Multistable Mechanical Metamaterials: A Brief Review

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Abstract: Over the past decade, multistable mechanical metamaterials have been widely investigated because of their novel shape reconfigurability and programmable energy landscape. The ability to reversibly reshape among diverse stable states with different energy levels represents the most important feature of the multistable mechanical metamaterials. We summarize main design strategies of multistable mechanical metamaterials, including those based on self-assembly scheme, snap-through instability, structured mechanism and geometrical frustration, with a focus on the number and controllability of accessible stable states. Then we concentrate on unusual mechanical properties of these multistable mechanical metamaterials, and present their applications in a wide range of areas, including tunable electromagnetic devices, actuators, robotics, and mechanical logic gates. Finally, we discuss remaining challenges and open opportunities of designs and applications of multistable mechanical metamaterials.

Key words: multistable mechanical metamaterials; self-assembly; snap-through; structured mechanism; geometrical frustration

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0 Introduction

Multistable mechanical metamaterials[1-30] are artificial structures with two or more different stable configurations that can be switched reversibly among each other. From the perspective of energy landscape, these stable configurations correspond to the minima of the system energy, and the energy barrier between different stable states confers the stability of multistable mechanical metamaterials at these configurations. Due to the need to be capable of switching among various geometric configurations, the multistable mechanical metamaterials mostly belong to the class of soft (or flexible) mechanical metamaterials with a high level of deformability. During the last decade, many intriguing systems of soft mechanical metamaterials[31-34] have been developed to offer unusual mechanical properties, such as J-shaped stress-strain curves, negative Poisson's ratios, negative thermal expansion and anisotropic swelling under large strains. Since many mechanical properties (e.g., phononic band gaps, stress-strain curves and Poisson's ratios) depend highly on the microstructure geometries of soft mechanical metamaterials, many researchers have investigated the tunability of these properties by mechanical loads, as well as by light, heat or magnetism. In many of these studies, the deformed configurations of mechanical metamaterials cannot be maintained after mechanical loads (or other stimuli) are removed, which sets certain limitations to practical applications. Developments of multistable mechanical metamaterials can overcome these limitations, since these metamaterials can maintain diverse stable configurations after removal of external loads/stimuli. As such, multistable mechanical metamaterials could significantly broaden the appli-

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cation fields of soft mechanical metamaterials, and enable novel developments of functional systems for energy absorption[17,19,22,35-37], logical operations[9,13-14,38-39], elastic wave control[13,15,40-46], soft robotics[47-55], and etc.

A diversity of multistable mechanical metamaterials[1,3-5,8,18,20-21,26,35-36,44] have been reported in the past decade. According to the different design strategies, these multistable mechanical metamaterials can be classified into four categories, including those based on the self-assembly scheme[4-5,20], snap-through instability[8,18,44], structured mechanism[21,35-36] and geometrical frustration[1,3,26], as shown in Fig.1. This paper aims to present an overview of state-of-art multistable mechanical metamaterials, covering these four important classes of de-

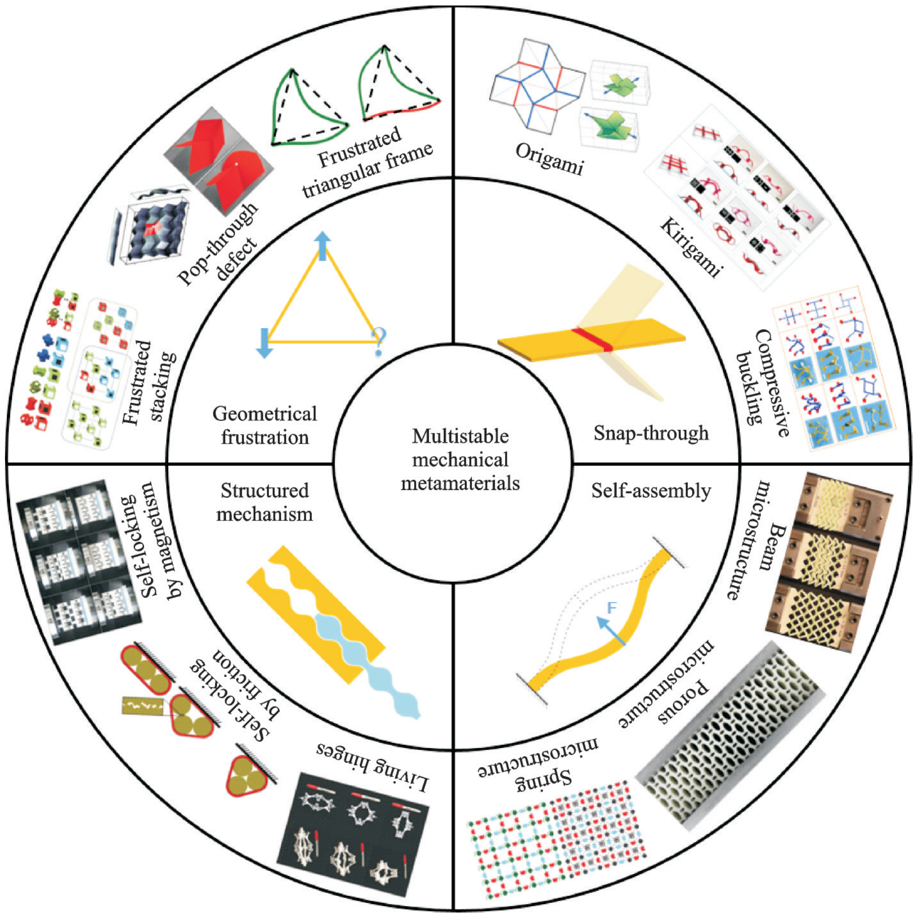


Fig.1 An overview of the four types for multistable mechanical metamaterials based on different design strategies. Self-assembly schemes usually rely on rolling, folding and buckling deformations of patterned 2D precursor structures to form well-controlled 3D architectures. Different loading paths could result in different 3D stable configurations for specially engineered 2D precursor structures. Snap-through instability occurs in the structure, when the minimum force of load-displacement curve becomes negative, providing another effective way for designing multistable mechanical metamaterials. Structural mechanism strategy relies on complex mechanical structures where self-locking usually occur by utilizing living hinges, frictional forces and magnetic forces. Geometrical frustration mainly refers to the use of geometrical constraints to break the propagation of the local order in lattice structures. The local "defect" in the lattice structure leads to a diversity of local deformations, resulting in the multistability. Adapted with permission from Ref.[4]. Copyright 2015, Springer Nature. Adapted with permission from Ref.[20]. Copyright 2020, Wiley-VCH. Adapted with permission from Ref. [5]. Copyright 2018, Springer Nature. Adapted with permission from Ref. [18]. Copyright 2015, Wiley-VCH. Adapted under the terms of the CC-BY 4.0 Creative Commons Attribution License from Ref.[8]. Copyright 2019, The Authors, published by Springer Nature. Adapted with permission from Ref. [44]. Copyright 2018, Elsevier. Adapted with permission from Ref.[21]. Copyright 2016, Wiley-VCH. Adapted with permission from Ref.[35]. Copyright 2019, Wiley-VCH. Adapted with permission from Ref.[36]. Copyright 2019, Elsevier. Adapted with permission from Ref.[3]. Copyright 2016, Springer Nature. Adapted with permission from Ref.[1]. Copyright 2017, AAAS. Adapted with permission from Ref.[26]. Copyright 2014, American Physical Society.

sign strategies. It begins with the discussions on the self-assembly scheme, followed by the introduction to snap-through instability, structured mechanism and geometrical frustration. Then, we present various applications of multistable mechanical metamaterials in soft robotics and mechanical devices Finally, we provide an outlook on the existing challenges and open opportunities.

