{\Delta F}{2\Delta\delta} \tag{1}$$

According to the Mohr integration, the variation of the bending moment distributed in each part can be expressed in Eq.(2), where $\varphi$ is the radians of arc $BC$, ranging from 0 to $(\pi-\alpha)$; $r$ the radius corresponding to arc $BC$; $a$ the length value of line $AB$, and $x_{ab}$ the distance from point $A$ to the calculation point.

$$\begin{cases} M_{ab}=\Delta F\sin\alpha\cdot x_{ab} \\ M_{bc}=\Delta Fa\sin\alpha+\Delta Fr\sin\alpha\cdot\sin\varphi+\Delta Fr\cos\alpha\cdot(1-\cos\varphi) \\ M_{cd}=\Delta Fa\sin\alpha+\Delta Fr(1+\cos\alpha) \end{cases} \tag{2}$$

$$\Delta\delta=\frac{\Delta Fa^2}{3E_mI_m}[a+3b+3(c-c_p)]\sin^2\alpha+\frac{\Delta Fa^2}{E_pI_p}c_p\sin^2\alpha \tag{3}$$

From Eq.(2), the bending moment $M_{cd}$ is a constant and evenly distributed in line $CD$. To simplify the theoretical model, it is assumed that $r\ll a$. The terms including $r$ are then ignored. Finally, $\Delta\delta$ can be calculated by Eq.(3), where $b$ is the length of arc $BC$, which equals to $r(\pi-\alpha)$; $c$ and $c_p$ the lengths of line $CD$ and piezoelectric element, respectively; $E_m$ the elastic modulus of the tempered spring steel, and $I_m$ the moment of inertia for the corresponding structure cross section. For the piezoelectric composite layer located at line $CD$, the equivalent elastic modulus and moment of inertia are denoted as $E_p$ and $I_p$, respectively.

Based on the Halpain-Tsai model[16], the equivalent elastic modulus $E_p$ can be calculated by

$$E_p=\frac{\phi_mE_m(q+E_{\text{piezo}})+\phi_{\text{piezo}}E_{\text{piezo}}(q+E_m)}{\phi_m(q+E_{\text{piezo}})+\phi_{\text{piezo}}(q+E_m)} \tag{4}$$

$$q=\phi_m\frac{E_m}{2(1+\mu_m)}+\phi_{\text{piezo}}\frac{E_{\text{piezo}}}{2(1+\mu_{\text{pizo}})} \tag{5}$$

where $\phi_m$ and $\phi_{\text{piezo}}$ are the volume fractions of the steel and the piezoelectric materials, respectively; $\mu_m$ and $\mu_{\text{piezo}}$ the Poisson's ratios of the steel and the piezoelectric materials, respectively; $E_{\text{piezo}}$ the elastic modulus of the piezoelectric ceramic; and $q$ the empirical fit parameter expressed in Eq.(5).

The stiffness of the piezoelectric clip-like spring can then be obtained by Eq.(1). Although the finial expression is complicated, the stiffness only has a relationship of $\alpha$ for a given piezoelectric clip structure. Consequently, the theoretical model of the stiffness can be written in Eq.(6), and the parameters $C_A$ and $C_B$ are constant.

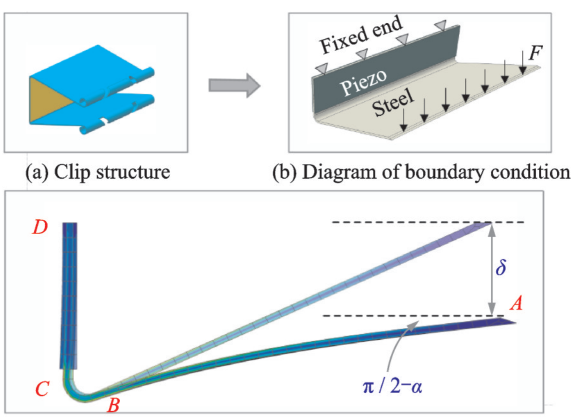
$$K=\frac{1}{C_A+C_B}\cdot\frac{1}{\sin^2\alpha} \tag{6}$$

$$\begin{cases} C_A=\dfrac{2a^2}{E_m I_m}\left[a+3b+3(c-c_p)\right] \\ C_B=\dfrac{2a^2}{E_p I_p}c_p \end{cases} \tag{7}$$

