# Impact of Capacity Parameters on Flexible Inventory Control Decision Model

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**Abstract:** From the mathematical point of view, the flexible inventory control model is proved in the practical problem application and the rationality of the capacity parameter selection and calculation. The purpose is to actively respond to demand fluctuations when there is a demand forecast error or a missing part of the demand information, and to avoid the risk of passive variable demand forecasting to set the immutable inventory capacity. At the same time, the game is controlled by the flexible and variable inventory control strategy and the customer's willingness to demand. The paper mainly studies the influence of the setting of capacity parameters on the booking-limit decision and its benefits under the control of flexible space with variable total capacity. Through the two trends of capacity increase flexibility and capacity reduction flexibility in the flexible inventory control model, the mathematical performance and marginal utility methods are introduced to change the performance of the booking-limit control decision model under different scenarios. The correlation analysis between the capacity limit level and the return under the optimal Booking-limit decision, and the above two flexibility parameters are obtained.

**Key words:** flexible inventory control; capacity parameter; booking-limit control decision model; revenue management

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## 0 Introduction

Driven by demand, airlines can quickly adjust the seat capacity of planned flights based on ticket bookings. For example, some major airlines in North America can achieve capacity adjustment by redistributing models within two weeks before the flight takes off. ULM Airlines can change the flight capacity by changing the model within 14—30 days before departure. This is the practical application of joint decision-making between flexible cabin control and cabin reservation control. If a larger aircraft is allocated after the model is redistributed, the operating cost will increase. If assigned to a smaller aircraft, the operating costs will be reduced.

The most widely used booking limits in the industry are the number of products that can be sold at a specified price level, that is, the booking-limit set for each price level. Once the number of products sold exceeds the predetermined limit, the price level is closed. It will no longer be sold at this price level. However, the process of making decisions based on booking-limit decision variables follows the corresponding decision-making principles. The decision-making principle consists of two levels: (1) The sale of a certain type of product is based on whether the quantity of the product has been sold or based on the time of sale or both. (2) The decision time point is a one-time decision made before the start of the sales cycle, or whether the sales process makes continuous decisions based on actual conditions. Conventional reservation limits generally refer to the number of seats that can be predetermined (sold) for any class of class, and are typically used in revenue management to allocate the appropriate capacity for each class of class for maximum benefit. For air-

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lines, traditional revenue management always considers that the flight capacity is fixed. Therefore, when the booking-limit decision of revenue management is implemented, the total capacity is used, and the capacity control of the capacity is adjusted only between the sub-tanks of each level.

This article looks at the issue of seat reservation restrictions for airlines with adjustable flight capacity. The main study is the control effect of the use of the booking-limit(the capacity protection). In this type of flexible capacity control, the adjustment trend of the capacity increase or decrease is flexible, and the effect of the booking-limit level and the use of the predetermined limit.

In the traditional income management study, once the calculation of the class capacity of each class is determined, the cabin control is usually carried out according to the calculated number. However, in actual operation, due to changes in the market situation, airlines must adjust their cabin capacity in real time, that is, the cabin control has a certain degree of capacity flexibility. Ref.[1] proposed that capacity control is a revenue management tool that can be dominated by the airline. Ref.[2] proposed that the seat reservation limit is the core of traditional revenue management, and it is applied to many aviation industries such as civil aviation, hotels, car rental, etc., adding hundreds of millions of dollars to it. The aviation industry has long used predetermined limits to achieve capacity control between different levels of fare. According to the recommendation of the scholar Post[3], Germanwings launched a blind-scheduled project, which was recognized by some consumers because of its low price and freshness. Both Ref.[4] and Ref.[5] have stated in the study that airlines can achieve flexible cabin control through a demand-driven scheduling. In the air cargo industry, Ref.[6] studied in 2008 that freight forwarders obtained most of the airline's freight capacity (air cargo space) through distribution contracts, and then sold freight capacity to customers with transportation needs at higher freight rates. Repatriate excess shipping capacity to the airline before the allowed time node. Refs.[7-8] considered the variable size of the seat capacity in the context of revenue management. Ref.[9] considered the company's decision on capacity, capacity and pricing at different time points, and verified the flexibility of the mix (i.e., the ability to adjust the production mix) and time flexibility (i.e., the latest) The impact of the ability to deliver the product at the time)

## 1 Model

Assuming that there are two grades of passengers on the flight, the low price demand and the high price demand are independent of each other and the low price passenger's reservation demand precedes the high price passenger(Fig.1).