1 Self-assembly Scheme

Self-assembly schemes usually rely on rolling, folding and buckling deformations of patterned 2D precursor structures to form well-controlled 3D architectures. In this process, different loading paths could result in different 3D stable configurations for specially engineered 2D precursor structures. This section focuses on discussions of three representative self-assembly schemes (including origami, kirigami and compressive buckling) that have been widely used to form 3D multistable mechanical metamaterials[2, 4-7, 10-12, 16, 20, 24, 25, 30, 38, 52, 54, 56-63].

1.1 Origami

With strategic designs of creases and folding deformations, a flat sheet can be transformed into 3D multistable metasurfaces. Fig.2(a1) demonstrates the configuration space of a foldable planar circular sheet and provides the schematics for folding motions[25]. The flat sheet here is composed of four rigid plates connected by four folds (marked by red, orange, purple and green in Fig.2(a1) with common endpoints. There is only one degree of freedom in the origami metamaterials, and the folding angles ρ_2, ρ_3 and ρ_4 can be represented by ρ_1, as shown in the top-right panel of Fig.2(a1). The stable configurations can be achieved by changing the folding angle (ρ_1). Differently, Fig. 2(a2) utilizes the square twist whose crease pattern has zero degree of freedom as the 2D precursor of multistable mechanical metamaterials[4]. The dark red and blue in Fig. 2(a2) represent the mountain and valley creases, respectively. By allowing bending deformations of facets, the square twist turns out to be foldable, and it offers bistability when the bending-to-crease energy ratio is large enough (i.e., $>$1 000). Through periodic tessellations of origami miltistable units (Fig. 2(a3)), a multistable metasurface that can be transformed into hyperbolic paraboloid is presented[10].

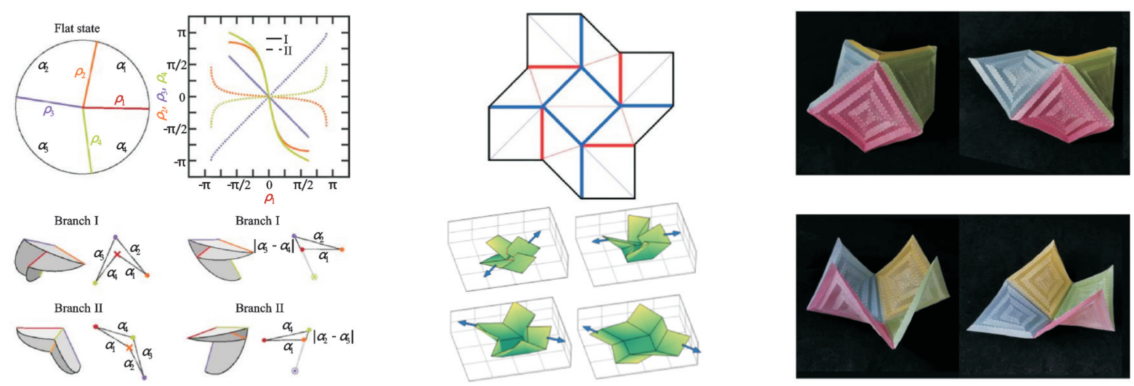
Origami techniques can be also utilized to design 3D multistable metamaterials with more complex geometric configurations (Fig.2(b)). Usually, a 3D multistable or monostable origami block is adopted as the building-block structure. Fig.2(b1) shows the stacking and bonding of translational periodic origami sheets (i.e., the classic Miura-ori) to form a space filling architecture[38]. The adjacent origami sheets are regarded as basic bistable unit cells that have stable convex and concave configurations. Instead of stacking the origami metasurface, Overvelde et al.[2] directly utilized space-filling tessellations of polyhedra to create 3D multistable mechanical metamaterials (Fig. 2(b2)). The cardboard and double-side tapes are used as the rigid face and flexible hinges, respectively. The prepared basic triangular, hexagonal, octahedral and cuboctahedra prims are also rigid. The tessellations of those prims determine the reconfigurability of architected metamaterials. Specially, the metamaterials in the top-right panel of Fig.2(b2) follow from a combination of octahedra and cuboctahedra, showing a completely rigid configuration. However, by combining the triangular and hexagonal prisms, the architected materials can be reconfigured by bending the edges, leading to two stable configurations. To enhance the multistability of such architected metamaterials, deformable prismatic structures are adopted as the basic building-block structures[11]. In Fig.2(b3), a multistable cuboctahedron is achieved by extruding the edges of the convex structure perpendicular to the faces. Then cubic tessellations of the prims are adopted to form multistable mechanical metamaterials with three additional stable states.

1.2 Kirigami

Kirigami represents another effective method to design multistable mechanical metamaterials. Here, the multistability mainly results from new deformation modes induced by geometrical cuts. Yang et al.[62] introduced periodic cuts in a flat sheet to form a stretchable metamaterial composed of repeating unit cells. Changing the kirigami patterns leads to different deformation modes, either with symmetric or antisymmetric deformed configurations. At the critical kirigami parameters, the structure of local unit cells is energetically metastable with two local minima, and the symmetric and antisymmetric states can coexist (Fig. 2(c1)). Except for the homogeneous sheet, "kirigami composites" were proposed to enable adaptive and shape-morphable composites[57], as well as achieving complicated shape change and mechanical property adaptations (Fig. 2 (c2)). The "zig-zag" pattern splits the composite paper into a row of kirigami module that consists of two patches with asymmetric fiber arrangements. The two stable configurations of the unit are shown in the bottom panel of Fig. 2(c2). Combining kirigami with origami represents another effective strategy to design multistable mechanical metamaterials. For example, Bobbert et al.[60] used kirigami cut pattern to create bistability in a flat sheet, and then folded them into deployable implants (Fig. 2(c3)). Due to the multistability, the deployable implants can switch freely between the compact retracted state and the deployed state, and maintain in a specified state, showing potential applications in orthopaedic surgeries.