Normally, the angle $\alpha$ between lines $AB$ and $DC$ should not be larger than $\pi/2$. If the original value of the $\alpha$ and the stiffness ($F=0$) are $\pi/3$ and $K_0$, the variation of $K$ will be lower than 25% of $K_0$. However, the clip-like spring usually has a static equilibrium position in which the initial value of the angle $\alpha$ is between $\pi/3$ to $\pi/2$. Hence, the maximal variation of $K$ is reduced to half.

## 2 Simulation Results and Discussion

The theoretical stiffness model of a single clip-like spring was established. However, the accuracy of the theoretical model needs to be validated due to the mentioned assumptions. Finite element method has been largely used to model various structures. This is also an efficient tool to test and analyze the mechanical properties of the designed piezoelectric clip-like spring. Fig.3 shows an equivalent finite element model, as well as its mechanical boundary conditions. Considering symmetry of the clip structure, the model is simplified to a half of the clip. The mesh of the model is generated and the simulations are performed using ABAQUS software. Geometrically nonlinear analysis is considered simultaneously. Fig.3(c) shows the expansion deformation of the structure under the action of force $F$, where the angle $\alpha$ is larger than its original value $\alpha_0$. Here, this angle $\alpha$ is considered as a function of deformation $\delta$, which is mainly due to the arc radius deformation of $BC$. At this moment, if the force $F$ changes to $F+\Delta F$, the following deflection $\Delta\delta$ is caused by the strain of the clip structure.




(a) Clip structure (b) Diagram of boundary condition

(c) Finite element model and its expansion deformation

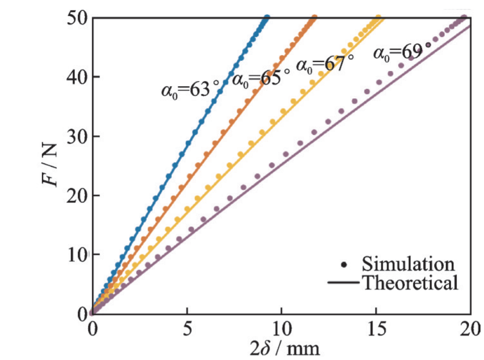
Fig.3 Single clip structure and its equivalent finite element model

A linearly increasing load $F$ which is selected to simulate the universal tensile testing machine is added on the free end of the clip. The key parameters of the tempered spring steel and the piezoelectric ceramic are given in Table 1. The force-deformation ($F$-$\delta$) results are therefore obtained from the ABAQUS software. The equivalent stiffness of the finite element model is finally calculated by Eq.(1).

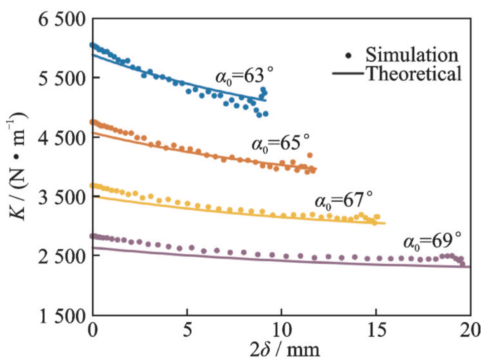
**Table 1 Key parameters of the selected clip structure**

| Parameter | Tempered spring steel | Piezoelectric ceramic |
|---|---|---|
| Young's modulus / GPa | 210 | 53 |
| Poisson's ratio | 0.28 | 0.36 |
| Width of the clip / mm | 50 | 50 |