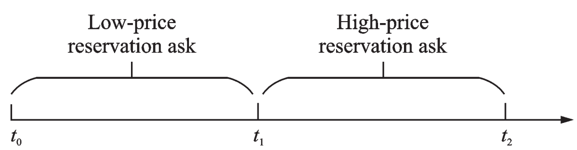



Fig.1 Booking horizon

At the fixed time node of the sales period, the airline can carry out the flight capacity according to the realized demand. Increased capacity will result in increased costs, and reduced costs will result in savings. The booking limit for low-cost passengers is determined prior to the start of the sales period. This article assumes that the oversold, no-show, and scheduled cancellations are not considered. Based on the above assumptions, we study that how the rise and fall of flexibility affect the predetermined limit level and its expected benefit level.

### 1.1 Parameter setting

Parameter setting is based on the above assumptions, as shown in Table 1.

### 1.2 Functions

Construct the objective Eq.(1) with the goal of maximizing the expected return of the airline. The total number of capacity adjustments can be expressed as Eq.(2)

$$\pi = \max_{v \geqslant 0} \mathrm{EXP}_{D_L}[\, p_L \min(D_L, v) + w(u - \min(D_L, v))\,] \tag{1}$$

$$m = \max(\min(D_H, x + \gamma u) - x, -\eta u) \tag{2}$$

**Table 1 Parameter setting**

| Parameter | Meaning |
|---|---|
| $p_H$ | Fares for high-priced passengers |
| $p_L$ | Fares for low-cost passengers |
| $t_0$ | The time when the reservation period begins |
| $t_1$ | The moment when low-cost passengers stop booking and high-priced passengers start booking |
| $t_2$ | Pre-periodical stop time |
| $D_L$ | Demand for low-priced tickets |
| $F_L(\cdot)$ | Demand accumulation function of low-priced tickets |
| $F_L(\cdot)$ | Demand density function for low-priced tickets |
| $D_H$ | High-priced ticket demand |
| $F_H(\cdot)$ | Demand accumulation function of high-priced tickets |
| $f_H(\cdot)$ | Demand density function for high-priced tickets |
| $u$ | Original number of seats on the flight |
| $m$ | Increase or decrease in the number of cabin capacities |
| $\pi^*$ | Maximum expected return |
| $w(x)$ | Expected return on high-priced seat capacity remaining $x$ after the moment $t_1$ |
| $v$ | Number of seats available for low-cost passengers |
| $v^*$ | The optimal number of seats that can be sold to low-cost passengers (optimal booking-limit level) |
| $v_b^*$ | Optimal value of $v$ when there is no constraint at the time point $t_2$ (optimal booking-limit level) |
| $\pi(v)$ | Expected revenue as a function of the predetermined number of seats reserved for low-price |
| $\Delta\pi$ | The difference between the expected return under the booking-limit and the expected return under the first come first service (FCFS) |
| $\pi^f$ | Expected benefits of FCFS decisions |
| $r$ | Maximum capacity increase rate |
| $\eta$ | Maximum capacity reduction rate |
| $c$ | Increase of the unit cost of one seat when making capacity adjustments |
| $s$ | When making capacity adjustments, the reduction of the unit savings of one seat |

1.$p_L<p_H$; $0\leqslant\eta\leqslant 1$ ; $c\leqslant p_L$; $c\geqslant s$; $v\geqslant 0$; $c\leqslant p_H$; $v^*=\max\left(v_b^*,0\right)$. 2. $\gamma,\eta,s,c$ are flexibility parameters.

Eq.(3) is the expected return of the remaining high-priced seat capacity $x$ at time $t_1$.

$$w(x)=\mathrm{EXP}_{D_H}[\,p_H\min(D_H,x+\gamma u)-c\cdot\max(m,0)+s\cdot\max(-m,0)\,] \tag{3}$$

The goal of the airline in this paper is to maximize the expected return by finding the optimal $v$ value. And the optimal booking-limit level is $v^*=(u-\lambda^*)^+$ obtained from Eqs.(4) and (5). Thus the optimal expected return Eq.(6) based on $v^*$ can be obtained

$$w'(x)=p_H\int_{x+\gamma u}^{\infty}f_H(\delta_H)\mathrm{d}\delta_H+c\int_{x}^{x+\gamma u}f_H(\delta_H)\mathrm{d}\delta_H+s\int_{x-\eta u}^{x}f_H(\delta_H)\mathrm{d}\delta_H \tag{4}$$

$$\pi'(v)=(1-F_H(\delta))(p_L-w'(u-v))$$
$$p_L=w'(\lambda^*)$$
$$v^*=(u-\lambda^*)^+$$
$$p_L=w'(\lambda^*) \tag{5}$$

$$\pi^*=\mathrm{EXP}_{D_L}[\,p_L\min(D_L,v^*)+w(u-\min(D_L,v^*))\,] \tag{6}$$

The time node of capacity adjustment is shown in Fig.2.