1.3 Compressive buckling

Mechanically guided assembly of 3D structures are of growing interest due to their widespread applications in bio-integrated electronics and optoelectronic devices. This method provides a new design strategy of multistable mechanical metamaterials that have significant applications in reconfigurable and tunable devices. For the cross-shaped 2D ribbon precursor[30] shown in Fig. 2(d1), the simultaneous compression in x and y directions leads to a pop-up configuration (shape I) while the sequential com-

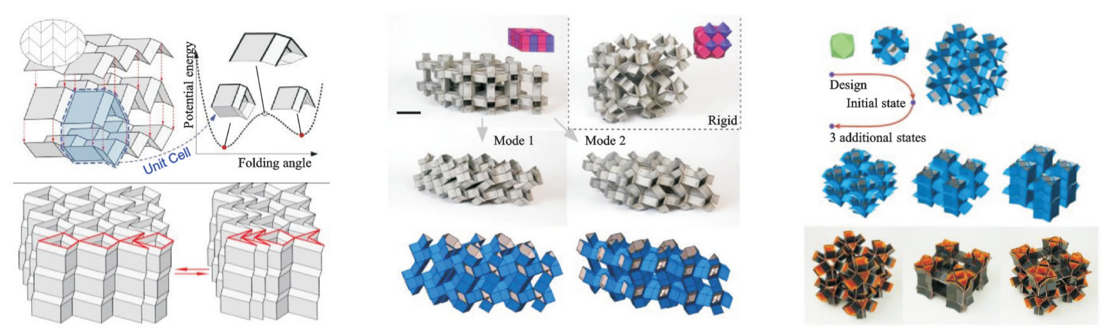


(a1) Origami with rigid facets. Adapted with permission from Ref. [25]. Copyright 2015, American Physical Society

(a2) Origami with non-rigid facets. Adapted with permission from Ref. [4]. Copyright 2015, Springer Nature

(a3) Tessellations of origami miltistable units. Adapted under the terms of the CC-BY 4.0 Creative Commons Attribution License from Ref. [10]. Copyright 2019, The Authors, published by Springer Nature

(a) Multistable metasurfaces realized by different strategies

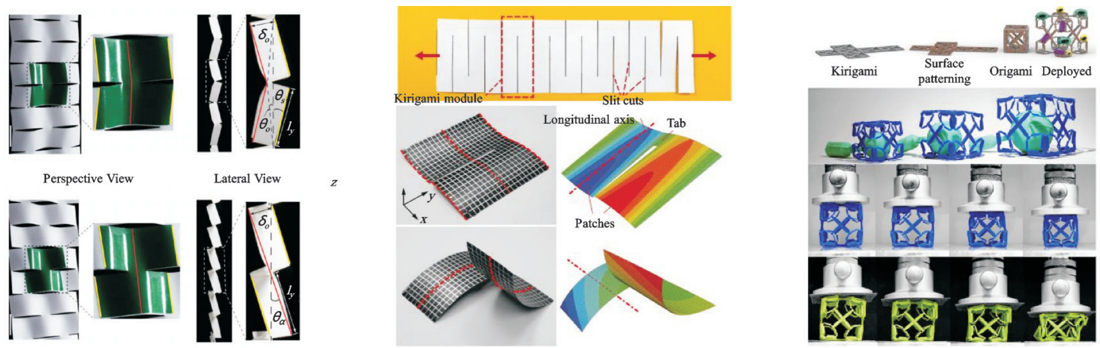


(b1) Periodical cellular solid by stacking and bonding of translational periodic origami sheets. Adapted with permission from Ref. [38]. Copyright 2017, Elsevier

(b2) 3D multistable mechanical metamaterial based on monostable. Adapted with permission from Ref. [2]. Copyright 2017, Springer Nature

(b3) 3D multistable mechanical metamaterial based on multistable origami building-block structures. Adapted under the terms of the CC-BY 4.0 Creative Commons Attribution License from Ref. [11]. Copyright 2019, The Authors, published by Springer Nature

(b) Multistable metamaterials with more complex geometric configurations

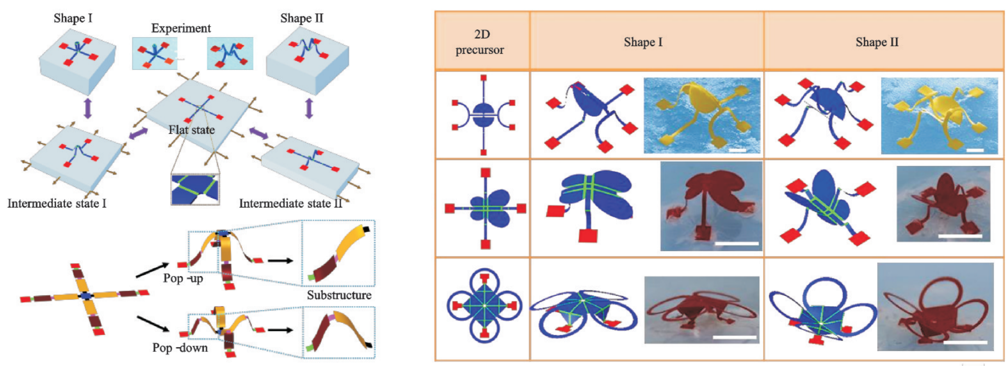


(c1) 2D multistable homogeneous sheets realized by periodical kirigam. Adapted with permission from Ref. [62]. Copyright 2018, American Physical Society

(c2) 2D multistable composite sheets realized by periodical kirigami. Adapted with permission from Ref. [57]. Copyright 2019, Elsevier

(c3) 3D multistable meta-biomaterials realized by the combination of kirigami and origami. Adapted under the terms of the CC-BY 4.0 Creative Commons Attribution License from Ref. [60]. Copyright 2020, The Authors, published by Elsevier

(c) 2D multistable sheets



(d1) Schematic illustration of the deformation mechanism.The top image in panel (d1) is adapted with permission from Ref.[5]. Copyright 2018, Springer Nature. The bottom image in panel (d1) is adapted with permission from Ref.[30]. Copyright 2019, Elsevier

(d2) Transformation of membrane-shaped 2D precursors into different 3D stable states. Adapted with permission from Ref.[5]. Copyright 2018, Springer Nature

(d) Multistable mechanical metamaterials based on mechanically guided assembly of 3D structures

Fig.2 Self-assembly scheme

pression (x direction first, then y) leads to a pop-down configuration (shape II). Luo et al.[30] established a finite-deformation model to analyze the stability of different buckling modes by using a perturbation method. Fig.2(d2) demonstrates a type of membrane-shaped 2D precursors that can be reconfigured into different stable states[5]. Here, shapes I and II correspond to simultaneous and sequential release, respectively. More interesting 3D multistable structures can be found in Ref. [5]. In addition to the translational displacements, Zhao et al.[12] utilized kirigami substrates to introduce local twisting deformations to form chiral 3D structures that cannot be realized by controlled compressive buckling.

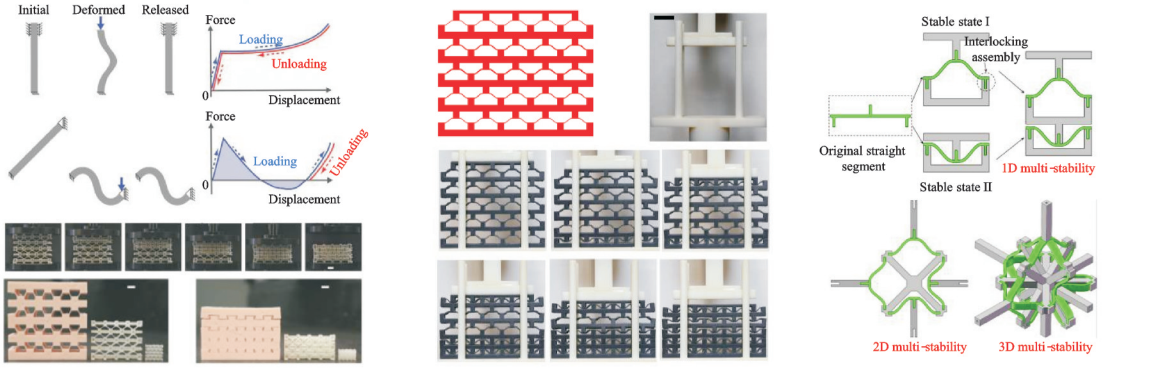
2 Snap-Through Instability

Negative stiffness structure is a type of specially engineered structure exhibiting increasing displacement with decreasing force, which is contrary to traditional structures. In the load-displacement curve of such structures, there are one or more minimum points, in addition to the starting point. Snap-through instability occurs in the structure, when the minimum force of load-displacement curve becomes negative, providing another effective way for designing multistable mechanical metamaterials[8-9,13-14,17-19,22,27-29,37,39,41-44,46,55,64-84].