Both the theoretical and simulation tensile curves of the selected clip models are shown in Fig.4(a), where the thicknesses of the steel and ceramic are 0.47 and 0.25 mm, respectively. The width of the clip $l$ is 50 mm, the half-length of the bottom side of the clip $c$ 13 mm, and the half-length of the piezoelectric ceramic $c_p$ 10 mm. To check the affecting relations between the equivalent stiffness and the length of the clip side (or the value of the original angle $\alpha_0$), $\alpha_0$ is assumed to change from 63° to 69°. From Fig.4(a), it is clearly seen that the simulation results agree well with the theoretical calculated ones. Besides, the larger the value of original angle $\alpha_0$ leads to the lower stiffness of the clip structure. Fig.4(b) gives the equivalent stiffness as a function of clip deformation, where the stiffness has a decreasing trend with the increase of deformation. Moreover, the variation of this trend is also affected by the original angle $\alpha_0$. The larger value of $\alpha_0$ leads to the smoother operation. Compared with the theoretical curves, the simulated stiffness seems a little bit larger than the corresponding calculated one. The major reason is the simplification of the theoretical model, especially the neglect of the bending moment along the arc $BC$. However, the error between the theoretical and simulated stiffnesses is acceptable according to the results shown in Fig.4(b).



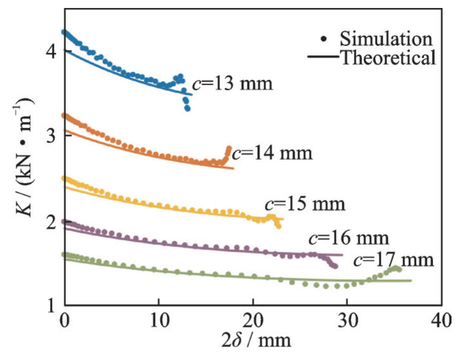

(a) Tensile curves



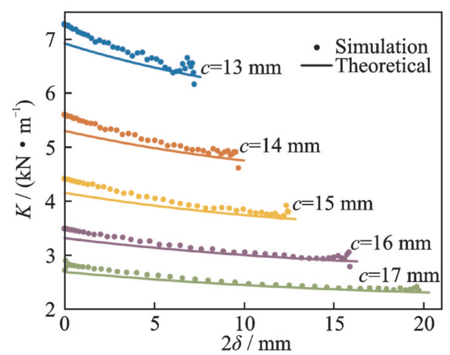

(b) Equivalent stiffness curves

Fig.4 Tensile curves and corresponding stiffness of clip-like structure with various original angles $\alpha_0$ ($c$ = 13 mm, $c_p$ = 10 mm, $l$ = 50 mm, $h$ = 0.47 mm)

The equivalent stiffness is also affected by the bottom side length and the thickness of the clip, as shown in Fig.5. The half-length of the bottom side $c$ changes from 13 mm to 17 mm, while the original angle $\alpha_0$ of the clip has a constant value of 66°. Since the original deformation $\delta$ of the clip-like structure is 0, the length of line $AB$ ($a$) also has a relationship with $c$ according to the geometric condition. Consequently, the constant parameters $C_A$ and $C_B$ expressed in Eq.(7) will increase as functions of $a$. Similarly, the thickness of the steel influences the moments of inertia $I_m$ and $I_p$, which finally also affects the constant parameters $C_A$ and $C_B$. In addition, the stiffness increases dramatically when the thickness of the steel changes from 0.47 mm to 0.57 mm.




(a) $h$=0.47 mm




(b) $h$=0.57 mm

Fig.5 Equivalent stiffness curves of clip-like structure with various bottom side lengths $c$ ($\alpha_0$ = 66°, $c_p$ = 10 mm, $l$ = 50 mm)

From the above simulation, it is found that the stiffness becomes rough at the end (the deformation $2\delta$ is the maximum under the corresponding condition). It is because the tensile force $F$ in the simulation software automatically becomes smooth at the end, hence the variation of the deformation $\Delta\delta$ has a small value or even close to zero. This variation will cause the undulation of the stiffness, and this undulating phenomenon does not mean that the practical stiffness is an unstable parameter. The average stiffness value in this part should be an acceptable result which is closer to the practical one.