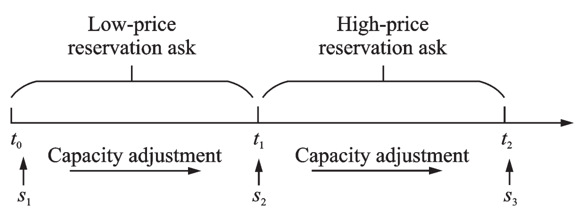



Fig.2 Capacity adjustment scenarios

In order to study the increase in revenue after using the booking-limit control decision, this paper compares it with the benefits of FCFS decision. And calculate $\Delta\pi$ of Eqs.(7) and (8). It can be seen in Eq.(8) that the benefit advantage of using the booking-limit control depends on the marginal benefit $w'(x)$ and the optimal booking-limit value $v^*$, both of which are related to the adjustment trend of increase or decrease.

$$\pi^{\mathrm{f}} = \mathrm{EXP}_{D_{\mathrm{L}}}[\, p_{\mathrm{L}} \min(D_{\mathrm{L}}, u+\gamma u) + w(u - \min(D_{\mathrm{L}}, u+\gamma u))] \tag{7}$$

$$\Delta\pi = \pi^* - \pi^{\mathrm{f}} = \mathrm{EXP}_{D_{\mathrm{L}}}\left[\int_{u-\min(D_{\mathrm{L}},v+\gamma u)}^{u-\min(D_{\mathrm{L}},v^*)} (w'(x) - p_{\mathrm{L}})\mathrm{d}x\right] \tag{8}$$

Eqs. (9) and (10) prove that $0 \leqslant \frac{\partial v_b^*}{\partial \gamma} \leqslant u$, $-u \leqslant \frac{\partial v_b^*}{\partial \eta} \leqslant 0$, $v^* = \max(v_b^*, 0)$, $v^*$ increases as $\gamma$ increases, and decreases as $\eta$, $c$, $s$ increase. It can be seen from the above proof that it is desirable to increase the degree of uplink flexibility by increasing the $\gamma$ value and decreasing the value of $c$. Similarly, increasing the degree of downlink flexibility, that is, reducing the optimal predetermined limit, can be achieved by increasing $\eta$ and $s$. On the basis of having a higher degree of uplink flexibility (i.e., $\gamma$ increases, or $c$ reduces), the flight has a strong capacity increase capability, and the ability to meet the demand of high-priced passengers is enhanced. From the perspective of marginal revenue, the marginal revenue of the unit that rejects low-priced passengers is guaranteed to protect the demand of high-priced passengers, which in turn leads to an increase in the optimal booking-limit $v^*$. Similarly, on the basis of a higher degree of downside flexibility (i.e., an increase in $\eta$ or $s$), the ability of a flight to reduce excess capacity increases, so that the marginal expected return of unit capacity reserved to protect the demand of high-priced passengers increases, which in turn leads to the reduction of the optimal booking-limit $v^*$

$$\begin{aligned}
\Phi &= f_{\mathrm{H}}((1+\gamma)u - v)\cdot(p_{\mathrm{H}} - c) + (c-s)\cdot f_{\mathrm{H}}(u-v) + s\cdot f_{\mathrm{H}}((1-\eta)u - v)\\
A &= f_{\mathrm{H}}((1+\gamma)u - v)\cdot(p_{\mathrm{H}} - c)\\
B &= (c-s)\cdot f_{\mathrm{H}}(u-v)\\
\Gamma &= s\cdot f_{\mathrm{H}}((1-\eta)u - v)\\
\Phi &= A + B + \Gamma
\end{aligned} \tag{9}$$

$$\frac{\partial v_b^*}{\partial \gamma} = \frac{A}{\Phi} \qquad \frac{\partial v_b^*}{\partial \eta} = \frac{-\Gamma}{\Phi} \tag{10}$$

Considering the relationship between the profit advantage of the booking-limit of the predetermined capacity and the flexibility coefficient in Eqs. (11) and (12), the sensitivity test method is used to explore the influence of various factors on the profit advantage. Prove that there is a positive number $L$ such that when $u+\gamma u \leqslant L$, $\Delta\pi$ increases with the increasing $\gamma$

$$\Delta\pi = \int_{v^*}^{(1+\gamma)u} f_{\mathrm{L}}(\delta_{\mathrm{L}})\mathrm{d}\delta_{\mathrm{L}} \int_{u-\delta_{\mathrm{L}}}^{u-v^*} (w'(x) - p_{\mathrm{L}})\mathrm{d}x + s\int_{(1+\gamma)u}^{\infty} f_{\mathrm{L}}(\delta_{\mathrm{L}})\mathrm{d}\delta_{\mathrm{L}} \int_{-\gamma u}^{u-v^*} (w'(x) - p_{\mathrm{L}})\mathrm{d}x \tag{11}$$