Fig.3(a1) introduces a typical design of bistable negative stiffness structure by combining "<" shaped topology with beam microstructures[17]. When a vertical displacement is applied, the straight tilted elastic beam bends, leading to the drop of the force. As the displacement increases further, the curved beam gradually becomes straight, and the force starts to increase, as shown in the top panel of Fig.3(a1). Shan et al.[17] stacked the bistable units through direct ink writing to form multistable mechanical metamaterials for trapping elastic strain energy. The middle panel of Fig.3(a1) demonstrates the deformation process of the metamaterial, and the bottom panel shows fabricated structures at different length scales. However, due to the homogeneous design, it is difficult to control the deformation sequence of bistable units at each layer, leading to the uncertainty of deformed configurations. Che et al.[73] introduced small variations in the unit cell geometry (i.e., the width of inclined beams in "<" shaped topology) to obtain a deterministic deformation sequence for such multistable mechanical metamaterials. Specifically, in the top-left panel of Fig.3(b2), the beam widths of each layer from bottom to top are 0.90, 1.02, 1.08, 1.15 and 0.96 mm. When the metamaterial is under a vertical compressive loading, each layer snaps into another stable state, in the order from small widths to large widths. Note that many reported multistable mechanical metamaterials show multistability in only one direction (e.g., the vertical direction of 3D structures), referred to as 1D multistable mechanical metamaterials in this paper. Great challenges exist in preparing mechanical metamaterials with controllable multi-directional multistability due to the complex topology. Instead of stacking "<" shaped bistable units in the vertical direction, Fig.3(a3) shows an array of bistable units in 3D space and use rigid frames to fix and connect these bistable units[80], similar to the design proposed in Ref.[17]. The realized 2D/3D multistable mechanical metamaterials show various unusual mechanical properties, including robust shape-reconfigurability and zero Poisson's ratio at large deformations. Also, the deterministic deformation sequence is achieved by using a gradient design in the geometry or material. However, it is challenging to control the deformed configuration of each bistable unit precisely. The local instability could be easily triggered in such metamaterials under compressive loadings, due to the independent flexible deformations of each "<" shaped bistable unit, which represents certain limitations for applications. Experimental demonstrations of multistable mechanical metamaterials with numerous controllable stable states also remain challenging.

Based the "<" shaped topology, another two types of bistable units can be achieved by replacing the beam microstructure with membrane[14,19,37,83] or spring microstructures[43-44,78,81], as shown in Fig.3(b) and Fig.3(c1). For example, Pan et al.[19] utilized "flexible drinking straw" to construct a type of multistable mechanical metamaterials capable of shape-reconfigurability and programmability. As shown in Fig.3(b1), the metamaterial is designed by parallel multistable mechanical pixels, which are constructed with a hollow multistable structure (i.e., drinking straw, architected by a series of bistable units) and a guide bar. Similarly, Fig.3(b2) utilizes a bistable hemispherical membrane to separate two cylindrical chambers[14]. The pressure input (P_M) at the top of the cylindrical chamber controls the switch of the bistable membrane. By inserting two pathways for airflow, the bistable structure can be used as a soft valve, where the output P_{out} (i.e., $P_{in,1}$ or $P_{in,0}$) can be controlled by P_M. Fig.3(c1) shows 2D multistable metamaterials with spring microstructures[44]. Utilizing the nonlinear bistable springs, the frequency and direction of elastic wave propagation can be tuned by switching the metamaterial among different stable configurations. More reconfigurable phononic crystals[43] are realized by adopting "<" shaped bistable elements with spring microstructures.

Since the mechanical response of structured materials usually depends on the boundary conditions, porous materials[8,27-28] could show multistability by exploiting special confinements. For example, the confined biholar sheets[27] are exploited to design programmable mechanical metamaterials with different types of mechanical behavior. When compressed, the holes in sheet undergo large deformations and change from circles into ellipses. Fig.3(c2) shows the force-displacement curves of the sheet with horizontal confinement and presents several configurations marked on the curves. The negative force F_y suggests the existence of multistability in this confined biholar sheet. Furthermore, Zhang et al.[8] investigated the mechanical response of porous rubber metamaterials with a regular array of elliptic holes. By applying pre-compression to the

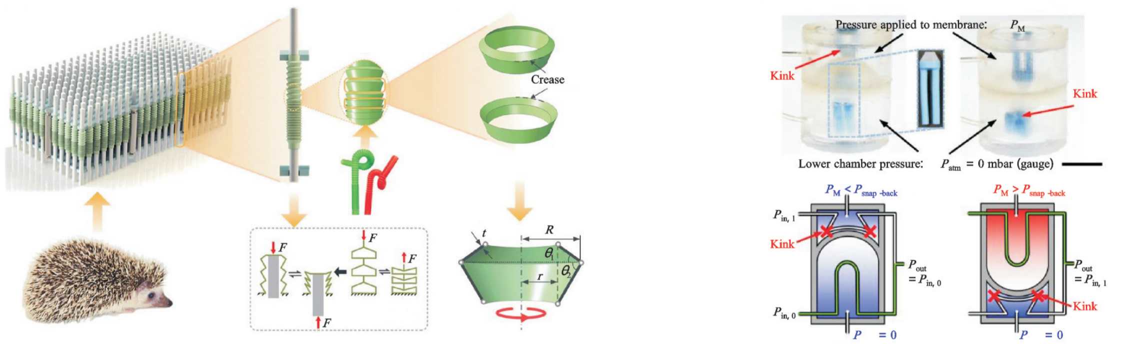


(a1) Schematics of bistable negative stiffness structure by combining "<" shaped topology with beam microstructures. Adapted with permission from Ref.[17]. Copyright 2015, Wiley-VCH

(a2) 1D multistable mechanical metamaterials with a deterministic deformation sequence. Adapted with permission from Ref.[73]. Copyright 2016, ASME

(a3) 3D metamaterials based on an array of bistable units in 3D space. Adapted under the terms of the CC-BY 4.0 Creative Commons Attribution License from Ref.[80]. Copyright 2020, The Authors, published by Elsevier

