Isotropic Optimization of a Stewart-Type Six-Component Force Sensor

ZHANG Tao, XIAO Xinyi, YANG Dehua*, WU Changcheng

College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 211106, P.R. China

(Received 12 May 2020; revised 23 July 2020; accepted 30 July 2020)

Abstract: This paper presents the isotropic optimization of a Stewart-type six-component force sensor. First, the static model of the sensor is built by the screw theory and the forward isotropy indexes and the inverse isotropy indexes are further presented. Second, a comprehensive evaluation function is established to evaluate the isotropic performance of the sensor. By compromising all the isotropy indexes and solving the extreme value of the function, the sensor optimization process is completed and an optimal solution of a set of sensor structure parameters is obtained. Finally, the design of the components and the assembly of the prototype are established by 3D modeling software Pro-E. The verification of the isotropic performance of the sensor is conducted by the finite element analysis software ANSYS. The results obtained by our research can provide useful reference to the isotropic performance evaluation and structure design of the stewart-type six-component force sensor.

Key words: Stewart; force sensor; isotropy; optimization; finite element analysis CLC number: TP212.1 Document code: A Article ID:1005-1120(2020)S-0102-07

0 Introduction

With the ability of measuring three force components and three torque components applied to it, the six-component force/torque sensor is one of the most important and challenging sensors for research. It is widely used in precision assembly, contour tracking, wind tunnel testing, and force control of robots. The Stewart platform is a classic six-degree-of-freedom (6-DOF) parallel mechanism proposed by Stewart^[1]. It has been successfully applied to the field of six-dimensional force sensors, thanks to its many advantages. First, Stewart platform possesses the distinguishing advantages of good stiffness, symmetric and compact design^[2]. Second, Stewart platform has been studied for a long time and has become a mature theoretical system. Most importantly, each elastic measurement limb of the Stewart platform-based force sensor just sustains tensile strain or compressive strain along its axis. With the neglect of the gravity of the legs and the

frictional moment in the spherical pairs, the sensor can realize the measurement of six-component force/ torque without stress coupling in theory^[3].

For optimal design, structural design is particularly important, since many of the sensor's performance are directly or indirectly related to the structural parameters. Isotropy is one of the most important performance of force sensor^[4]. It leads to the minimum relative error in the force mapping^[2]. How to design a sensor with excellent performance has been extensively studied by many authors. Zhao^[5] proposed a design method based on the performance indices atlases. Yao et al.^[6] listed the analytical relationship between the comprehensive performance index and the structural parameters, which proved that the sensor can not satisfy both the forward isotropic and the inverse isotropic. Ref.[7] proposed a design method based on optimized triangular cones.

In this paper, an optimization method is discussed. By establishing a comprehensive performance evaluation function and solving the extremity

^{*}Corresponding author, E-mail address: dhyang@nuaa.edu.cn.

How to cite this article: ZHANG Tao, XIAO Xinyi, YANG Dehua, et al. Isotropic optimization of a Stewart-type six-component force sensor[J]. Transactions of Nanjing University of Aeronautics and Astronautics, 2020, 37(S): 102-108. http://dx.doi.org/10.16356/j.1005-1120.2020.S.013

of it, the optimal set of structural parameters is obtained when the comprehensive performance index is the best. The prototype is further modeled according to the structural parameters.

1 Statics Model of Stewart-Type **Force Sensor**

As the foundation for the further research of the sensor, the static mathematical model of the sixcomponent force sensor should be built first. As show in Fig.1, the classical six-component force/ torque sensor based on Stewart platform is composed of an upper platform, a lower platform and six elastic legs connecting the tow platforms with spherical joints.

The Cartesian coordinate $OX_BY_BZ_B$ is set up with its origin located at the geometrical center of the upper platform. The X_B axis is perpendicular to the line connecting B_1 and B_6 . Symbols are defined as follows: B_i ($i=1, 2, \dots, 6$) and A_i ($i=1, 2, \dots, 6$) 6) denote the position vectors of the center of the ith spherical joint on the upper platform and the lower platform with respect to the coordinate system, respectively; R_B and R_A denote the radius of the circles, with which the centers of spherical joint located, on the upper platform and the lower platform, respectively; H denotes the distance between the upper and the lower platform; α_B denotes the twist angle between B_1 and B_6 ; α_A denotes the twist angle between A_1 and A_6 ; In addition, the twist angles between B_1 and B_3 , between B_3 and B_5 , between A_1 and A_3 , between A_3 and A_5 are $\frac{2\pi}{3}$.