## 3 Experimental Results and Discussion

Experimental measurements have also been performed as a second validation of the proposed analytical model. The basic experimental setup is shown in Fig.6. Here, an Instron 3343 universal tensile testing machine is selected to produce the force-deformation ($F$-$2\delta$) curves of the single piezoelectric clip-like spring. It is considered as a good method to obtain the equivalent stiffness of spring structures. The generated tensile force $F$ slowly increases according to the given program, where the configuration value is 5 N/min, and the deformation $2\delta$ can then be measured automatically by the universal tensile testing machine at the same time. In addition, one selected clip-like spring should be tested for more than one time to avoid the random errors caused by the spring structure and the testing system.

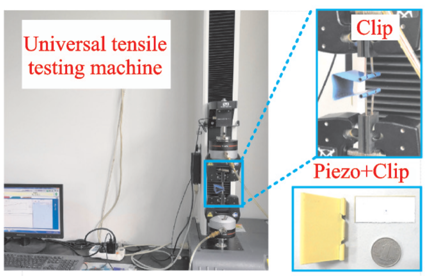



Fig.6 Experimental setup and photograph of one piezoelectric ceramic and one metal clip

Finally, Fig.7 illustrates six experimental curves derived from the universal tensile testing machine, where the selected piezoelectric clip is tested for six times. The original angle of the practical clip is 66.5°, its bottom side length 28 mm, the thickness of the clip 0.47 mm, the width of the clip and the piezoelectric ceramic 50 mm, and the length and the thickness of the piezoelectric ceramic 20 and 0.25 mm, respectively. Although the six experimental tensile curves do not coincide completely, we still consider these experimental values agree well with the theoretical results, the errors between these two kinds of values are acceptable. It is worthy of noting that the practical deformation $2\delta$ should not be larger than the length of the bottom side (28 mm), this is mainly due to the assumption of the theoretical stiffness model.

The average tensile force-deformation result is

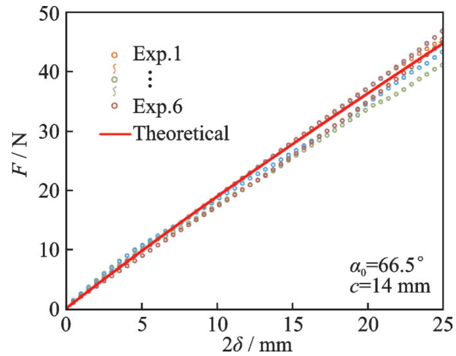
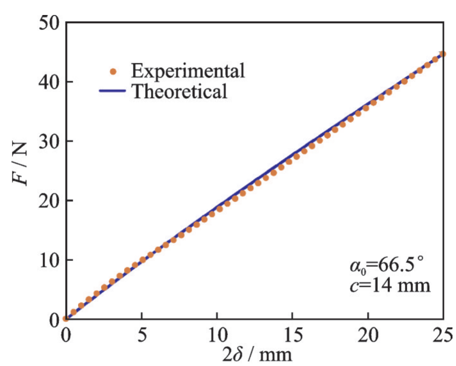


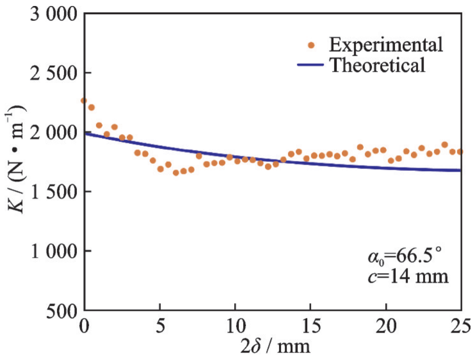

Fig.7 Six experimental tensile curves of the selected piezoelectric clip-like structure

obtained from the six original tensile values. As shown in Fig.8(a), the experimental random error is further reduced. It can be found that the average tensile result agrees well with the analytical model, the accuracy of the model is validated again. From the tensile curves, we can also see that the tensile force is roughly proportional to the deformation of the clip, hence it is possible to deal with the clip-like structure as a linear spring, the equivalent stiffness is approximately considered as a constant value.