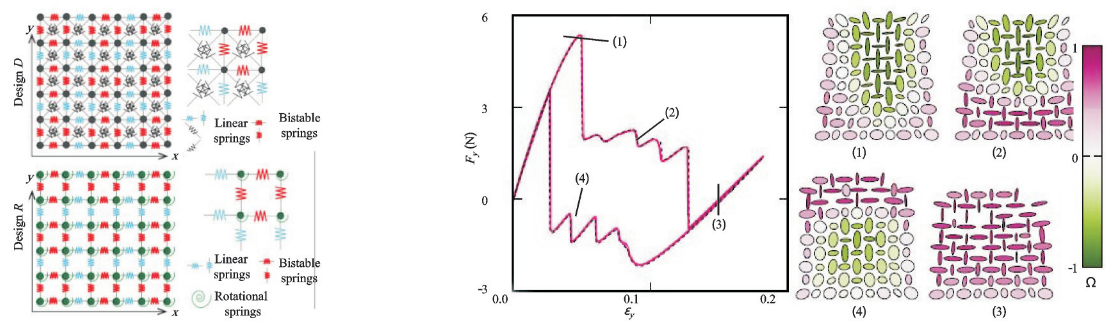
(a) 1D to 3D multistable mechanical metamaterials based on "<" shaped bistable units with beam microstructures



(b1) Pixel mechanical metamaterials by multistable drinking straw. Adapted with permission from Ref.[19]. Copyright 2019, Wiley-VCH

(b2) Soft pneumatic multistable valve by hemispherical membrane. Adapted with permission from Ref.[14]. Copyright 2019, National Academy of Sciences

(b) Multistable mechanical metamaterials based on "<" shaped bistable units with membranes microstructures



(c1) 2D multistable mechanical metamaterials with spring microstructures. Adapted with permission from Ref.[44]. Copyright 2018, Elsevier

(c2) 2D multistable mechanical metamaterials realized by confined biholar sheets. Adapted with permission from Ref.[27]. Copyright 2014, American Physical Society

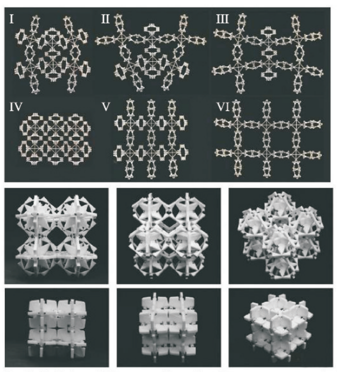
(c) Multistable mechanical metamaterials based on other negatives stiffness structures

Fig.3 Snap-through instability

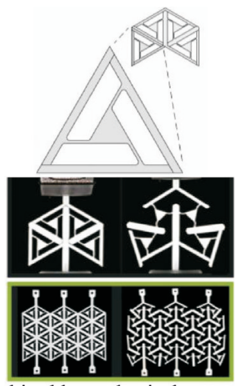
metamaterials in the vertical direction, the cells change from monostability to bistability in the horizontal direction. A theoretical model is also established to explain the unusual mechanical behavior.

3 Structured Mechanisms

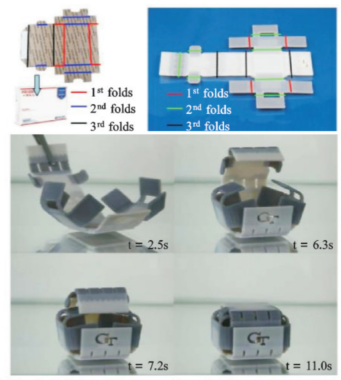
Another common strategy to design multistable mechanical metamaterials relies on the development of structured mechanisms[15,21,23,35-36,40,45,85-87], where self-locking usually occur by utilizing living hinges, frictional forces and magnetic forces. Fig.4(a1—a3) shows three representative examples that adopt living hinge to realize multistable mechanical metamaterials[15,21,87]. Specifically, Haghpanah et al.[21] utilized bistable triangular units as the building block, and realized 2D multistable mechanical metamaterials with large numbers of stable states based on hinge mechanisms (Fig. 4(a1)). Except for the beam-hinge design, the panel-hinge design also provides an effective route to multistable mechanical metamaterials, but with denser configurations (bottom panel of Fig.4(a1)). Furthermore, Jin et al.[15] introduced a novel building-block design strategy composed of four triangular units, also based on the panel-hinge mechanisms. Differently, the building-block structure (top panel of Fig.4(a2)) is monostable, while the configuration (bottom panel of Fig.4(a2)) with a 3×3 tessellation of building-block structures is multistable. Mao et al.[87] utilized shape memory polymers as living hinges to realize self-folding structures. By controlling the heating position, they controlled the sequence of self-folding and formed a self-locked box (Fig.4(a3)) capable



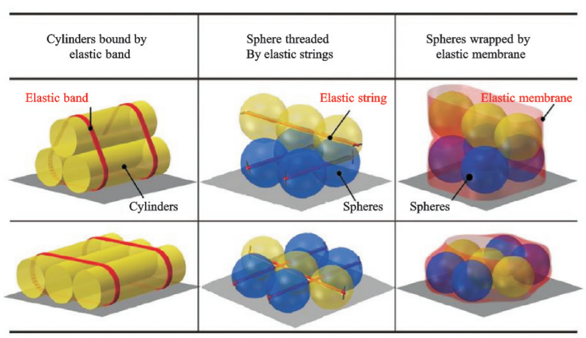
(a1) Multistable mechanical metamaterial based on beam-hinge design. Adapted with permission from Ref. [21]. Copyright 2016, Wiley-VCH



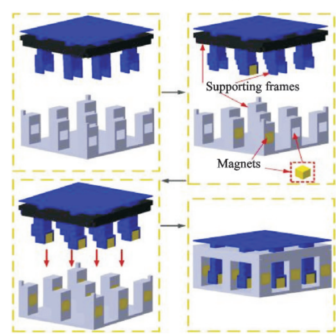
(a2) Multistable mechanical metamaterial based on panel-hinge design. Adapted with permission from Ref. [15]. Copyright 2020, National Academy of Sciences



(a3) Multistable mechanical metamaterial based on self-folding structures. Adapted under the terms of the CC-BY 4.0 Creative Commons Attribution License from Ref. [87]. Copyright 2015, The Authors, published by Springer Nature

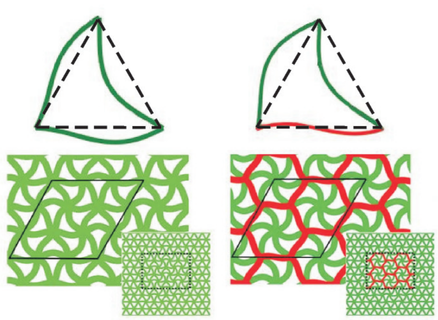


(a4) Multistable mechanical metamaterial which is self-locked by frictional force. Adapted with permission from Ref. [35]. Copyright 2019, Wiley-VCH

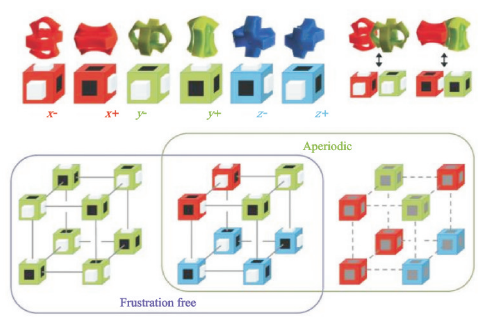


(a5) Multistable mechanical metamaterial which is self-locked by magnetic force. Adapted with permission from Ref. [36]. Copyright 2019, Elsevier

(a) Five representative multistable mechanical metamaterials based on structured mechanism



(b1) Frustrated stacking of triangular building-blocks. Adapted with permission from Ref. [26]. Copyright 2014, American Physical Society