For the equilibrium of the upper platform, the following equation can be obtained by screw theory^[8]

 $\sum_{i=1}^{6} f \, \mathbf{x} = \mathbf{F}_{-} + \mathbf{F}_{+}$



Fig.1 Statics model of Stewart-type force sensor

where f_i represents the reaction produced on the *i*th elastic leg; $\$_i$ the unit line vector alone the *i*th leg; F_F and F_M represent the force vector and the moment vector applied on the center of the upper platform, respectively.

Eq. (1) can be rewritten in the form of matrix equation as

$$F = G \cdot f \tag{2}$$

where $F = \{F_F, F_M\}^{T} = \{F_x \; F_y \; F_z \; M_x \; M_y \; M_z\}^{T}$ is a six-dimensional vector composed of the external force and moment; $f = \{f_1 \ f_2 \ f_3 \ f_4 \ f_5 \ f_6\}^T$ a vector composed of the axial forces of the six legs; G is the force Jacobian matrix^[9] which is given by

$$G = \begin{bmatrix} S_{1} & S_{2} & S_{3} & S_{4} & S_{5} & S_{6} \\ S_{01} & S_{02} & S_{03} & S_{04} & S_{05} & S_{06} \end{bmatrix}$$

$$\begin{cases} S_{i} = \frac{B_{i} - A_{i}}{|B_{i} - A_{i}|} \\ S_{oi} = \frac{A_{i}B_{i}}{|B_{i} - A_{i}|} \end{cases}$$
(3)

where

$$B_{i} - A_{i} = [R_{B} \cos\varphi_{i} - R_{A} \cos\beta_{i}, R_{B} \sin\varphi_{i} - R_{A} \sin\beta_{i} H]^{T}$$
$$R_{A} \sin\beta_{i} H]^{T}$$
$$A_{i} \times B_{i} = [R_{B}H \sin\varphi_{i}, - R_{B}H \cos\varphi_{i}R_{A}R_{B} \sin(\varphi_{i} - \beta_{i})]^{T}$$

$$\sum_{i=1} f_i \$_i = F_F + F_M \qquad (1) \qquad -R_B H \cos \varphi_i R_A R_B \sin (1) \qquad -R_B H$$

L

If G is not singular, Eq.(2) can be expressed as

 $f = G^{-1} \bullet F = C \bullet F \tag{4}$

where C is the inverse mapping matrix.

Obviously, the element of the matrix *G* is just determined by the five parameters of the Stewart platform, including $R_B, R_A, \alpha_A, \alpha_B$ and *H*.

And the characteristics of matrix G will directly impact the performance of the sensor, which is the basis of the force sensor design. Therefore, structural design is particularly important for optimal design.

2 Isotropic Characterization of Sensor

Different from common uniaxial force sensors, the six-component force sensor based on Stewart platform is designed to measuring all applying force components applied to the center of the upper platform. So it is expected that the sensor have the same measurement sensitivity in six-force component. In another word, the sensor needs to be isotropic. The isotropic sensor may be considered as the most slightly affected by the interferential noise, the machining error, and other error sources^[2]. From the perspective of mathematical analysis, when the condition number^[5] of G matrix is 1, the sensor is isotropic. Generally speaking, it is non-sense in physics to measure isotropic performance between the force and the moment. Now, it is usually to discuss the force isotropy and moment isotropy seperately. Based on the discussion above, matrixes Gand *C* can be expressed as

$$G = [G_F G_M]^{\mathrm{T}}$$
(5)

$$C = [C_F C_M] \tag{6}$$

where

$$G_{F}G_{F}^{T} = \begin{bmatrix} \psi_{F1} & 0 & 0 \\ 0 & \psi_{F2} & 0 \\ 0 & 0 & \psi_{F3} \end{bmatrix}$$
$$G_{M}G_{M}^{T} = \begin{bmatrix} \psi_{M1} & 0 & 0 \\ 0 & \psi_{M2} & 0 \\ 0 & 0 & \psi_{M3} \end{bmatrix}$$

