Improved Whale Optimization Algorithm Based on Mirror Selection

*LI Jingnan**, *LE Meilong*

College of Civil Aviation, Nanjing University of Aeronautics and Astronautics, Nanjing 211106, P.R. China

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Abstract: Since traditional whale optimization algorithms have slow convergence speed, low accuracy and are easy to fall into local optimal solutions, an improved whale optimization algorithm based on mirror selection (WOA-MS) is proposed. Specific improvements includes: (1) An adaptive nonlinear inertia weight based on Branin function was introduced to balance global search and local mining. (2) A mirror selection method is proposed to improve the individual quality and speed up the convergence. By optimizing several test functions and comparing the experimental results with other three algorithms, this study verifies that WOA-MS has an excellent optimization performance.

Key words: inertia weight; mirror selection; whale optimization algorithm(WOA)

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0 Introduction

The whale optimization algorithm (WOA) is a bionic intelligent algorithm that simulates the humpback whale predation process. In the optimization process, different hunting strategies with fixed probability are selected[1]. Due to its simple setup and less parameters, it has been widely used in many fields. Like other heuristic algorithms, however, it has shortcomings. For example, the convergence accuracy is low, the convergence speed is slow and it is easy to fall into local optimum. Therefore, many scholars have proposed improvements. Ding et al.[2] proposed an elite-oriented learning strategy for the whale optimization algorithm. The position of the individual whale is updated by a formula sort of similar to that in the particle swarm optimization (PSO) algorithm to obtain a new individual whale to increase the search ability. Yousri et al.[3] proposed an improved solution for population initialization. Based on the ergodicity of the chaotic system, the chaotic sequence is randomly generated to initialize the population and the random vector in the whale optimization algorithm is also generated by the chaotic sequence. The method effectively improves the randomness of the initial state of the population. Gaganpreet et al.[4] also initialized the population by using chaotic sequences and compared the results of 20 mapping experiments. They pointed out that Tent mapping had the best effect on population initialization. Elaziz et al.[5] proposed a method for automatically selecting chaotic maps using integral evolutionary algorithms to initialize populations.

In the above studies, different scholars have improved the whale optimization algorithm to tailor to the characteristics of their respective research fields, but their ways of improvement are the unilateral and not significant. Further research is needed to balance the global search and local mining, maintain population diversity and improve convergence speed to improve the overall optimization ability.

This paper specifically targets the above shortcomings. Firstly, based on the standard whale optimization algorithm, two improved strategies are proposed to enhance its performance in different aspects. Then the improved algorithm is simulated. The experimental results are compared with other three classical algorithms in many aspects. The results demonstrate that the above measures can effectively improve the convergence speed and accuracy

*Corresponding author, E-mail address: lijingnan@163.com.

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of the algorithm. The computational experiments show that the improved algorithm has an excellent overall performance.

1 Standard Whale Optimization Algorithm

1.1 Predatory behavior and strategy of humpback whales

WOA is a bionic intelligent algorithm that simulates the predation behavior and strategy of humpback whales. During the humpback whale predation, each individual in the whale group has three ways to update their position.

(1) Search for prey: Whales search for prey by randomly updating their locations which is called the exploration process.

(2) Encircling prey: When an individual discovers a prey, other individuals in the whale group move to the individual through information sharing.

(3) Bubble-net attacking: After discovering a prey, humpback whales emit different sizes of bubbles spirally around the prey at a certain interval. Through the difference in the rising speed of different bubbles, all the bubbles reach the prey at the same time to realize the attack on the prey.

Through the three ways, each individual in the population keeps updating its position until it catches the prey or reaches the maximum number of iterations.

1.2 Mathematical modeling of WOA

The hunting behavior of whales can be described by the following mathematical models.

(1) Search for prey

Humpback whales can update their positions in the random walk phase by

$$\boldsymbol{D}=|\boldsymbol{C}\cdot\boldsymbol{X}_{\mathrm{rand}}-\boldsymbol{X}(t)| \tag{1}$$

$$\boldsymbol{X}(t+1)=\boldsymbol{X}_{\mathrm{rand}}-\boldsymbol{A}\cdot\boldsymbol{D} \tag{2}$$

where $\boldsymbol{X}_{\mathrm{rand}}$ indicates a randomly selected location vector in current population; $\boldsymbol{X}(t)$ the tth position vector of the current population; and "·" element multiplication. $\boldsymbol{A}$, $\boldsymbol{C}$ are the coefficient vectors which is defined by

$$\boldsymbol{A}=2\boldsymbol{a}\cdot\boldsymbol{r}-\boldsymbol{a} \tag{3}$$

$$\boldsymbol{C}=2\cdot\boldsymbol{r} \tag{4}$$

where $\boldsymbol{a}$ represents a vector that decreases linearly from 2 to 0 in the whole process of population iteration. $\boldsymbol{r}$ a random vector that is uniformly distributed within [0, 1].