$$\begin{aligned}
\frac{\partial \Delta\pi}{\partial \gamma} = -u\cdot\Big\{&(p_{\mathrm{H}} - c)\int_{v^*}^{(1+\gamma)u} f_{\mathrm{L}}(\delta_{\mathrm{L}})\mathrm{d}\delta_{\mathrm{L}} \int_{(1+\gamma)u-\delta_{\mathrm{L}}}^{(1+\gamma)u-v^*} \frac{\partial w'(x)}{\partial \gamma} f_{\mathrm{H}}(\delta_{\mathrm{H}}) d\delta_{\mathrm{H}} +\\
&(1 - F_{\mathrm{H}}((1+\gamma)u))\cdot(p_{\mathrm{H}} - p_{\mathrm{L}}) + (p_{\mathrm{H}} - c)\cdot\\
&(1 - F_{\mathrm{L}}((1+\gamma)u))\cdot F_{\mathrm{H}}((1+\gamma)u - v^*)\Big\}
\end{aligned} \tag{12}$$

Similarly, for $w'(x)$, the partial derivative of the parameters $\eta$, $c$, $s$ can be obtained in Eq.(13)

$$\begin{aligned}
\frac{\partial w'(x)}{\partial \eta} &= s\cdot u\cdot f_{\mathrm{H}}(x - \eta x) & \frac{\partial w'(x)}{\partial \eta} &\geqslant 0\\
\frac{\partial w'(x)}{\partial c} &= \int_{x}^{x+\gamma u} f_{\mathrm{H}}(\delta_{\mathrm{H}})\mathrm{d}\delta_{\mathrm{H}} & \frac{\partial w'(x)}{\partial c} &\geqslant 0\\
\frac{\partial w'(x)}{\partial s} &= \int_{x-\eta u}^{x} f_{\mathrm{H}}(\delta_{\mathrm{H}})\mathrm{d}\delta_{\mathrm{H}} & \frac{\partial w'(x)}{\partial s} &\geqslant 0
\end{aligned} \tag{13}$$

And because $(w'(u-v^*) - p_{\mathrm{L}})\frac{\partial v^*}{\partial \gamma} = 0$, we can obtain $(w'(u-v^*) - p_{\mathrm{L}})\frac{\partial v^*}{\partial \eta} = 0$, $(w'(u-v^*) - p_{\mathrm{L}})\frac{\partial v^*}{\partial c} = 0$, $(w'(u-v^*) - p_{\mathrm{L}})\frac{\partial v^*}{\partial s} = 0$, thus it can derive Eq.(14). $\frac{\partial \Delta\pi}{\partial \eta}$, $\frac{\partial \Delta\pi}{\partial c}$, $\frac{\partial \Delta\pi}{\partial s}$ are both greater than 0. In summary, it can be concluded that if there is a positive $M$ such that the probability of $D_{\mathrm{L}} > M$ is 0, then $u^+$ When $\gamma u \geqslant M$, $\Delta\pi$ decreases as $\gamma$ increases. $\Delta\pi$ increases as $\eta$, $c$ and $s$ increase.

$$\begin{aligned}
\frac{\partial \Delta\pi}{\partial \eta} &= \int_{\delta_L}^{u+\gamma u} f_L(\delta_L)\mathrm{d}\delta_H \int_{x=u-\delta_L}^{u-v^*} \frac{\partial w'(x)}{\partial \eta}\mathrm{d}x + \int_{\delta_L=u+\gamma u}^{\infty} f_L(\delta_L)\mathrm{d}\delta_L \int_{x=-\gamma u}^{u-v^*} \frac{\partial w'(x)}{\partial \eta}\mathrm{d}x \\
\frac{\partial \Delta\pi}{\partial c} &= \int_{\delta_L=v^*}^{u+\gamma u} f_L(\delta_L)\mathrm{d}\delta_L \int_{x=u-\delta_L}^{u-v^*} \frac{\partial w'(x)}{\partial c}\mathrm{d}x + \int_{\delta_L=u+\gamma u}^{\infty} f_L(\delta_L)\mathrm{d}\delta_L \int_{x=-\gamma u}^{u-v^*} \frac{\partial w'(x)}{\partial c}\mathrm{d}x \\
\frac{\partial \Delta\pi}{\partial s} &= \int_{\delta_L=v^*}^{u+\gamma u} f_L(\delta_L)\mathrm{d}\delta_L \int_{x=u-\delta_L}^{u-v^*} \frac{\partial w'(x)}{\partial s}\mathrm{d}x + \int_{\delta_L=u+\gamma u}^{\infty} f_L(\delta_L)\mathrm{d}\delta_L \int_{x=-\gamma u}^{u-v^*} \frac{\partial w'(x)}{\partial s}\mathrm{d}x
\end{aligned} \tag{14}$$

For the advantage of using the booking-limit control $\Delta\pi$, this paper points out that $\Delta\pi$ increases or decreases with respect to the parameter $\gamma$ depending on the upper bound of the capacity $u+\gamma u$, and $\Delta\pi$ increases with respect to $\gamma$ within a certain range. The analysis $\Delta\pi$ increases with respect to the parameters $\eta$, $s$, and $c$. When $\eta$, $s$, or $c$ increases, the optimal booking-limit decreases, which means that more low-priced passengers' reservation requests are rejected, and the FCFS decision control is low. The price of passengers has not decreased. Rejecting the seats vacated by low-cost passengers can be used to protect the demand of high-priced passengers, which will result in higher expected returns, resulting in an increase in $\Delta\pi$.

