Safety Assessment for Autonomous Aerial Refueling Based on Reachability Analysis

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(Received 6 January 2021; revised 12 April 2021; accepted 15 April 2021)

Abstract: Autonomous aerial refueling (AAR) has demonstrated significant benefits to aviation by extending the aircraft range and endurance. It is of significance to assess system safety for autonomous aerial refueling. In this paper, the reachability analysis method is adopted to assess system safety. Due to system uncertainties, the aerial refueling system can be considered as a stochastic system. Thus, probabilistic reachability is considered. Since there is a close relationship between reachability probability and collision probability, the collision probability of the AAR system is analyzed by using reachability analysis techniques. Then, the collision probability is accessed by using the Monte-Carlo experiment method. Finally, simulations demonstrate the effectiveness of the proposed safety assessment method.

Key words: aerial refueling; safety assessment; collision probability; probabilistic reachability; Monte-Carlo method

CLC number: V328.5 **Document code:** A **Article ID:** 1005-1120(2021)02-0216-09

0 Introduction

Autonomous aerial refueling (AAR) is an important method to increase the voyage and endurance of unmanned aerial vehicles and avoid the conflict between the takeoff weight and the payload weight[1-2]. Among the aerial refueling methods in operation today, the probe-drogue refueling (PDR)[3-4] is the most widely adopted one, owing to its flexibility and simple requirement for equipment. There is plenty of studies on the control design of AAR, such as linear quadratic regulator (LQR)[5], Nonzero setpoint (NZSP)[6], active disturbance rejection control (ADRC)[7], adaptive control[8], backstepping control[9], etc.

However, AAR is one of the most dangerous operations in the aviation field. Safety hazards in aerial refueling mainly exist in the "unsafe contact" between the receiver and the tanker. Under unsafe conditions, once the receiver and the tanker collide, the refueling equipment may be damaged. What is worse, the aircraft body may be damaged. During the aerial refueling, due to the close distance between the receiver and the tanker, the aerodynamic disturbances are serious. As a result, the control design is difficult and thus the collision probability is high. Therefore, it is necessary to evaluate system safety of the autonomous aerial refueling system in advance, which can guide the implementation of the aerial refueling mission.

Due to the influence of the environment, the state space of the AAR system can be divided into a safe area and an unsafe area as shown in Fig.1. In order to ensure safety, the system state should be kept outside of the unsafe area, and the control input should be selected to prevent the system from entering the unsafe area. Safety assessment can be divided into two categories: One is the worst-case setting, and the other is the stochastic setting[10]. In the worst environment, the system has bounded disturbance inputs, and the purpose of safety assessment is to prove that for all possible disturbance in-

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How to cite this article: REN Jinrui, MA Haibiao, QUAN Quan, et al. Safety assessment for autonomous aerial refueling based on reachability analysis[J]. Transactions of Nanjing University of Aeronautics and Astronautics, 2021, 38(2): 216-224.

http://dx.doi.org/10.16356/j.1005-1120.2021.02.003

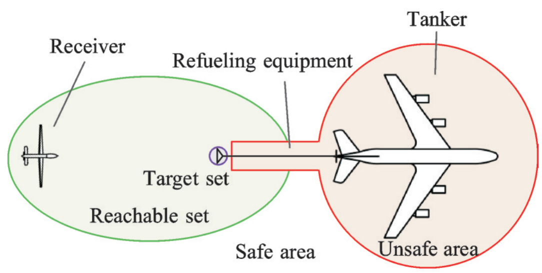

Fig.1 Airspace division for aerial refueling

puts, the system will not enter the unsafe area. In the random environment, there is a random disturbance in the system, and the purpose of safety assessment is to prove that the probability of the system entering the unsafe area is sufficiently small. A new solution to the problem of safety assessment is proposed in Ref.[11], that is, the "barrier certificate" is used to separate the safe area from the unsafe area. In the worst environment, this method does not need to calculate the reachable set and can be directly applied to continuous hybrid systems with nonlinearity, uncertainty and dynamic constraints. In a random environment, the safety assessment problem of random continuous hybrid systems can be solved and the upper bound of the probability of the system reaching the unsafe area can be calculated.