(2) Encircling prey

During the encirclement of the predation, the humpback whale can update its position by

$$\boldsymbol{D}=|\boldsymbol{C}\cdot\boldsymbol{X}^*(t)-\boldsymbol{X}(t)| \tag{5}$$

$$\boldsymbol{X}(t+1)=\boldsymbol{X}^*(t)-\boldsymbol{A}\cdot\boldsymbol{D} \tag{6}$$

where $\boldsymbol{X}^*(t)$ represents current position of the optimal individual. It is constantly updated in each iteration.

(3) Bubble-net attacking

After humpback whales discover their prey, they hunt through bubble attack. The location of humpback whales can be updated

$$\boldsymbol{D}=|\boldsymbol{X}^*(t)-\boldsymbol{X}(t)| \tag{7}$$

$$\boldsymbol{X}(t+1)=\boldsymbol{X}^*(t)+\boldsymbol{D}e^{bl}\cos(2\pi l) \tag{8}$$

where e represents the bottom number of natural logarithms; b the logarithmic helix shape constant; l the random numbers uniformly distributed within [0, 1]. The whale launches bubble attacks on the prey along a spiral trajectory in the process of enclosure and contraction. It is assumed that whales have a probability P to update their position by different methods. P is a random number uniformly distributed within [0, 1]. The flow chart of the standard whale optimization algorithm is shown in Fig.1.

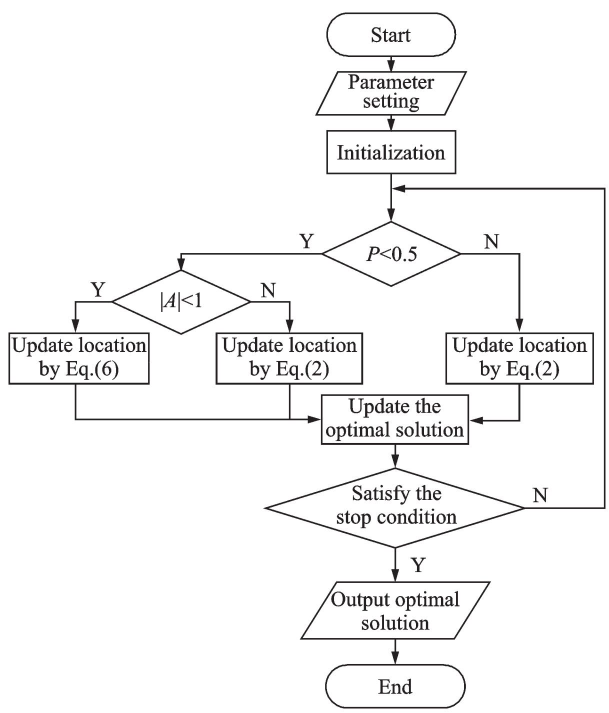


Fig.1 Standard flow of the whale optimization algorithm

2 Improvement of WOA

Given the fact that the whale optimization algorithm is easy to fall into the local optimal solution and the convergence speed is slow, this paper proposes an improved whale optimization algorithm based on mirror selection (WOA-MS) with two improved methods which will improve the overall performance of the algorithm from different aspects.

2.1 Adaptive inertia weight

The concept of inertia weight is introduced in WOA. At the beginning of the iteration, the algorithm should focus on the global search and use the limited number of individuals to explore the region of the solution space as much as possible to obtain enough solution space information. In the latter part of the iteration, the algorithm should focus on local mining and continuously update the position in a small area to find the precise position of the global optimal solution. Therefore, the inertia weight should decrease as the number of iterations increases. For a generation of populations, the centrally located individuals focus on local mining work while the marginal individuals focus on global search work. Therefore, the inertia weight should increase as the distance increases between the individual and the population center. In order to realize the two different dynamic changes of the inertia weight, this paper introduces the Branin function and constructs the above inertia weight into WOA.