Eqs.(11) — (14) results show that when achieving a higher degree of downlink flexibility, the yield advantage $\Delta\pi$ using the booking-limit control is always incremented. In the case of achieving a higher degree of uplink flexibility, there are two cases of increasing and decreasing using the profit advantage of the booking-limit $\Delta\pi$. In particular, when the capacity upper limit is lower than the smaller level, the ability to increase the capacity adjustment becomes stronger, and the influence of the reservation limit control income is more obvious. The relationship between the reservation restriction control and the uplink flexibility and the downlink flexibility is that the reservation restriction control and the downlink flexibility are complementary, and the reservation restriction control and the uplink flexibility are alternative or complementary.

Eqs. (15) — (17) study the relationship between $\pi^*$ and the flexibility parameters $\gamma$ and $\eta$. It is necessary to use $\pi^*$ to find the second-order partial derivatives for the four parameters. $\pi^*$ is a concave function about the parameters $\gamma$ and $\eta$. $\frac{\partial^2\pi^*}{\partial\eta^2}\leqslant 0$ and $\frac{\partial^2\pi^*}{\partial\gamma^2}\leqslant 0$.

$$\begin{aligned}
Q(\delta) = &\int_{\delta_L=0}^{v^*} \mathrm{d}F_L(\delta_L) \int_{\delta_H=u+\gamma u-\delta_L}^{\infty} f_H(\delta_H)\mathrm{d}\delta_H + \\
&\int_{\delta_L=v^*}^{\infty} \mathrm{d}F_L(\delta_L) \int_{\delta_H=u+\gamma u-v^*}^{\infty} f_H(\delta_H)\mathrm{d}\delta_H
\end{aligned} \tag{15}$$

$$\begin{aligned}
\frac{\partial\pi^*}{\partial\gamma} &= E_L(v^*)(p_L - w'(u-v^*))\cdot \\
&\quad \frac{\partial\pi^*}{\partial\gamma} + Q(\delta)\cdot(p_H - c)u \\
\frac{\partial\pi^*}{\partial\eta} &= s\cdot u\cdot\Bigg\{\int_{\delta_L=0}^{v^*}\int_{\delta_H}^{u-\eta u-\delta_L} f_L(\delta_L)\mathrm{d}\delta_L\mathrm{d}\delta_H + \\
&\quad \int_{\delta_L=v^*}^{\infty}\int_{\delta_H=0}^{u-\eta u-\delta_L} f_H(\delta_H)f_L(\delta_L)\mathrm{d}\delta_L\mathrm{d}\delta_H + \\
&\quad E_L^{-1}(v^*)(p_L - w'(u-v^*))\frac{\partial v^*}{\partial\eta}
\end{aligned} \tag{16}$$

$$\begin{aligned}
\frac{\partial^2\pi^*}{\partial\gamma^2} &= -(p_H-c)u\Bigg\{\int_{\delta_L=0}^{v^*} uf_H(u+\gamma u-\delta_L)\mathrm{d}F_L(\delta_L) + \\
&\quad \int_{\delta_L=v^*}^{\infty}(u-\frac{\partial v^*}{\partial\gamma})f_H(u+\gamma u-v^*)\mathrm{d}F_L(\delta_L)\Bigg\} \\
\frac{\partial^2\pi^*}{\partial\eta^2} &= -su\Bigg\{\int_{\delta_L=0}^{v^*} f_H(u-\eta u-\delta_L)uf_L(\delta_L)\mathrm{d}\delta_L + \\
&\quad \int_{\delta_L=v^*}^{\infty} f_H(u-\eta u-v^*)(u+\frac{\partial\pi^*}{\partial\eta})f_L(\delta_L)\mathrm{d}\delta_L\Bigg\}
\end{aligned} \tag{17}$$

Eq. (18) shows that $\pi^*$ is a convex function with respect to the parameters $s$ and $c$, respectively. That is, $\frac{\partial^2\pi^*}{\partial s^2}\geqslant 0$, $\frac{\partial^2\pi^*}{\partial c^2}\geqslant 0$. Similarly Eq. (19) can prove that $\frac{\partial^2\pi^*}{\partial\gamma\partial\eta}\leqslant 0$ and $\frac{\partial^2\pi^*}{\partial c\partial s}\geqslant 0$.