$$C_{F}C_{F}^{T} = \begin{bmatrix} \varsigma_{M1} & 0 & 0 \\ 0 & \varsigma_{F2} & 0 \\ 0 & 0 & \varsigma_{F3} \end{bmatrix}$$

$$C_{M}C_{M}^{T} = \begin{bmatrix} \varsigma_{M1} & 0 & 0 \\ 0 & \varsigma_{M2} & 0 \\ 0 & 0 & \varsigma_{M3} \end{bmatrix}$$

$$= \sqrt{R_{A}^{2} + R_{B}^{2} - 2R_{A}R_{B}\cos\left[(\alpha_{A} - \alpha_{B})/2\right] + H^{2}}$$

$$m = R_{A}^{2} + R_{B}^{2} - 2R_{A}R_{B}\cos\left[(\alpha_{A} - \alpha_{B})/2\right]$$

$$n = 2R_{A}^{2}\sin^{2}\left[(\alpha_{A} - \alpha_{B})/2\right]$$

$$\mu_{F1} = \psi_{F2} = 3m/L^{2}, \quad \psi_{F3} = 6H^{2}/L^{2}$$

$$\psi_{M1} = \psi_{M2} = 3H^{2}R_{B}^{2}/L^{2}, \quad \psi_{M3} = 3R_{B}^{2}n/L^{2}$$

$$\varsigma_{F1} = \varsigma_{F2} = 2L^{2}/3n \ \varsigma_{F3} = L^{2}/6H^{2}$$

 $\begin{bmatrix} \zeta_{F1} & 0 & 0 \end{bmatrix}$

$$\varsigma_{M1} = \varsigma_{M2} = 2mL^2/3H^2R_B^2n$$

 $\varsigma_{M3} = L^2/3R_B^2n$
and C_F are the first three rows of

where G_F and C_F are the first three rows of the G matrix and the first three columns of the C matrix, respectively, which are the force transfer factors. G_M and C_M are the last three rows of the G matrix and the last three columns of the C matrix, respectively, which are the torque transfer factors. The four matrices G_F , G_M , C_F , and C_M comprehensively reflect the isotropic performance of the sensor, and are the basis for studying the isotropic performance and structural design of the sensor.

Many factors should be considered in the design of the six-component force sensor, Including isotropy, sensitivity, stiffness, magnification, etc. Among them, many indexes are mutually constrained. Not all sensors can achieve isotropic. So we need to define some indexes to measure the force isotropy performance and torque isotropy performance of the sensor. Based on the mathematical model established in the second part and four matrices, the six-component force/torque sensor based on Stewart platform isotropy indexes are defined as follows

(1) Forward force isotropy index I_{GF}

$$I_{GF} = \frac{1}{\operatorname{cond}(G_F)} = \frac{\left[\lambda_{\min}(G_F G_F^{\mathrm{T}})\right]^{1/2}}{\left[\lambda_{\max}(G_F G_F^{\mathrm{T}})\right]^{1/2}} \qquad (7)$$

(2) Forward torque isotropy index I_{GM}

$$I_{GM} = \frac{1}{\operatorname{cond}(G_M)} = \frac{\left[\lambda_{\min}(G_M G_M^{\mathrm{T}})\right]^{1/2}}{\left[\lambda_{\max}(G_M G_M^{\mathrm{T}})\right]^{1/2}} \qquad (8)$$

(3) Inverse force isotropy index I_{CF}

$$I_{CF} = \frac{1}{\operatorname{cond}(C_F)} = \frac{\left[\lambda_{\min}(C_F C_F^T)\right]^{1/2}}{\left[\lambda_{\max}(C_F C_F^T)\right]^{1/2}} \qquad (9)$$

(4) Inverse torque isotropy index I_{CM}

$$I_{CM} = \frac{1}{\operatorname{cond}(C_M)} = \frac{\left[\lambda_{\min}(C_M C_M^{\mathrm{T}})\right]^{1/2}}{\left[\lambda_{\max}(C_M C_M^{\mathrm{T}})\right]^{1/2}} \quad (10)$$

where $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denotes the minimum and maximum eigenvalue of matrix, respectively.