Branin is a two-dimensional complex function and its image is uneven. Its function expression is

$$f(x,y)=\left(y-\frac{5.1}{4\pi^2}x^2+\frac{5}{\pi}x-6\right)^2+10\left(1-\frac{1}{8\pi}\right)\cos x+10 \tag{9}$$

In order to achieve the dynamic adjustment effect of the above inertia weight, this paper transforms the Branin function into

$$f(x,y)=\left[\left(y-\frac{5.1}{4\pi^2}x^2-\frac{5}{\pi}x-6\right)^2+10\left(1-\frac{1}{8\pi}\right)\cos(-x)+10\right]\Big/100 \tag{10}$$

The deformed function image is shown in Fig.2.

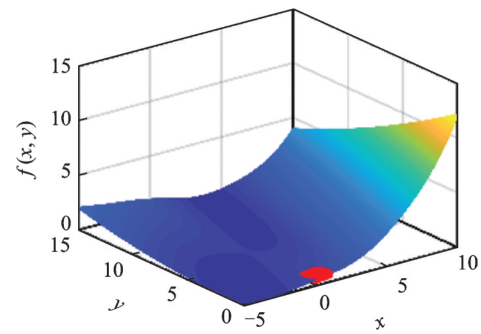


Fig.2 Deformed Branin function

It can be seen that in the interval [0, 1] (the red area in Fig.2), the function satisfies the variation law described above.

The adaptive inertia weight calculation process is as follows.

(1) In each generation of the iterative process, the average value vector of each dimension is calculated as

$$\boldsymbol{X}_{\text{avg}}(j)=\sum_{i=1}^{N}\boldsymbol{X}(i,j)/N \tag{11}$$

where $X(i,j)$ indicates the position on the jth dimension of the ith whale; N the population size.

(2) Calculate the distance of each individual in the population is calculated from the mean in each dimension by

$$\boldsymbol{d}(j)=|\boldsymbol{X}(j)-\boldsymbol{X}_{\text{avg}}(j)| \tag{12}$$

where $\boldsymbol{X}(j)$ represents the position vector of the population on the j-th dimension.

(3) $\boldsymbol{x}$ and $\boldsymbol{y}$ are calculated by Eqs.(13), (14) which are substituded into Eq.(10) to obtain $\boldsymbol{f}(j)$.

$$\boldsymbol{x}=\frac{\boldsymbol{d}(j)}{\boldsymbol{d}_{\max}(j)+10^{-200}} \tag{13}$$

$$y=\frac{t}{T} \tag{14}$$

where $\boldsymbol{f}(j)$ represents the inertia weight vector of the individual in the population on the jth dimension. T represents the maximum number of iterations of the population; $\boldsymbol{d}_{\max}(j)$ the farthest distance between all individuals and the average value on the jth dimension. To prevent denominator from getting to zero, we add a small constant after $\boldsymbol{d}_{\max}(j)$ to ensure that x exists in any case. The image of the adaptive Branin coefficient function is shown in Fig.3.

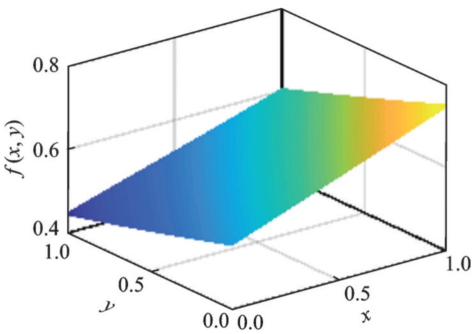


Fig.3 Adaptive Branin coefficient

Position updates are carried out by Eqs.(15), (16), (17), which replace Eqs.(2), (6), (8), respectively

$$X(t+1)=F(t)\cdot X_{\text{rand}}-A\cdot D \tag{15}$$

$$X(t+1)=F(t)\cdot X^*(t)-A\cdot D \tag{16}$$

$$X(t+1)=X^*(t)+F(t)\cdot De^{bl}\cos(2\pi l) \tag{17}$$

where $F(t)$ represents the inertia weight vector of the tth generation of an individual on each dimension.

The inertia weight updated by this method is large at the beginning of the iteration which is conducive to global search. At the later stage of the iteration, it becomes small, which is beneficial to local mining. For a certain generation of groups, the inertia weight of individuals in the center is smaller, which is conducive to improving the local mining capacity. The inertia weight of individuals in the edge is larger, which is conducive to global search.