$$\begin{aligned}
\frac{\partial^2\pi^*}{\partial c^2} &= -E_L^{-1}(v^*)\frac{\partial v^*}{\partial c}\int_{u-v^*}^{u+\eta u-v^*} f_H(\delta_H)\mathrm{d}\delta_H \\
\frac{\partial^2\pi^*}{\partial s^2} &= -E_L^{-1}(v^*)\frac{\partial v^*}{\partial s}\int_{u-\eta u-v^*}^{u-v^*} f_H(\delta_H)\mathrm{d}\delta_H
\end{aligned} \tag{18}$$

$$\begin{aligned}
\frac{\partial^2\pi^*}{\partial\gamma\partial\eta} &= -s\cdot u\cdot\frac{\partial v^*}{\partial\gamma}E_L^{-1}(v^*)\cdot f_H((1-\eta)u-v^*) \\
\frac{\partial^2\pi^*}{\partial c\partial s} &= -E_L^{-1}(v^*)\frac{\partial v^*}{\partial s}\int_{u-v^*}^{u+\gamma u-v^*} f_H(\delta_H)\mathrm{d}\delta_H
\end{aligned} \tag{19}$$

In order to further compare the relationship between the expected benefit and the marginal utility of the flexible parameters of flexible capacity control decision and FCFS decision, the effect of the benefit $\pi^{\mathrm{N}}$ on the flexible parameter marginal utility without using the flexible booking-limit control decision is also considered here. And Eq. (19) can prove that $\frac{\partial^2 \pi^{\mathrm{N}}}{\partial\gamma\partial\eta}=0$ and $\frac{\partial^2 \pi^{\mathrm{N}}}{\partial s\partial c}=0$. That is, in the FCFS decision, the marginal utilities of the capacity parameters $\gamma$ and $\eta$ are independent of each other, and the marginal utilities of the capacity parameters $s$ and $c$ are independent of each other (Eq.(20))

$$\begin{aligned}\frac{\partial \pi^{\mathrm{N}}}{\partial\eta} &= -s\cdot u\cdot\int_{\delta_{\mathrm{L}}=0}^{u-\eta u} f_{\mathrm{L}}(\delta_{\mathrm{L}})\mathrm{d}\delta_{\mathrm{L}}\cdot \\ &\quad \int_{\delta_{\mathrm{H}}=0}^{u-\eta u-\delta_{\mathrm{L}}} f_{\mathrm{H}}(\delta_{\mathrm{H}})\mathrm{d}\delta_{\mathrm{H}} \\ \frac{\partial^2 \pi^{\mathrm{N}}}{\partial\gamma\partial\eta} &= \frac{\partial}{\partial\gamma}\Big[s\cdot u\cdot\int_{\delta_{\mathrm{L}}=0}^{u-\eta u} f_{\mathrm{L}}(\delta_{\mathrm{L}})\mathrm{d}\delta_{\mathrm{L}}\cdot \\ &\quad \int_{\delta_{\mathrm{H}}=0}^{u-\eta u-\delta_{\mathrm{L}}} f_{\mathrm{H}}(\delta_{\mathrm{H}})\mathrm{d}\delta_{\mathrm{H}}\Big]\end{aligned} \tag{20}$$

This paper proves that when the capacity parameter is large (the capacity can be increased or decreased in a larger range), the growth rate of $\pi^*$ is decreasing, indicating that the increase of the flexible parameters $\gamma$ and $\eta$ will cause the revenue to decrease. At the same time, when the unit cost $c$ decreases and the unit saving $s$ increases, the growth rate of $\pi^*$ is increased, that is, optimizing the capacity adjustment behavior of $s$ and $c$ will increase the profit. In the FCFS decision, $c$ reduction and $s$ increase will not change the profit. In the booking-limit control decision, as $c$ decreases, the optimal reservation limit increases, thus meeting more low-price passenger demand, and at $t_2$ increasing the capacity at all time makes the further decrease of the $c$ value more favorable, that is, the return of the $c$ decrease is increasing. Similarly, as $s$ increases, the expected decrease in time $t_2$ increases (due to the reduction of the predetermined limit), which makes the extra of $s$ adding more value. Conversely, under FCFS decisions, the total amount of capacity expected to increase or decrease is independent of $c$ and $s$. Therefore, the expected return under FCFS decision is linear with $c$ and $s$.

## 2 Case Study

We consider an example shown in Table 2 to examine the impact of capacity flexibility.