Fig.8(b) plots the experimental and theoretical clip stiffness as a function of the deformation $2\delta$, the various range of the stiffness is from 1 700 N/m to 2 000 N/m theoretically. As mentioned above, if the clip-like spring operates at its static equilibrium position ($\delta \approx c$), the variation of the practical equivalent stiffness will be very smaller, especially when the deformation $2\delta$ is in the range of 6 mm to 25 mm. Consequently, the geometric nonlinearity of the clip-like spring can be ignored in some engineering simplifications. Such kind of springs can be assumed as linear springs.




(a) Tensile curves




(b) Equivalent stiffness

Fig.8 Tensile curves and corresponding stiffness of the selected piezoelectric clip-like structure

## 4 Conclusions

A new geometry to lower stiffness of a piezoelectric spring is proposed without increasing the complication of piezoelectric integration, making the piezoelectric spring compatible with the low resonant frequency oscillating structure suitable for low frequency vibration control or energy harvesting. A simple analytical method to calculate the equivalent stiffness is introduced, whose predictions and accuracy are also validated by finite element simulation and experimental tests. The results from the analytical model confirm the possibility of designing a compact piezoelectric clip-like spring, and the equivalent stiffness can be tuned through the several key parameters, for instance, the length of the bottom side, the original angle between the bottom and the side of the clip, and the thickness of the clip. Based on the experimental measurement results, the geometric nonlinearity of the clip-like spring can be ignored, which means it is possible to linearize the clip-like spring and simplify the following dynamic model of the corresponding piezoelectric oscillators.

**References**

[1] WANG L, HOU Y, ZHAO K, et al. A novel piezoelectric inertial rotary motor for actuating micro underwater vehicles[J]. Sensors and Actuators A: Physical, 2019, 295: 428-438.

[2] KURUSINGAL V, DEAN C, GATSIOS G. Lead-free ceramic transducers for sonar applications[C]// Proceedings of 2018 IEEE International Ultrasonics Symposium (IUS). [S.l.]: IEEE, 2018: 1-4.

[3] WU Y, QIU J, KOJIMA F, et al. Design methodology of a frequency up-converting energy harvester based on dual-cantilever and pendulum structures[J]. AIP Advances, 2019, 9(4): 045312.

[4] WU Y, QIU J, ZHOU S, et al. A piezoelectric spring pendulum oscillator used for multi-directional and ultra-low frequency vibration energy harvesting[J]. Applied Energy, 2018, 231: 600-614.

[5] FAN K, TAN Q, LIU H, et al. Improved energy harvesting from low-frequency small vibrations through a monostable piezoelectric energy harvester[J]. Mechanical Systems and Signal Processing, 2019, 117: 594-608.

[6] TANG G, YANG B, HOU C, et al. A piezoelectric micro generator worked at low frequency and high acceleration based on PZT and phosphor bronze bonding[J]. Scientific Reports, 2016, 6: 38798.

[7] BAO B, GUYOMAR D, LALLART M. Electron-phonon metamaterial featuring nonlinear tri-interleaved piezoelectric topologies and its application in low-frequency vibration control[J]. Smart Materials and Structures, 2016, 25(9): 095010.

[8] JI H, QIU J, CHENG L, et al. Semi-active vibration control based on unsymmetrical synchronized switch damping: Analysis and experimental validation of control performance[J]. Journal of Sound and Vibration, 2016, 370: 1-22.

[9] TSUKAMOTO T, UMINO Y, SHIOMI S, et al. Bimorph piezoelectric vibration energy harvester with flexible 3D meshed-core structure for low frequency vibration[J]. Science and Technology of Advanced Materials, 2018, 19(1): 660-668.

[10] KARAMI M A, INMAN D J. Analytical modeling and experimental verification of the vibrations of the zigzag microstructure for energy harvesting[J]. Journal of Vibration and Acoustics, 2011, 133 (1) : 011002.