2.2 Mirror selection

Mirror selection strategy is proposed as a way to mutate the whole population through mirror method. It can greatly increase the diversity of the population, but at the same time increases the number of individuals in the population. Therefore, the individuals after the variation are selected to input into the next generation of iterations to enhance the diversity of the population, improve the individual quality and accelerate the convergence speed. Mirror selection strategy can be divided into two parts: Mirror variation and survival of the fittest. Specifically, the symmetric solution is generated in the solution space by

$$X(t)'=B_u+B_l-X(t) \tag{18}$$

where $X(t)'$ represents the location of mirror individuals; B_u and B_l represent individual upper and lower bound vectors; $X(t)$ represents original individual position vector.

After that, the number of the population increased to twice the number of the original individuals and then entered the survival of the fittest. The newly generated mirror solution is merged with the original solution and sorted according to the fitness. Individuals in the top N in the fitness ranking are selected as the preferred individuals and enter the next calculation and iteration.

2.3 Computational complexity analysis

The time complexity of the bionic intelligent algorithm is related to the population size N, the problem dimension D and the number of iterations T[6]. The time complexity of WOA is $O(T*N*D)$. WOA-MS adds inertia weight calculation and mirror selection on the basis of WOA which increases the time complexity of $O(T*N*D)$ and $O(T*N*D)$. Although WOA-MS calculation is increased, the magnitude of the calculation has not changed and the time complexity remains $O(T*N*D)$.

The spatial complexity of the bionic intelligent algorithm is mainly related to population size N and problem dimension D. The spatial complexity of WOA is $O(N*D)$. In WOA-MS, the mirror selection makes the population size increase to $2N$ and the dimension of the problem remains unchanged. So it adds space complexity $O(N*D)$. The order of magnitude of spatial complexity has not changed, so the spatial complexity of WOA-MS is still $O(N*D)$.

In summary, based on the standard whale optimization algorithm, combined with the two improved methods proposed in this paper, we propose the basic implementation steps of WOA-MS, as shown in Fig.4. The blue step is the improvements of this study.

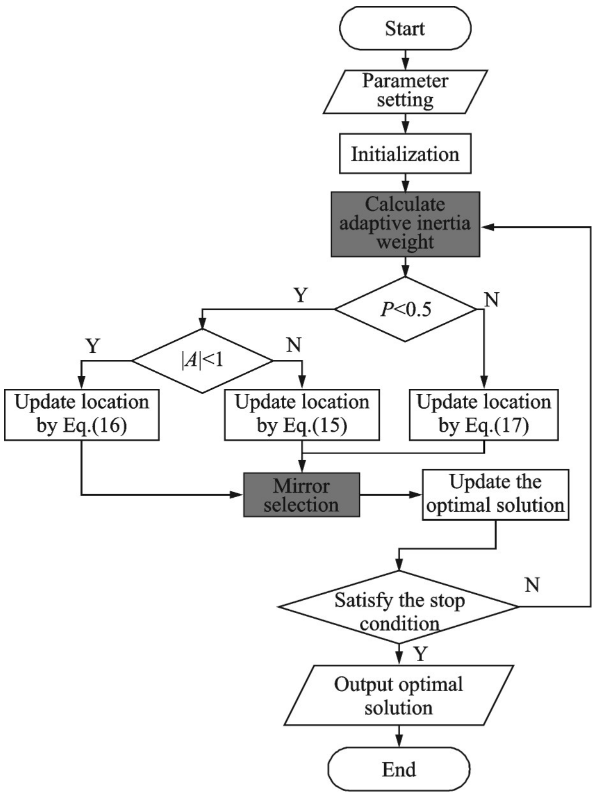


Fig.4 Flowchart of WOA-MS

3 Algorithmic Performance Testing

3.1 Test function

In order to verify the performance of WOA-MS proposed in this paper, 12 typical test functions are taken as examples. The benchmark functions in two-dimensional 3D images are shown in Fig. 5. Functions 1—4 are single-peak benchmark functions with 100 dimensions. Functions 5—8 are multimodal benchmark functions with 100 dimensions. Functions 9—12 are fixed-dimensional functions and the dimensions are two-dimensional.

WOA belongs to the swarm intelligence evolutionary algorithm. Its essence is to search the solution space by updating the individual position of each generation to find the optimal solution. In the swarm intelligence evolutionary algorithm, particle swarm optimization (PSO) is one of the classical algorithms with strong robustness and wide application. It also searches by constantly adjusting the position of individuals in the swarm. Although the position change rules of the two algorithms are different, they optimize through the change of individual position in each iteration process. The essence of the two algorithms is similar. Therefore, this paper compares standard PSO, standard WOA, improved particle swarm optimization (IPSO)[7] and WOA-MS. Among them, IPSO improvement measures include adding dynamic self-learning factor C_1, dynamic group learning factor C_2 and dynamic inertia weight W. The performance of WOA-MS is tested under the same parameters.