Reachability is an important method to analyze system safety, which is commonly-used in air traffic management. Reachability analysis is to evaluate whether the system state can reach a specific set in a certain time range starting from given initial conditions and limited by control inputs. For a deterministic system, the motion characteristics of the system can be completely determined, and the reachability is a "0/1" binary problem. However, for a stochastic system, due to system uncertainties including modelling uncertainties, observation uncertainties and environmental uncertainties, etc., different trajectories generated from each initial state have different probabilities. The probability of the system arriving at the target set from an initial state with an initial distribution is a kind of probabilistic reachability. When the system change is affected by some control inputs, the control inputs should be selected appropriately to minimize the probability of the system state entering the unsafe area. For deterministic systems, the reachable set can be calculated based on the system model, and the system safety can be assessed by the "model checking" method. The common methods for calculating reachable sets are the Hamilton-Jacobi equation method and the approximation method. The feasible state space of aircraft is analyzed by the Hamilton-Jacobi equation method in Ref.[12], and the safe control range of quadrotor aircraft is studied by the reachable set analysis method in Ref.[13]. For a stochastic system, the safety must be repeatedly assessed online according to the updated information of the system state, and the probability of the system state entering the unsafe area is taken as the criterion to measure the risk degree. The approximation theory related to the Markov process can be used to calculate the probability of the system entering the unsafe area. In Refs.[14-15], the over-approximation method is used to calculate the reachability of the system. The asymptotic approximation method based on the Markov process is used in Ref.[16] to calculate the reachability probability of the system.

The aerial refueling system is a complex system with a variety of uncertainties. The aerial refueling process is abstracted into six maneuvers and six modes in Ref.[17]. Based on the relationship between mode transition, a hybrid system model of the whole process of aerial refueling is established. Then, the capture set and the unsafe set are defined to solve the numerical solution of each maneuvering reachable set in the aerial refueling by the Hamilton-Jacobi equation method. The capture set and unsafe set can be used as a guide for the design of mode transition and safety assessment in different stages of aerial refueling. In addition, great progress has been made in assessing the safety of uncertain systems based on the Monte Carlo method. The safety of robot trajectory in uncertain environment is studied in Ref.[18]. The Monte-Carlo sampling method is used to calculate the collision probability between the robot and obstacles. In Ref.[19], a Monte-Carlo motion planning algorithm is designed, which can be used to evaluate the collision probability between the robot trajectory and obstacles. The algorithm reduces Monte-Carlo variance by combining the sampling method and the control variable method, and also improves the calculation accuracy. The calculation speed is improved to meet the real-time performance. On the whole, the safety of autonomous aerial refueling is rarely assessed in the existing research. Reachability calculations for automated aerial refueling have been conducted in Ref.[20], but the Monte Carlo simulation method was not used.

In this paper, the safety assessment problem for the AAR system is solved by using the reachability analysis method. First, the trajectory space of the receiver can be determined based on the closed-loop receiver model. Then, the collision model of aerial refueling is built according to the aircraft size and flight trajectory. Next, the reachability probability and the collision probability of the AAR system are obtained by the Monte-Carlo experiment method, which can be used to assess the system safety of the AAR system.

The main contribution of this paper is that the safety assessment of the AAR system can be conducted based on the reachability probability and the collision probability. Monte-Carlo simulation is adopted to investigate the collision probability by using reachability analysis. The analysis results can provide guidance in advance to decide whether autonomous aerial refueling should be carried out. The proposed method is simple and effective.

The paper is organized as follows. Estimation of collision probability based on the Monte-Carlo method is presented in section 1. Section 2 gives the reachability analysis for the aerial refueling system. Illustrative simulations are provided in section 3 to show the effectiveness of the proposed method. Section 4 concludes the paper.

1 Estimation of Collision Probability Based on Monte-Carlo Method

In practical engineering, there is a close relationship between reachability probability and collision probability. For evaluating the path reachability, given the initial state and target state, if the system can safely transit from the initial state to the target state, the system is reachable. If the system collides during the transition, we consider the system to be unreachable. Similarly, for stochastic systems, due to the existence of system uncertainties, the occurrence of collision is stochastic. Collision probability can be analyzed by using reachability analysis techniques.