Obviously, the larger the value of the four isotropy indexes, the better the isotropic performance of the sensor. In particular, when the sensor satisfies both the forward force isotropy and the forward torque isotropy, it is said to satisfy the forward isotropy; when the sensor satisfies both the reverse force isotropy and the reverse torque isotropy, it is said to satisfy the inverse isotropy. When the sensor satisfies both the forward isotropy and the reverse isotropy, it is said to satisfy the complete isotropy.

3 Isotropic Optimization of Sensor

For Stewart platform-based force sensors, Ref. [5] has demonstrated that it is impossible to satisfy the forward isotropy and the reverse isotropy simultaneously. In another word, it is impossible to achieve the complete isotropy. Therefore, optimizing the design requires both consideration of design requirements and trade-offs of multiple performance indexes, in order to achieve the best overall isotropy performance.

As can be seen from the definition of the above isotropy indexes, I_{GF} , I_{GM} , I_{CF} and I_{CM} all range from 0 to 1. The closer to 1 is, the better the isotropic performance of the sensor. In order to obtain the best comprehensive isotropy performance, multi-objective optimization based on various performance indicators is required. The optimization problem is formulated as follows

$$\operatorname{Min}\left(I = \frac{K_1}{I_{GF}} + \frac{K_2}{I_{GM}} + \frac{K_3}{I_{CF}} + \frac{K_4}{I_{CM}}\right) \quad (11)$$

where K_1 and K_2 denote the weight of the forward force isotropy index and the forward torque isotropy index, respectively; K_3 and K_4 the weight of the inverse force isotropy index and the inverse torque isotropy index, respectively. Engineers can choose different weight ratios based on concrete design goals. In this paper, all isotropy indexes here are considered together, the weights are set as

$$K_1 = K_2 = K_3 = K_4 = 1 \tag{12}$$

By solving the extreme of the function I, the best comprehensive isotropy index of Stewart platform-based force sensor is achieved. Meanwhile, one set of structural parameters with the best comprehensive isotropy index is obtained.

The specific optimization process is reversed. A cluster of feasible solutions within the design constraints is given. The values of the isotropy index and the optimization function I corresponding to each feasible solution are calculated, based on the above discussion, the optimal solution is obtained when the optimization function gets the minimum. The design constraints are set as follows

$$\begin{cases} L = 84 \text{ mm} \\ \alpha_B = 20^{\circ} \\ 50 \text{ mm} < R_B = R_A < 300 \text{ mm} \\ 30^{\circ} < \alpha_A < 90^{\circ} \end{cases}$$
(13)

As seen from Fig.2, each point on the graph is a feasible design and the Y-coordinate gives corresponding optimization function value of that design (point). Obviously, the minimum value of the Ycoordinate is not 4. This is because that the classic Stewart structure six-component force sensor cannot achieved the completely isotropy. In another word, $I_{GF}=I_{GM}=I_{CF}=I_{CM}=1$ is impossible. So the minimum value of the optimization function always greater than 4. Under this premise, it is known from the optimization results that the minimum value of the function is 5.656 9 when $I_{GF}=I_{GM}=I_{CF}=I_{CM}=$ $\sqrt{2}/2$ and at this point the sensor's comprehensive isotropy performance is optimal.

It is worth noting that from the engineering

point of view, the solution close to the minimum value can also be regarded as the optimal solution. These solutions are listed in Table 1.