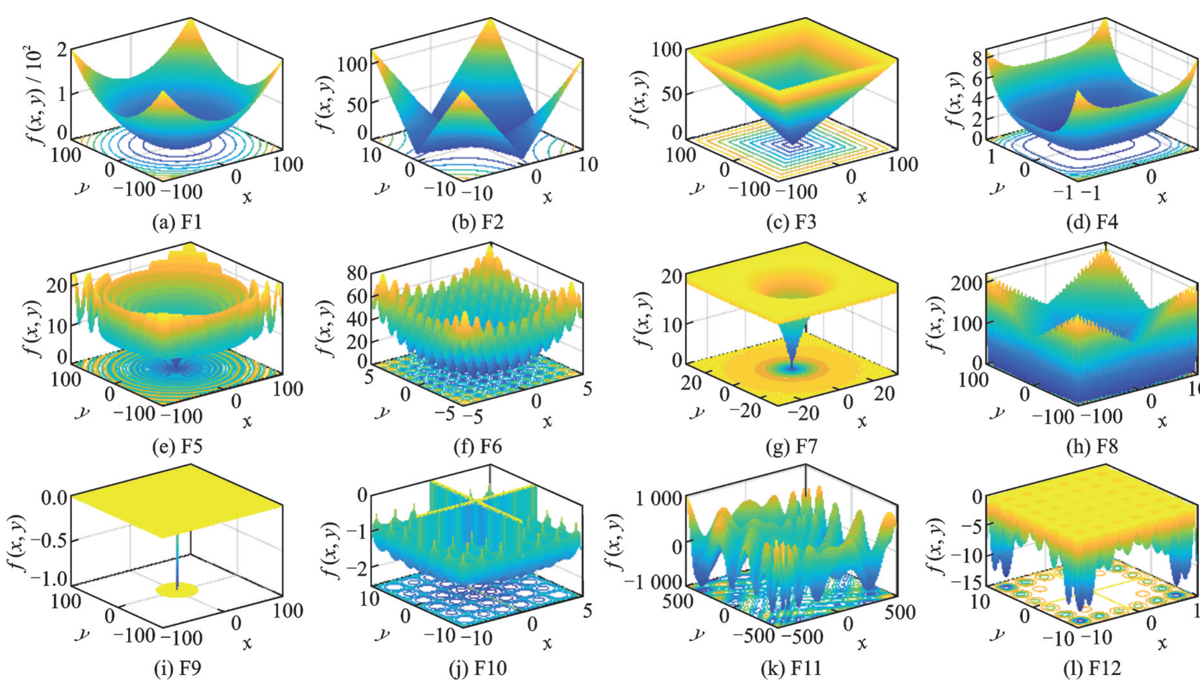


Fig.5 3D images

3. 2 Parameter setting

The parameters of each algorithm are set as shown in Table 1, where N is the number of individuals and T the number of iterations.

Each algorithm runs 20 times independently and compares the average value of the optimization results and the standard deviation. The results are shown in Table 2, where S.D. means standard deviation.