(b2) Frustrated stacking of cubic building-blocks. Adapted with permission from Ref. [3]. Copyright 2016, Springer Nature

(b) 2D and 3D multistable metamaterials based on frustrated stacking

Fig.4 Structured mechanism and geometrical frustration of maintaining its configuration after cooling. Fu et al.[35] utilized stretchable components to connect rough rigid cylinders/spheres to form multistable granular metamaterials (Fig.4(a4)). When the structure is compressed, the elastic band is stretched, and the structure would recover to the initial configuration, if there is no friction between the cylinders. In this design, the frictional forces between rough cylinders limit the mutual movement between the cylinders, and lock deformed configurations (left column in Fig. 4 (a4)). The same self-locking mechanism exists in granular metamaterials in middle and right columns of Fig. 4 (a4). Using functional materials that respond to external stimulus in metamaterial designs, various novel multistable mechanical metamaterials controlled by multi-physical fields were proposed[36,87]. For example, Tan et al.[36] introduced magnetic systems in multistable mechanical metamaterials for trapping energy (Fig.4(a5)). Specifically, the bistable unit consists of three magnets, including two outer magnets and one central magnet, and the repulsive magnetic force exists between the central and outer magnets with appropriate arrangement. It is well known that the direction of magnetic field force is always consistent with the connection between the central and outer magnets. As the middle magnet crosses the line of two outer magnets, the repulsive magnetic force would push central magnets move further until another stable configuration is reached. Based on the design of structural mechanisms, various mechanical multistable metamaterials with abundant controllable stable states can be achieved.

4 Geometrical Frustration

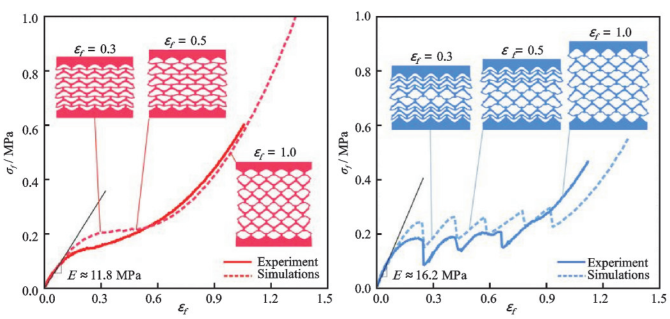
Geometrical frustration mainly refers to the use of geometrical constraints to break the propagation of the local order in lattice structures[1,3,26]. The local "defect" in the lattice structure leads to a diversity of local deformations, resulting in the multistability. Fig.4(b) shows two representative examples of multistable mechanical metamaterials based on geometrical frustration[3,26]. Compared with the square cell, the edges of the triangle cell (Fig.4(b1)) cannot be bent in the same mode, due to geometric constraints. Kang et al.[26] investigated the two deformation modes (i. e., symmetric and chiral configurations) of triangular cell and analyzed the deformation behavior of geometrically frustrated triangular lattice materials. The design strategy of multiple deformation modes provides a useful guideline for designing multistable mechanical metamaterials. Differently, Coulais et al.[3] changed the way of stacking cells in cubic lattice to realize multiple deformation modes. Each flexible building block structure can be oriented independently, and by frustrated stacking (Fig. 4 (b1)) of building block structures, texture metamaterials with multistability and programmability were realized.

5 Unusual Mechanical Properties and Applications

In previous sections, we introduce various design strategies of multistable mechanical metamaterials. This section discusses the unusual mechanical properties[13,15,17-19,22,35-37,40-46,57-58,65,67-68,79-80,83] of these multistable mechanical metamaterials and demonstrates their representative applications[5,9,13-14,16,38,47-55].

5.1 Unusual mechanical properties

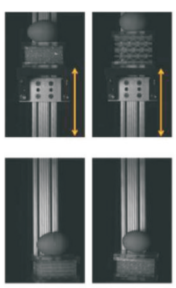
For 1D multistable mechanical metamaterials with "＜" shaped bistable elements[18], they expand layer by layer under stretch (Fig.5(a1)). Each layer of bistable units shows a special load-displacement response where the minimum force is negative as shown in Fig.2(a1). Consequently, stacking of them leads to a stepped load-displacement curve of lattice materials. By tuning the geometric parameters of bistable building-block structures, the number of steps in the stress-strain curve and effective elastic modulus can be tuned in this type of multistable mechanical metamaterials. By 3D stacking of "＜" shaped bistable elements, multistable mechanical metamaterials[78] with zero Poisson's ratio and controllable thermal expansion can be realized as well (Fig.5(a2)). The interlocking assembly method with multimaterials and bistable unit is utilized to ensure completely symmetric multistable mechanisms, resulting in a robust shape-reconfigurability. It is known that multistable mechanical metamaterials have complex energy landscape, and each stable state corresponds to an energy minimum. The metamaterials need to absorb energy from the external system during the process of switching from low-energy stable state to high-energy stable state. Fig. 5 (b1—b3) shows three representative examples on using multistable mechanical metamaterials to absorb impact energy[17,19,22]. They all adopt "＜" shaped bistable units as the basic building-block structures, and stack them in the height direction. Different from traditional energy-absorbing materials based on dissipation or destruction, when the metamaterials are impacted, each layer of bistable units sequentially switches to high-energy stable states, and the impact energy can be stored in the structure in the form of elastic energy. Compared



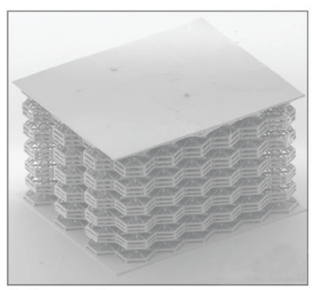
(a1) Tunable stress-strain curves. Adapted with permission from Ref. [18]. Copyright 2015, Wiley-VCH

(a2) Zero Poisson's ratio. Adapted under the terms of the CC-BY 4.0 Creative Commons Attribution License from Ref. [80]. Copyright 2020, The Authors, published by Elsevier

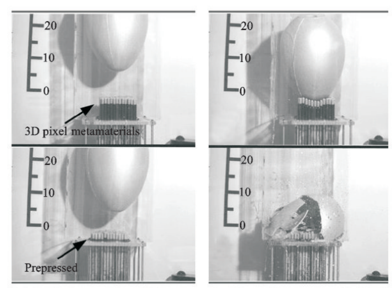
(a) Multistable mechanical metamaterials with unusual mechanical properties



(b1) Beam microstructures. Adapted with permission from Ref. [17]. Copyright 2015, Wiley-VCH

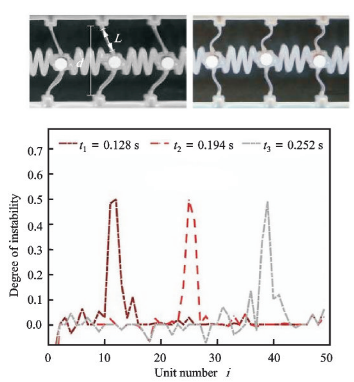


(b2) Membrane microstructures. Adapted with permission from Ref. [22]. Copyright 2016, Wiley-VCH