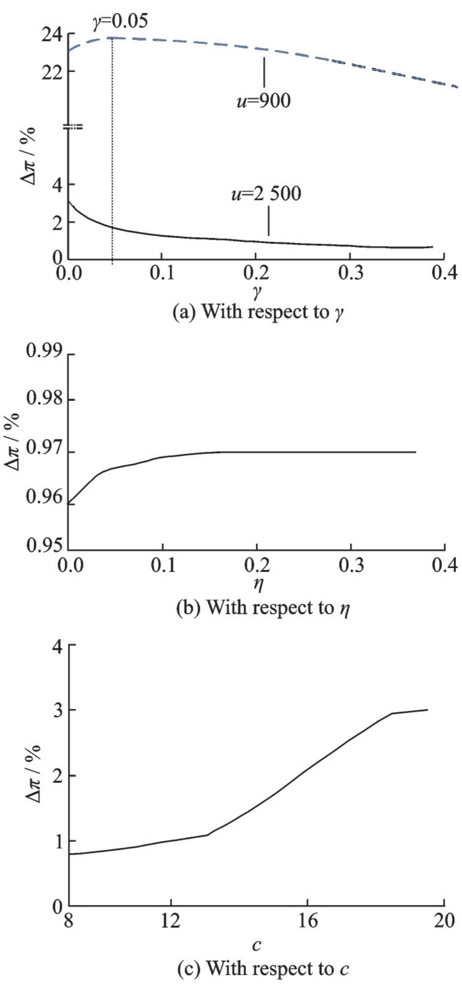
**Table 2 Function parameters of numercical example**

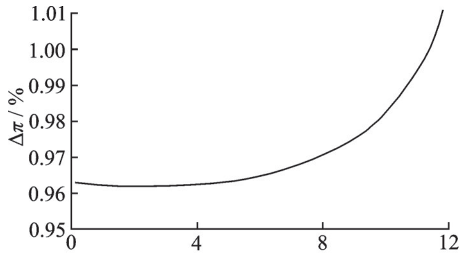
| Capacity $u$ | Flexibility parameter | | | | Ticket price | | |
|---|---|---|---|---|---|---|---|
| | $\gamma$ | $\eta$ | $c$ | $s$ | $p_1$ | $p_2$ | $p_3$ |
| 2 500 | 0.2 | 0.3 | 12 | 8 | 24 | 18 | 14 |

**Table 3 Constant parameters of numercical example**

| Time | Arrival probability | | | Capacity demand | | |
|---|---|---|---|---|---|---|
| $t_2$ | $\varphi_{t_1}$ | $\varphi_{t_2}$ | $\varphi_{t_3}$ | $d_1$ | $d_2$ | $d_3$ |
| 80 | 0.1 | 0.1 | 0.1 | $N(100, 30)$ | $N(100, 30)$ | $N(100, 30)$ |

In each experiment, we vary the value of one capacity flexibility parameter while holding the values of the other parameters constant, as shown in Fig.3.






(d) With respect to $s$

Fig.3 $\Delta\pi$ of booking-limit control as a function of capacity flexibility parameters $\gamma, \eta, c, s$

Based on this example (Table 2), we find that as upside flexibility increases (as $\gamma$ increases or $c$ decreases), booking-limit control can become more and less beneficial (Figs.3(a) and (b)), the percentage benefit of booking-limit control increases with respect to $\gamma$ when $u=900$ and $\gamma \leqslant 0.05$ (the dotted line in Fig.3(a)). As downside flexibility increases (as $\eta$ or $s$ increases), booking-limit control always becomes more beneficial (Figs.3(a)—(d)).

Regarding the value of upside and downside flexibility, it is shown that in this example, increasing upside flexibility through reducing unit cost exhibits increasing returns under booking-limit control regardless of whether $t$ is 50 or 100 (see Fig.4).

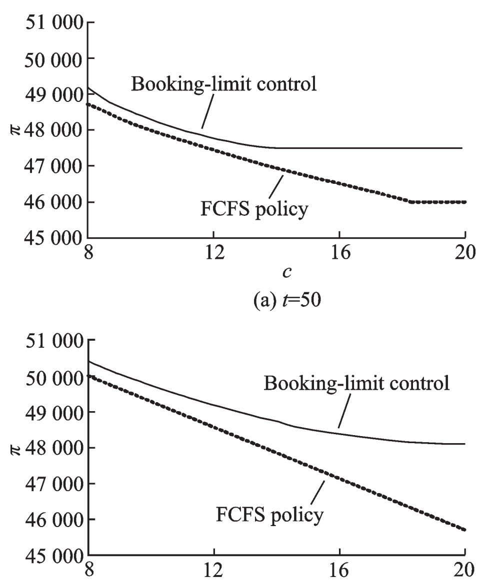



Fig.4 $\pi$ as a function of unit cost of adding capacity

Under the booking-limit control, the marginal value of upside flexibility extent $\gamma$ decreases with respect to downside flexibility extent $\eta$, and the marginal value of a reduction in unit cost $c$ decreases with respect to unit saving $s$ (see Fig.5).