Table 1 Parameter settings of the algorithms

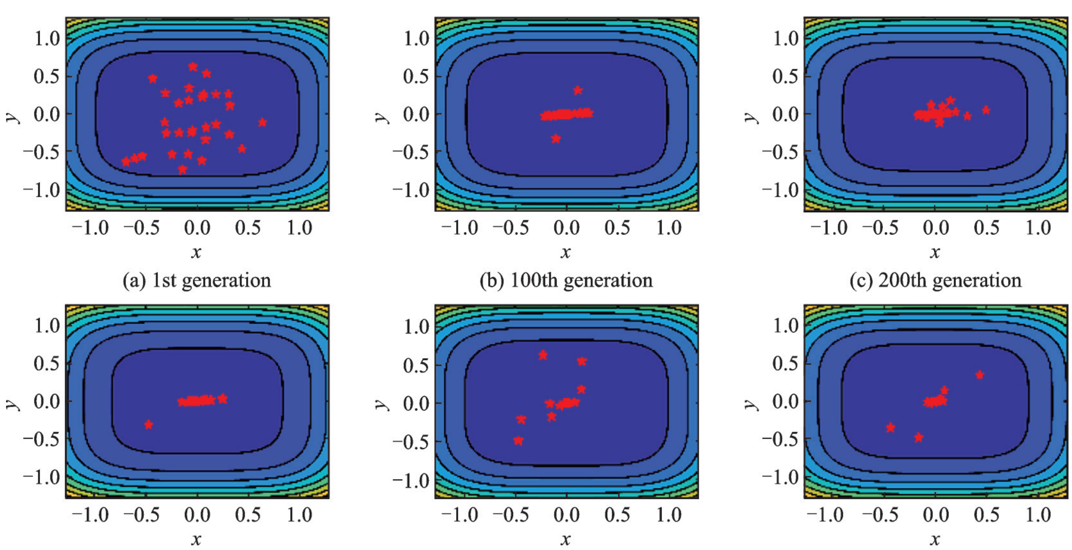
Algorithm	Parameter setting
PSO	N=30, T=500, C_1=2, C_2=2, W=1.05
WOA	N=30, T=500
IPSO	N=30, T=500, C_1: 1.8—0.9, C_2: 0.9—1.8, W: 0.95—0.45
WOA-MS	N=30, T=500

Table 2 Experimental results of the four algorithms

Benchmark function	PSO		WOA		IPSO		WOA-MS	
	mean	S.D.	mean	S.D.	mean	S.D.	mean	S.D.
F1	25.251 6	3.644 4	1.02E−145	4.27E−145	2.61E−163	0	0	0
F2	44.498 1	6.100 5	6.61E−88	2.96E−87	2.52E−81	8.86E−81	7.56E−248	0
F3	11.700 0	1.226 0	5.04E−20	2.02E−19	3.99E−81	1.58E−80	2.83E−230	0
F4	45.100 1	50.942 5	3.56E−04	4.28E−04	1.10E−04	8.40E−05	8.47E−06	1.20E−05
F5	1.36E+04	2.07E+03	1.93E−34	3.46E−34	2.54E−40	4.99E−40	0	0
F6	578.717 3	46.621 3	0	0	0	0	0	0
F7	7.735 5	1.048 5	2.66E−15	2.16E−15	8.88E−16	0	8.88E−16	0
F8	208.172 5	31.627 8	5.69E−66	2.39E−65	2.08E−80	8.53E−80	1.44E−253	0
F9	−1	8.98E−06	−0.996 1	0.005 1	−1	5.86E−06	−0.967 9	0.04
F10	−2.062 6	4.59E−07	−2.062 6	1.46E−05	−2.062 6	4.97E−07	−2.062 6	0
F11	−800.215 3	124.251 5	−938.821 2	24.151 1	−869.065 0	65.162 2	−949.902	12.541 3
F12	−16.782 3	4.310 9	−19.208 5	1.252 2	−19.086 2	0.160 2	−19.208 5	1.01E−04

3. 3 Comparison of population diversity

Population diversity can be reflected in the individual location of the iterative process. The ideal state of individual distribution is that some individuals gather around the current optimal solution for local mining. In order to prevent the algorithm from falling into local optimum, the remaining individuals are required to travel globally in other locations in the solution space. Take F4 as an example. The individual distribution during the WOA-MS iteration is shown in Fig.6.



(a) 1st generation (b) 100th generation (c) 200th generation

(d) 300th generation (e) 400th generation (f) 500th generation

Fig.6 Iterative process of WOA-MS on test function 4

According to the theory of population diversity[8], in the iteration process, not all individuals have a certain working mode, but each has its own division of labor, balancing the global search and local mining.

In Fig.6(c), most of the individuals have gathered around the optimal solution for local mining activities, but there are still individuals far from the optimal solution which are searching globally and this phenomenon always exists.

By comparison, the population diversity of WOA shows a significant downward trend, as shown in Fig.7.

By the time of iteration to the 300th generation (Fig.7(d)), the diversity of the population have sharply decreased and almost all individuals are engaged in local mining. This kind of situation will make the algorithm extremely easy to fall into the local optimal solution in the multi-peak function optimization, and lead to premature phenomenon.