[11] CLEMENTINO M A, NITZSCHE F, DE MARQUI C. Experimental verification of a semi-active piezoelectric pitch link for helicopter vibration attenuation[C]//Proceedings of the 25th AIAA/AHS Adaptive Structures Conference. [S. l.] : AIAA, 2017: 0057.

[12] LI M, ZHOU J, JING X. Improving low-frequency piezoelectric energy harvesting performance with novel X-structured harvesters[J]. Nonlinear Dynamics, 2018, 94(2): 1409-1428.

[13] WEI C, JING X. Vibrational energy harvesting by ex-

ploring structural benefits and nonlinear characteristic[J]. Communications in Nonlinear Science and Numerical Simulation, 2017, 48: 288-306.

[14] LIU C, JING X, LI F. Vibration isolation using a hybrid lever-type isolation system with an X-shape supporting structure[J]. International Journal of Mechanical Sciences, 2015, 98: 169-177.

[15] WU Y, QIU J, JI H, et al. Piezoelectric spring pendulum oscillator for animal/human motion energy harvesting[C]//Proceedings of 2018 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM). [S.l.]: IEEE, 2018: 774-779.

[16] YUNG K C, WANG J, YUE T M. Modeling Young's modulus of polymer-layered silicate nanocomposites using a modified Halpin—Tsai micromechanical model[J]. Journal of Reinforced Plastics and Composites, 2006, 25(8): 847-861.

**Acknowledgements** This work was supported by the National Natural Science Foundation of China (No. 51705251) and the Introduction of Talent Research Start-up Fund of Nanjing Institute of Technology (No. YKJ201960).

**Authors** Dr. **BIAN Kan** received his M. S. and Ph. D. degrees in Engineering Mechanics from Nanjing University of Aeronautics and Astronautics, China, in 2008 and 2013, respectively. He is currently a lecture at the School of Materials Science and Engineering in Nanjing Institute of Technology. His research interests include intelligent drive mechanism, functional materials and structures, etc.

Dr. **WU Yipeng** received his M.S. degree in flight vehicle design and engineering from Nanjing University of Aeronautics and Astronautics, China, in 2011. He received his Ph.D. degree in science of engineering from University of Grenoble, France, in 2014. He is currently an associate professor at the State Key Laboratory of Mechanics and Control of Mechanical Structures from Nanjing University of Aeronautics and Astronautics. His research mainly focuses on vibration energy harvesting based on piezoelectric transducers.

**Author contributions** Dr. **BIAN Kan** designed the study and wrote the manuscript. Mr. **WANG Yue** contributed to the simulation and experimental data for the validation. Mr. **HUANG Xuewen** established the theoretical model. Dr. **WU Yipeng** wrote, reviewed and edited the submitted manuscript. All authors commented on the manuscript draft and approved the submission.

**Competing interests** The authors declare no competing interests.

(Production Editor: ZHANG Tong)

# 一种金属夹片式压电弹簧结构的等效刚度分析

卞侃[1],王越[2],黄学文[3],吴义鹏[2]

(1.南京工程学院材料科学与工程学院,南京 211167,中国;
2.南京航空航天大学机械结构力学及控制国家重点实验室,南京 210016,中国;
3.华能山东如意(巴基斯坦)能源(私人)有限公司,济南 250014,中国)

**摘要:**压电陶瓷通常很难与普通弹簧结构集成。为了解决这一问题,本文受长尾票夹结构的启发,提出了一种新的金属夹片式弹簧结构。通过建立理论模型、有限元仿真以及实验测试,对所设计的压电夹片式弹簧结构的等效刚度进行了深入分析和讨论。研究结果验证了该结构紧凑的夹片式压电弹簧的设计可行性,且可通过改变几个关键几何参数来调整其等效刚度。最后,由实验结果证实的理论模型预测表明,弹簧结构的等效刚度与夹片结构变化的瞬时角度具有函数关联性,而这种由几何非线性引起的刚度变化在某些实际工程应用中可忽略不计。据此可将夹片弹簧结构线性化,用于后续结构,如压电振子的结构动力学建模简化。

**关键词:**压电弹簧;低刚度;莫尔定理;有限元仿真