Table 1 Optimal solution set

Parameter set	Isotropic indexes	Value of function I
$\alpha_A = 50^{\circ}$ $R_B = 228 \text{ mm}$ $H = 59.27 \text{ mm}$	$I_{CF} = 0.7100$	5.656 9
	<i>I</i> _{см} =0.710 3	
	$I_{GF} = 0.7040$	
	$I_{GM} = 0.704.2$	
$\alpha_A = 55^{\circ}$ $R_B = 196 \text{ mm}$ $H = 59.16 \text{ mm}$	$I_{CF} = 0.712.7$	
	<i>I_{см}</i> =0.709 8	5.657 0
	$I_{CF} = 0.700 \ 4$	
	$I_{CM} = 0.7015$	
$\alpha_A = 60^{\circ}$ $R_B = 172 \text{ mm}$ $H = 59.06 \text{ mm}$	$I_{CF} = 0.715.2$	
	<i>I_{см}</i> =0.709 9	5.657 1
	$I_{CF} = 0.700 \ 4$	
	<i>I_{см}</i> =0.699 1	
$\alpha_A = 65^{\circ}$ $R_B = 153 \text{ mm}$ $H = 59.09 \text{ mm}$	$I_{CF} = 0.714.3$	
	$I_{CM} = 0.7317$	5.657 1
	$I_{CF} = 0.700.6$	
	$I_{CM} = 0.7000$	
$\alpha_A = 70^{\circ}$ $R_B = 139 \text{ mm}$ $H = 58.61 \text{ mm}$	$I_{CF} = 0.725 \ 9$	5.657 8
	I _{CM} =0.700 5	
	I_{CF} =0.708 7	
	$I_{CM} = 0.688 8$	
$\alpha_A = 80^{\circ}$ $R_B = 116 \text{ mm}$ $H = 58.74 \text{ mm}$	$I_{CF} = 0.722.8$	
	$I_{CM} = 0.716 \ 1$	5.657 8
	$I_{CF} = 0.698.2$	
	$I_{CM} = 0.691.7$	
$\alpha_A = 90^{\circ}$ $R_B = 100 \text{ mm}$ $H = 58.64 \text{ mm}$	$I_{CF} = 0.725.2$	5.658 4
	$I_{CM} = 0.7230$	
	$I_{CF} = 0.691.6$	
	<i>I</i> _{см} =0.685 9	
$\alpha_A = 100^{\circ}$ $R_B = 88 \text{ mm}$ H = 58.59 mm	$I_{CF} = 0.7265$	5.659 6
	$I_{CM} = 0.732.4$	
	$I_{CF} = 0.682~7$	
	$I_{CM} = 0.688 2$	

4 Isotropic Verification by Finite Element Analysis

Finally, in order to make the mechanical assembly more convenient, we select a set of structural parameters in Table 1 as the structural parameters of the prototype: $\alpha_A - \alpha_B = 70^\circ$, $R_B = R_A = 100$ mm, L = 84 mm, H = 58.64 mm.

The following points should be noted in the design and selection of components.

(1) The connection pair is chosen to use flexible hinge with three degrees of rotational freedom. The reason why the real ball joint is not used is that it causes void and friction, which affects the measurement accuracy of the sensor. In addition to this, the real ball hinge also limits the miniaturization of the sensor.

(2) Measurement of the reaction produced on the six elastic legs is obtained by six uniaxial force sensors. Uniaxial force sensors require further calibration to ensure measurement accuracy.

The parameters of the specific uniaxial force sensor are shown in Table 2.

Parameter	Value
$Sensitivity/(mV {\scriptstyle \bullet} V^{{\scriptstyle -1}})$	1.0 ± 0.05
Nonlinearity/(%F.S)	± 0.5
Repeatability/(%F.S)	± 0.5
Lag/(%F.S)	± 0.5
Zero output/(%F.S)	± 1
Excitation voltage/V	10—15

 Table 2
 Parameter of uniaxial force sensor

The model is built through the 3D modeling software Pro-E as shown in Fig.3. The detailed exploded view is shown in Fig.4. The 3D model was imported into the finite element analysis software ANSYS for further verification of the isotropic performance of the sensor. The better the isotropic performance of the sensor, the closer the deformation caused by the force/torque of different dimensions acts on the sensor. Since the sensor is symmetrical (the same performance in the X and Z directions), only the following four cases need to be simulated.



Fig.3 Stewart platform-based force/torque sensor



Explosion diagram of the component. Fig.4

Condition 1 100 N force applied to the center of the upper platform (point *O*) along the *X* axis, as shown in Fig.5.

Condition 2 100 N force applied to the O along the Y axis, as shown in Fig.6.

Condition 3 1 N·m moment applied to the Oaround the X axis which is shown in Fig.7.

Condition 4 1 N·m moment applied to the Oaround the Y axis, as shown in Fig.8.

It has been verified that the deformation caused by the force/torque in the X and Z directions is almost equal. Figs. 5-8 illustrate that the deformation caused by the force/torque in the X/Z and Y directions is also very close. It is proved that the sensor designed based on the optimization result has good isotropic performance.