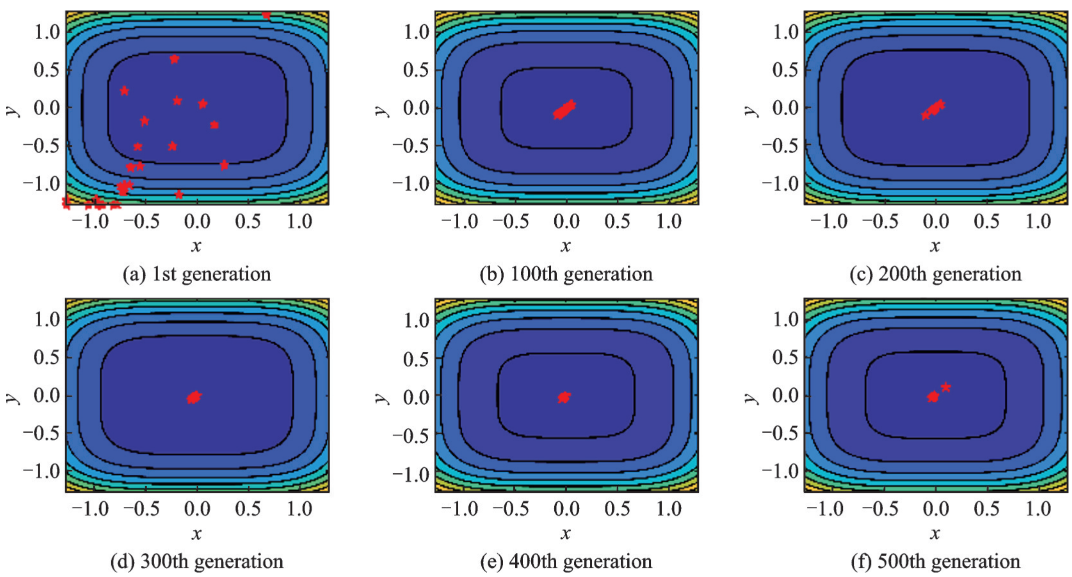


Fig.7 Iterative process of WOA on test function 4

In summary, WOA-MS has significant advantages in population diversity.

3.4 Comparison of convergence rate

Another shortcoming of WOA is that the convergence speed is slow. In the finite iteration process, the faster the convergence speed, the earlier the local mining, and the higher the convergence accuracy. Taking the function F12 as an example, the theoretical optimal solution is −19.208 5. The fitness curve of each algorithm is shown in Fig.8.

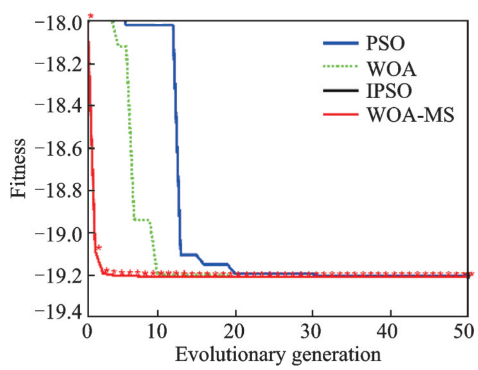


Fig.8 Contrast of convergence speed

It can be clearly seen from Fig.8 that WOA-MS's fitness curve (red) has the fastest decline rate, close to −19.2, when iterates by about the 4th generation.

The fitness curves of WOA (green) and PSO (blue) are close to −19.2 in the 11th and 30th generations, respectively. It can be seen from the fitness record table that WOA-MS finds the optimal solution −19.208 5 in the 215th generation while WOA finds the optimal solution in the 485th generation and PSO does not find the optimal solution.

In summary, the convergence speed of WOA-MS has obvious advantages, far faster than other three algorithms, and its local mining ability is strong.

3.5 Comparison of the ability of jumping out of local optimal solutions

We take the function F11 as an example. Its function image is shown in Fig. 9. There are many local optimal solutions for this function in the graph. This requires high searching ability of the algorithm. If the algorithm falls into one of the local optimal solutions and does not jump out, it is very likely that the solution will be considered as the optimal solution which will lead to premature phenomena. Fig. 10 is the fitness image of each algorithm under F11.