For stochastic systems, it is necessary to analyze the reachability from the perspective of probability. The Monte-Carlo method is usually used to approximate the mathematical expectation of functions containing random variables. Consider a random variable $\boldsymbol{X} \in \boldsymbol{R}^n$, and a bounded function f: $\boldsymbol{R}^n \rightarrow \boldsymbol{R}$, for m independent and identically distributed samples of the random variable $\{\boldsymbol{X}^{(i)}\}_{i=1}^{m}$. According to the central limit theorem, when $m \rightarrow \infty$

$$\sqrt{m}\left(\frac{1}{m} f(\boldsymbol{X}^{(i)}) - E[f(\boldsymbol{X})]\right) \xrightarrow{D} N(0,\tau^2) \tag{1}$$

where $\xrightarrow{D}$ means convergence in distribution; $N(0,\tau^2)$ a Gaussian distribution with mean value 0 and variance τ^2. When $m \rightarrow \infty$, we have

$$\frac{1}{m} f(\boldsymbol{X}^{(i)}) \xrightarrow{p} E[f(\boldsymbol{X})] \tag{2}$$

where $\xrightarrow{p}$ refers to convergence in probability.

Let the random variable $\boldsymbol{X}$ represent the trajectory of the receiver, A a collision event, and f the indicator function to indicate that there is a collision, then $f(A)=1$. Therefore, Eq.(2) can be expressed as

$$P(A) = E[f(\boldsymbol{X})] \tag{3}$$

The collision probability is expressed as p and it can be estimated by

$$\hat{p} = \frac{1}{m}\sum_{i=1}^{m} f(\boldsymbol{X}^{(i)}) \tag{4}$$

Then τ^2 can be estimated according to the sampling variance of $f(\boldsymbol{X}^{(i)})$, namely, when $m \rightarrow \infty$

$$\hat{\tau}^2 = \frac{1}{m}\sum_{i=1}^{m}\left(f(\boldsymbol{X}^{(i)}) - \hat{p}\right)^2 \xrightarrow{p} \tau^2 \tag{5}$$

The uncertainty of $\hat{p}$ can be quantified by approximating the variance of the collision probability estimate $\hat{p}$. Therefore, the Monte-Carlo method can be used to estimate the collision probability of the given trajectory by sampling the motion trajectory of the considered plant.

2 Reachability Analysis for the AAR System

Reachability can be divided into two types, namely forwards reachable set (FRS) and backwards reachable set (BRS). FRS refers to the state set that the system can reach in a certain period of time from a given initial state set. BRS is the set of all initial states that can be controlled to a given target state set in a certain period of time. In this part, FRS is considered.

For AAR, due to the existence of system uncertainties, the trajectory of the receiver will deviate from the nominal trajectory, as shown in Fig. 2. In order to compensate the system uncertainties, a controller needs to be designed to make the receiver track the nominal trajectory quickly[21]. It is worth noting that the controller can only suppress the system uncertainty to some extent, rather than eliminate it completely. When the system uncertainties are very serious, it may lead to an uncontrollable system. Therefore, it is of significance to evaluate system safety based on reachability probability and collision probability.

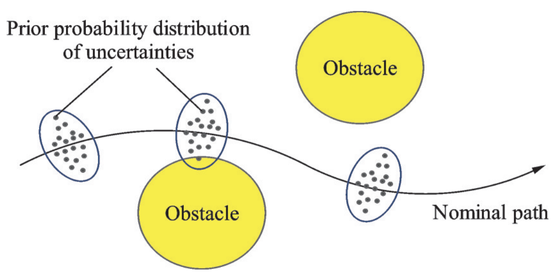

Fig.2 Influence of system uncertainties on a trajectory

2.1 Closed-loop model of the receiver

In the aerial refueling system, the dynamic equation of the receiver is expressed as

$$\dot{\boldsymbol{x}}_{\mathrm{r}}=f(\boldsymbol{x}_{\mathrm{r}},\boldsymbol{u}_{\mathrm{r}},\boldsymbol{d}),\boldsymbol{x}_{\mathrm{r}}(0)=\boldsymbol{x}_{\mathrm{r}0} \tag{6}$$

where $\boldsymbol{x}_{\mathrm{r}}\in\boldsymbol{R}^{12}$ is the state of the receiver; $\boldsymbol{u}_{\mathrm{r}}\in\boldsymbol{R}^{4}$ the input of the receiver; and $\boldsymbol{d}\in\boldsymbol{R}^{d}$ the uncertainty of the system, including modelling uncertainties and environmental disturbances.