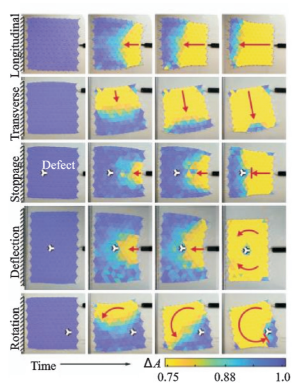


(b3) Adapted with permission from Ref. [19]. Copyright 2019, Wiley-VCH

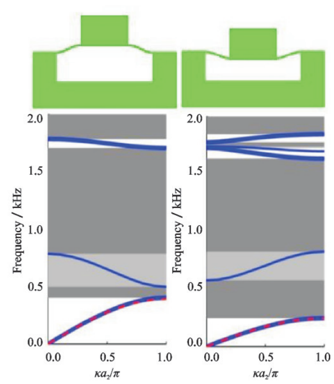
(b) Energy absorption for multistable mechanical metamaterials with "<" shaped bistable units with different structures



(c1) Stable propagations of 1D nonlinear transition wave. Adapted with permission from Ref.[13]. Copyright 2016, National Academy of Sciences



(c2) Propagations of 2D transition wave, including wave pinning, wave deflection and rotation. Adapted with permission from Ref.[15]. Copyright 2020, National Academy of Sciences



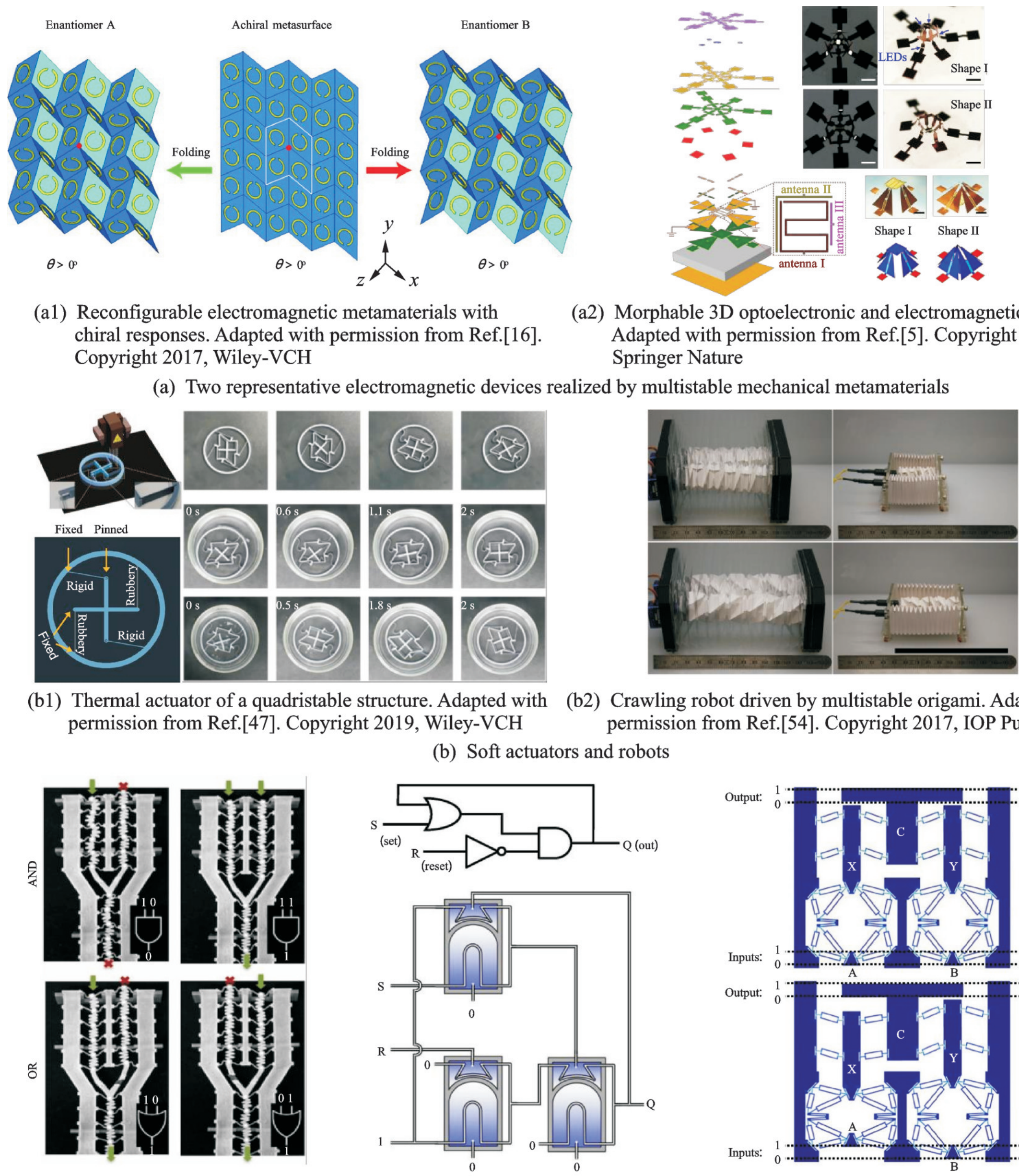
(c3) Dispersion relations for bistable unit cells for the initial and deformed stable configurations. Adapted with permission from Ref.[42]. Copyright 2017, Elsevier

(c) Multistable mechanical metamaterials for controlling the propagations of elastic wave

Fig.5 Unusual mechanical properties of multistable mechanical metamaterials with Fig.5(b1), Frenzel et al.[22] reduced the lattice constant by two orders of magnitude by laser lithography, and realized a faster energy absorption while maintaining a much lower effective mass density (Fig.5(b2)). Pan et al.[19] utilized flexible drinking straw to realize low-cost multistable mechanical metamaterials with a good capacity of energy absorption (Fig.5(b3)). On the contrary, when the multistable mechanical metamaterials are switched from high-energy stable states to low-energy stable states, they release energy. Based on this mechanism, Raney et al.[13] realized the stable propagation of mechanical signals in soft dissipative systems. Specifically, they used a series of "＜" shaped bistable elements (Fig.5(c1)) and set all of them at high-energy stable state. The input signal pushed the first bistable unit back to low-energy state and the released energy pushed the next bistable unit to low-energy state. Continuing to cycle until the end, the stable nonlinear transition wave could propagate through the system with a high fidelity and controllability. Furthermore, the shape reconfigurability of multistable mechanical metamaterials allows applications in controlling wave propagations. Specifically, Jin et al.[15] adopted tessellated multistable units to tailor propagating transitions fronts. The defects and boundary conditions of structures show combined influences on the propagations of transition wave (Fig.5(c2)). Meaud et al.[42] investigated the phononic properties of multistable mechanical metamaterials at different stable configurations (Fig. 5(c3)). They observed that the low frequency band gaps that do not exist in the initial configuration could emerge in deformed configurations.