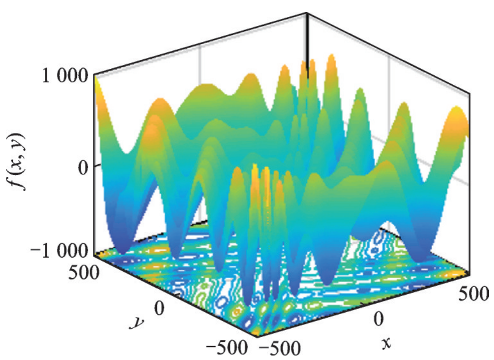


Fig.9 3D image of F11

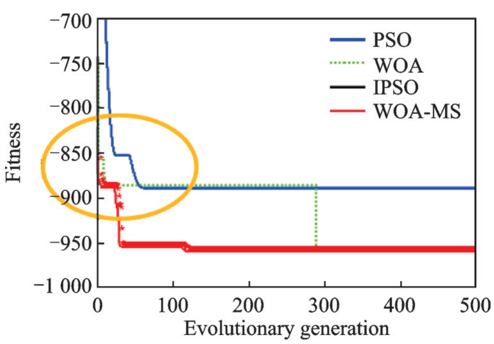


Fig.10 Fitness curve of F11

Near the fitness value −890, all the three algorithms are trapped in the local optimal solution and the fitness has different degrees of stagnation. After about 40 iterations, WOA-MS shows a cliff-like decline, indicating that individuals have jumped out of the local optimal solution and found a better fitness position so that the group could effectively avoid premature phenomenon. The WOA fitness curve (green) also experiences the above process, but its process is slow and an effective descent process occurs after about 200 iterations. The whole process has multiple stagnations and the algorithm is less efficient. The PSO's fitness curve (blue) is stagnant at an earlier position of around −850, but no precocity occurred. In the vicinity of fitness −890, the curve tends to be flat and the fitness stagnates once again. It can be seen that the algorithm falls into local optimum and the phenomenon continues until the end of the iteration. Therefore, a serious premature phenomenon has occurred, resulting in a large deviation between the optimization result and the global optimal solution. The ability of PSO of jumping out of the local optimal solution is insufficient.

In summary, WOA-MS's optimization performance is better than those of PSO, WOA and IPSO in terms of jumping out of local optimum and searching global optimal solution.

3.6 Run time comparison

As noted above, WOA-MS's time complexity and spatial complexity are not increased in magnitude, but the addition of multiple strategies increases the amount of computation for each iteration. Algorithm runtime is expected to increase. It has been verified by many experiments that under the same test function and the same variable value range, the running speed of WOA-MS is about 40.0% higher than that of WOA, about 46.6% lower than PSO and about 47.8% lower than IPSO. Although WOA-MS has no change in time complexity and space complexity, the calculation amount increases, resulting in a 40% increase in the running time, compared to WOA, and the solution results are greatly improved, showing that an extra 40% of the computing time is worthy.

4 Conclusions

(1) WOA-MS has a good population diversity. WOA-MS introduces the adaptive inertia weight to well balance the global search and local mining. Therefore, in the process of optimization, there is no phenomenon that the population is highly concentrated and the diversity is sharply reduced, while the diversity is maintained to improve the global search

ability.

(2) WOA-MS has a fast convergence speed and strong local mining capacity. The mirror selection method greatly improves the individual quality of each generation, speeds up the convergence speed of the algorithm and improves the local mining efficiency.

In summary, WOA-MS has an excellent overall optimization performance, compared to the other three traditional algorithms.

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Authors Mr. **LI Jingnan** is currently pursuing the Master degree in transportation planning and management at Nanjing University of Aeronautics and Astronautics, Nanjing, China. His current research interests include air transportation system optimization and evolutionary optimization algorithms.

Prof. **LE Meilong** received his Ph.D. degree from Shanghai Jiao Tong University. In 2015, he joined Nanjing University of Aeronautics and Astronautics as a full professor and doctoral supervisor. His main research interests include airlines operation optimization, airport operation optimization and revenue management.

Author contributions Mr. **LI Jingnan** designed the experiment, compiled the program and wrote the manuscript. Prof. **LE Meilong** has carried out supervision and guidance.

Competing interests The authors declare no competing interests.

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基于镜像选择的改进鲸鱼优化算法

李璟楠，乐美龙

(南京航空航天大学民航学院,南京 211106,中国)

摘要:针对鲸鱼优化算法收敛速度慢、精度低、易陷入局部最优解的缺点,提出了一种基于镜像选择的改进鲸鱼优化算法(Whale optimization algorithm based-on mirror selection, WOA-MS)。具体改进包括:(1)为了平衡全局搜索和局部开采,提出了一种基于Branin函数的自适应非线性惯性权重;(2)为了提高算法的个体质量和收敛速度,提出了一种镜像选择方法。通过对若干种测试函数进行优化,并与其他三种算法的实验结果进行比较,证明了WOA-MS具有良好的优化性能。

关键词:惯性权重;镜像选择;鲸鱼优化算法(Whale optimization algorithm based-on mirror selection, WOA)