Fig.5 Total deformation of sensor in Condition 1





Fig.7 Total deformation of sensor in Condition 3



Fig.8 Total deformation of sensor in Condition 4

Conclusions 5

This paper introduces the isotropic optimization of the Stewart-type six-component force sensor. Based on the static model of the sensor, the mathematical expression of the sensor's isotropic performance index is derived. The optimal solution of the sensor parameters is obtained by establishing an evaluation function and finding the extreme value. The optimization results show that the sensor has the best isotropic performance when $I_{GF}=I_{GM}=I_{CF}=I_{CM}=\sqrt{2}/2$. The finite element analysis results further prove this conclusion. The contents of this paper should be useful for the optimization design works of the Stewart platform-based force sensor.

References

- [1] STEWART D. A platform with six degrees of freedom[J]. Proc Instn Mech Engrs, 1965, 180: 371-386.
- [2] MERLET J P. Parallel robots[M]. Dordrecht, Netherlands: Kluwer Academic Publishers, 2002.
- [3] HOU Y, YAO J, LU L, et al. Performance analysis and comprehensive index optimization of a new configuration of Stewart six-component force sensor[J]. Mechanism and Machine Theory, 2009, 44 (2): 359-368.
- [4] XIONG Y. On isotropy of robot's force sensors [J]. Acta Automatica Sinica, 1996, 22(1):10-18. (in Chinese)
- [5] ZHAO Xianchao. Design theory and application research of stewart structure six-dimensional force sensor
 [D]. Qinhuangdao: Yanshan University, 2003. (in Chinese)
- [6] YAO J, HOU Yulei, MAO S, et al. Analytical analysis and optimization design of six-dimensional force sensor for Stewart structure [J]. Journal of Mechanical Engineering, 2009, 45(12): 22-28.
- [7] DWARAKANATH T A, DASGUPTA B, MRUTHYUNJAYA T S. Design and development of a Stewart platform based force-torque sensor[J].

Mechatronics, 2001, 11(7): 793-809.

- [8] HUANG Z. Space agency science [M]. Beijing: Mechanical Industry Press, 1991.(in Chinese)
- [9] NGUYEN C C, ANTRAZI S S, ZHOU Z L. Analysis and implementation of a 6 DOF Steward platformbased force sensor for passive compliant robotic assembly[C]//Proceedings of IEEE Proceedings of the SOUTHEASTCON'91. Williamsburg, USA: IEEE, 1991: 880-884.

Acknowledgements This work was supported by the Fundation of Graduate Innovation Center in Nanjing University of Aeronautics and Astronautics (No.kfjj20190308) and the Fundamental Research Funds for the Central Universities.

Authors Mr. **ZHANG** Tao is a postgraduate of Nanjing University of Aeronautics and Astronautics. He majors in instrumentation engineering. His research interests include the design and application of the sensor.

Prof. YANG Dehua obtained his Ph.D. degree from the Chinese Academy of Sciences, the main research direction is the design of parallel robots and optical electromechanical systems.

Author contributions Mr. ZHANG Tao is responsible for the design and optimization of the sensor. Prof. YANG Dehua is responsible for answering questions in the design and manufacturing process, Mr. XIAO Xinyi is responsible for the simulation of sensor stiffness, and Mr. WU Changcheng is responsible for software answering.

Competing interests The authors declare no competing interests.

(Production Editor: ZHANG Bei)

基于各向同性的 Stewart 型六维力传感器的优化设计

张 韬,肖心怡,杨德华,吴常铖

(南京航空航天大学自动化学院,南京 211106,中国)

摘要:介绍了Stewart型六维力传感器基于各向同性的优化设计。首先,基于螺旋理论建立了传感器的静态数学模型,并进一步提出了正向各向同性指标和逆向各向同性指标。接下来,建立了一个综合评价函数来评估传感器的综合各向同性性能的优劣。通过折衷所有各向同性指标并求解函数的极值,完成了传感器优化过程,获得了一组传感器结构参数的最优解。最后,零部件的设计和样机模型的建立由3D建模软件Pro-E完成。传感器各向同性性能的验证由有限元软件ANSYS完成。研究结果对Stewart型六分量力传感器的各向同性性能评估和结构优化设计具有重要意义。

关键词:Stewart;力传感器;各向同性;优化;有限元分析