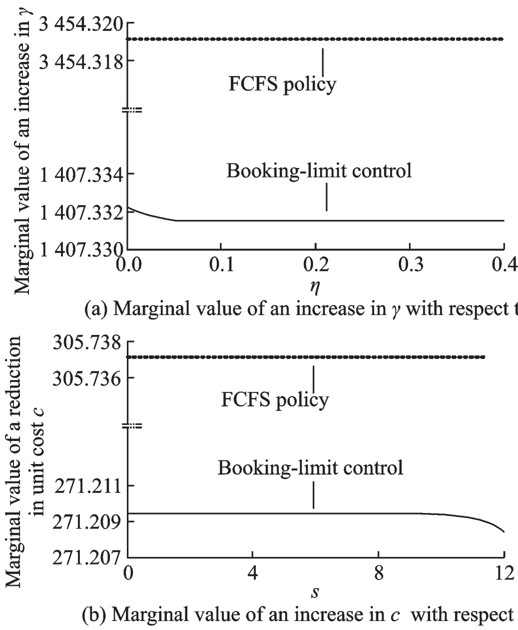



Fig.5 Marginal value of upside flexibility as a function of downside flexibility

We have also examined numerical examples with other parameter values, where the insights regarding the impact of capacity flexibility still hold.

Regarding the benefit of using booking-limit control over FCFS policy, Table 4 shows that the $\Delta\pi$ always decreases as capacity adjustment is postponed from $t_1$ to $t_2$, and always increases as the adjustment is postponed from $t_2$ to $t_3$. This is consistent with the finding. We have examined several other numerical examples and observed that the finding in equations always holds.

**Table 4 Results of numerical example**

| $u$ | Time | Booking-limit control | | $\pi^{\mathrm{f}}$ | $(\Delta\pi/\pi^{\mathrm{f}})/\%$ |
|---|---|---|---|---|---|
| | | $v^*$ | $\pi^*$ | | |
| 1 800 | $t_1$ | 394 | 28 260 | 25 014 | 12.98 |
| | $t_2$ | 394 | 28 260 | 25 224 | 12.04 |
| | $t_3$ | 373 | 29 786 | 25 357 | 17.47 |
| 2 500 | $t_1$ | 1 094 | 36 611 | 35 564 | 2.94 |
| | $t_2$ | 1 094 | 36 618 | 36 084 | 1.48 |
| | $t_3$ | 1 085 | 38 456 | 37 339 | 2.99 |
| 3 200 | $t_1$ | 1 724 | 43 651 | 43 551 | 0.23 |
| | $t_2$ | 1 794 | 44 092 | 44 059 | 0.07 |
| | $t_3$ | 1 808 | 46 194 | 46 150 | 0.10 |

## 3 Conclusions

Regarding the relationship between capacity uplink flexibility and downlink flexibility, it is pointed out in the foregoing that as $\eta$ increases, the marginal value of $\gamma$ decreases, and as the unit cost saving $s$ increases, the marginal value of unit cost $c$ decreases. It shows that when the flight has strong downlink flexibility, the marginal value of increasing the uplink flexibility ($\gamma$ increases or $c$ decreases) will decrease. Therefore, it can be considered that the uplink flexibility and the downlink flexibility are mutually substitutable. We find that the booking-limit control decision makes the uplink flexibility and downlink flexibility related. When the capacity downlink flexibility increases ($\eta$ or $s$ increases), the optimal booking-limit is reduced, resulting in an increase in rejected low-cost passenger reservation requests, thereby making the capacity ceiling expansion capability smaller, and thus increasing the uplink flexibility (increasing $\gamma$) or reducing the value of $c$. Therefore, under the FCFS decision, the marginal value of the uplink flexible increase is independent of the downlink flexibility due to no predetermined limit. The establishment of capacity flexibility will generate corresponding costs, so the value evaluation of capacity flexibility is also an important issue worthy of study. The above studies show that the booking-limit affects the marginal value of capacity up-and-down flexibility and down-line flexibility. When using the predetermined limits, the revenue can be increased by increasing $c$ and s to increase flexibility. At the same time, uplink flexibility and downlink flexibility can be substituted for each other in terms of improving revenue, that is, when one of the flexibility is increased, the other flexibility will be weakened.

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# 容量参数对柔性库存控制决策模型的影响

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**摘要**：灵活舱位控制模型中关于灵活度参数的选取均基于假设进行选取，从数理角度对容量参数选取合理性给予证明使得模型更符合实际问题情景。当存在需求预测误差或需求信息部分缺失时，合理的灵活度参数能够主动应对需求波动，规避被动依赖需求预测设定不可变舱位容量所带来的风险。本文主要研究总容量可变的灵活舱位控制下，容量参数的设置对预定限制决策及其收益的影响。通过灵活舱位控制模型中的容量增加灵活度、容量减少灵活度两个变化趋势，引入数理证明及边际效用的方法对预定限制舱位控制决策模型在不同情景下的性能变化。得出最优预定限制决策下的容量限制水平及收益与上述两种灵活度参数的相关性分析。

**关键词**：灵活舱位控制；容量参数；预定限制决策模型；收益管理