5.2 Applications

Considering the abundant unusual mechanical properties of multistable mechanical metamaterials, they have shown promising potential for applications in diverse fields. Fig.6(a) demonstrates two representative electromagnetic devices[5,16] realized by multistable mechanical metamaterials. Wang et al.[16] periodically arranged split-ring resonators on Miura-ori chiral metamaterials. As shown in Fig.6(a1), the flat sheet can be reconfigured into two different 3D metasurfaces with parallel and antiparallel net electric and magnetic dipoles of split-ring resonators, resulting in strong chiral responses. Therefore, the electromagnetic properties of the metamaterials can be easily controlled by tuning the configurations. Fu et al.[5] utilized the compressive buckling to realize morphable 3D optoelectronic devices and electromagnetic devices. As shown in top panel of Fig.6(a2), a flat precursor can be transformed into two different stable states (i.e., concave or convex configurations) by controlling the loading path, resulting in two different working state (i.e., on or off states) of the optoelectronic device. By integrating antennas on the 2D precursor, electromagnetic device with reconfigurable shielding capability is realized (bottom panel of Fig.6(a2)). In addition, multistable mechanical metamaterials are widely used to design actuators and robots (Fig.6(b)). Many mechanical actuators have been reported based on the mechanism of energy release from high-energy states to low-energy states. Since this type of mechanical actuators show low efficiency, many responsive materials are introduced in the design of multistable mechanical metamaterials. For example, Jeong et al.[47] extended bistable structures into quadristable rotational structures, and utilized shape memory polymers as the four connections (i.e., two rigid and two rubbery beams) between central rotational units and outer cycle. By controlling the heating temperature and time, the shapes of multistable structures could be changed, and various thermal actuation behavior could be obtained for tailored design parameters. Pagano et al.[54] utilized multistable mechanical metamaterials based on the origami to realize a crawling robot. An origami tower was formed by folding sheets and can produce longitudinal and rotational deformation. As shown in Fig.6(b2), two origami towers were connected by another origami paper, and DC motors were utilized to actuate the deformation of the origami robots. Since multistable mechanical metamaterial could be designed to offer a rich number of controllable stable states, they show significant applications in information processing and logic operations[9,13-14,38]. For example, Raney et al.[13] merged two series of bistable units into output bistable units. These bistable units are initially in a high-energy state (i. e., logic state 0) and the states of input chains have a combinational effect on the output. By changing design parameters of the bistable units at the output, the energy landscape of output bistable units can be tuned to realize different logic gates. Specifically, for bistable units with a high energy barrier at the output, the released energy could activate the output chain, only when both input chains are activated, behaving as an AND gate (top panel of Fig. 6(c1)). For bistable units with low energy barrier at the output, the output chain is activated as long as one input chain is activated (bottom panel of Fig.6(c1)). However, it is difficult to switch the input/output chain from logic state 1 (i.e., low-energy state) to logic state 0 (i.e., high-energy state), due to the flexible connections between bistable units. Preston et al.[14] introduced a novel design of soft valve based on single bistable structures. They



(a1) Reconfigurable electromagnetic metamaterials with chiral responses. Adapted with permission from Ref.[16]. Copyright 2017, Wiley-VCH

(a2) Morphable 3D optoelectronic and electromagnetic devices. Adapted with permission from Ref.[5]. Copyright 2018, Springer Nature

(a) Two representative electromagnetic devices realized by multistable mechanical metamaterials

(b1) Thermal actuator of a quadristable structure. Adapted with permission from Ref.[47]. Copyright 2019, Wiley-VCH

(b2) Crawling robot driven by multistable origami. Adapted with permission from Ref.[54]. Copyright 2017, IOP Publishing

(b) Soft actuators and robots

(c1) AND and OR gate. Adapted with permission from Ref.[13]. Copyright 2016, National Academy of Sciences

(c2) NAND gate. Adapted with permission from Ref.[14]. Copyright 2019, National Academy of Sciences

(c3) Set-reset latch. Adapted under the terms of the CC-BY 4.0 Creative Commons Attribution License from Ref.[9]. Copyright 2019, The Authors, published by Springer Nature

(c) Mechanical logic operators

Fig. 6 Applications of multistable mechanical metamaterials used the pressure to control the switch of stable states, thereby controlling the on/off state of the airflow inside the bistable valves. In addition to the three basic logic gates (i.e., AND, OR and NOT gates) and their combinations, more complex mechanical devices, including set-reset latch, shift register, leading-edge detector and digital-to-analog converter, were realized by the bistable valves (Fig. 6(c2)). Note that most of the previous mechanical logic devices are all in centimeter or even larger scales. Based on the additive manufacturing technology, Song et al.[9] has realized the fabrication of micro-mechanical logic gates (Fig. 6(c3)) based on bistable structures. Since more complex logic operations require more stable states, it remains challenging to effectively integrate basic logic gates in a small space for complex logic operations.

6 Conclusions

This paper provides an overview of recent developments of multistable mechanical metamaterials. Four different classes of multistable mechanical metamaterials are highlighted, according to their design strategies. The unusual mechanical properties of these multistable mechanical metamaterials enable many novel applications, as discussed in this review.

Despite this significant progress, several challenges remain in the designs and applications of multistable mechanical metamaterials, as detailed in the following.

(1) For multistable mechanical metamaterials based on the self-assembly mechanism, the number of stable states in the demonstrated experiments is not very high, due to the limited scalability and yield of the fabrication process. Development of high-precision, micro/nano-scale self-assembly technology with improved yield is essential to solve this problem. The resulting periodical array of 3D multistable structures with hundreds of unit cells has promising potentials for uses in tunable electromagnetic/mechanical devices whose properties are adjustable to meet requirements of different application scenarios.

(2) For multistable mechanical metamaterials based on the snap-through instability, the reported structures are mostly based on "<" shaped bistable structures and show multistablility mostly in a single direction. Additionally, the flexible connections are required between bistable elements to ensure a large number of stable states. However, these flexible connections make it challenging to individually address/switch the stable state of each unit cell in the multistable mechanical metamaterial. Novel designs of multistable building-block structures are needed to realize multistable mechanical metamaterials with massive, controllable stable states in multiple directions.

(3) Most of the existing multistable mechanical metamaterials did not incorporate functional materials as their component materials. Thereby, the switch of stable states is mostly through mechanical loading, and the reported unusual properties are mainly mechanical properties, including tunable stress-strain curves, unusual Poisson's ration, energy absorption and elastic wave propagation control. By adding active materials that respond to external stimuli, multistable mechanical metamaterials could be designed to offer more different types of unusual physical properties (e.g., tunable thermodynamic, electromagnetic or acoustic properties).

(4) Currently, a majority of current research in this area focus on the metamaterial design and demonstration of their multistability. Although Fig.6 introduces some representative applications of multistable mechanical metamaterials, there are still many open opportunities for future developments. Considering the freely switchable stable states, multistable mechanical metamaterials have broad opportunities in the design of adjustable meta-devices that exploit the properties of metamaterials. Also, the ample numbers of stable states ensure the applications of multistable mechanical metamaterials in information processing, memory and transmission. Achieving a huge quantity of controllable stable states in a limited space is the key to this application.

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多稳态力学超材料研究综述

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摘要：多稳态力学超材料最重要的特点是能够在具有不同能级的稳态之间进行可逆的构型切换。本文总结了多稳态力学超材料的主要设计策略，包括基于自组装、突跳失稳、结构化机构以及几何失措的策略；重点讨论了可以实现的稳态数量以及稳态可控性；进而，集中分析了这些多稳态力学超材料的非常规力学性质，并详述其在可调控电磁器件、驱动器、机器人以及机械逻辑运算等领域的应用。最后，展望了多稳态力学超材料设计和应用中存在的挑战及机遇。

关键词：多稳态力学超材料；自组装；突跳失稳；结构化机构；几何失措