The linear model is obtained by linearizing Eq.(6) at the equilibrium point $(\boldsymbol{x}_{\mathrm{r}}^{*},\boldsymbol{u}_{\mathrm{r}}^{*})$, and it is obtained as

$$\begin{cases}\dot{\bar{\boldsymbol{x}}}_{\mathrm{r}}=\boldsymbol{A}\bar{\boldsymbol{x}}_{\mathrm{r}}+\boldsymbol{B}\bar{\boldsymbol{u}}_{\mathrm{r}}+\bar{\boldsymbol{d}}\\ \bar{\boldsymbol{y}}=\boldsymbol{C}\bar{\boldsymbol{x}}_{\mathrm{r}}\quad \bar{\boldsymbol{x}}_{\mathrm{r}}(0)=\bar{\boldsymbol{x}}_{\mathrm{r}0}\end{cases} \tag{7}$$

where the matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ are the obtained linearized matrices. The output vector $\bar{\boldsymbol{y}}$ is the trajectory of the receiver; $\bar{\boldsymbol{d}}=[\bar{d}_1\quad \bar{d}_2\quad\cdots\quad \bar{d}_{12}]^{\mathrm{T}}$ is the lumped disturbance of the system. Noteworthy, $\bar{\boldsymbol{d}}$ can be modelled based on experience or can be estimated by an observer. The reference trajectory of the receiver generated by the trajectory generator is expressed as $\bar{\boldsymbol{y}}_{\mathrm{r}}^{\mathrm{d}}=[\bar{x}_{\mathrm{r}}^{\mathrm{d}}\quad \bar{y}_{\mathrm{r}}^{\mathrm{d}}\quad \bar{h}_{\mathrm{r}}^{\mathrm{d}}]^{\mathrm{T}}$, and the actual trajectory of the receiver is $\bar{\boldsymbol{y}}_{\mathrm{r}}=[\bar{x}_{\mathrm{r}}\quad \bar{y}_{\mathrm{r}}\quad \bar{h}_{\mathrm{r}}]^{\mathrm{T}}$, so the tracking error can be written as

$$\bar{\boldsymbol{y}}_{\mathrm{r}}^{\mathrm{e}}=\bar{\boldsymbol{y}}_{\mathrm{r}}-\bar{\boldsymbol{y}}_{\mathrm{r}}^{\mathrm{d}} \tag{8}$$

where $\bar{\boldsymbol{y}}_{\mathrm{r}}^{\mathrm{e}}\in\boldsymbol{R}^{3}$. Suppose that the trajectory tracking controller is

$$\bar{\boldsymbol{u}}_{\mathrm{r}}=-\boldsymbol{K}_{x}\bar{\boldsymbol{x}}_{\mathrm{r}}-\boldsymbol{K}_{\mathrm{e}}\int_0^t[\bar{\boldsymbol{y}}_{\mathrm{r}}(\tau)-\bar{\boldsymbol{y}}_{\mathrm{r}}^{\mathrm{d}}(\tau)]\mathrm{d}\tau \tag{9}$$

where $\boldsymbol{K}_{x}\in\boldsymbol{R}^{4\times 12},\boldsymbol{K}_{\mathrm{e}}\in\boldsymbol{R}^{4\times 3}$. By substituting Eq.(9) into the dynamic equation of the receiver, the closed-loop model of the receiver can be obtained as

$$\begin{cases}\dot{\bar{\boldsymbol{x}}}_{\mathrm{r}}=(\boldsymbol{A}-\boldsymbol{B}\boldsymbol{K}_{x})\bar{\boldsymbol{x}}_{\mathrm{r}}-\\ \qquad \boldsymbol{B}\boldsymbol{K}_{\mathrm{e}}\int_0^t[\bar{\boldsymbol{y}}_{\mathrm{r}}(\tau)-\bar{\boldsymbol{y}}_{\mathrm{r}}^{\mathrm{d}}(\tau)]\mathrm{d}\tau+\bar{\boldsymbol{d}}\\ \bar{\boldsymbol{y}}_{\mathrm{r}}=\boldsymbol{C}\bar{\boldsymbol{x}}_{\mathrm{r}}\quad \bar{\boldsymbol{x}}_{\mathrm{r}}(0)=\bar{\boldsymbol{x}}_{\mathrm{r}0}\end{cases} \tag{10}$$

2.2 Collision model of the AAR system

The complete aerial refueling process is shown in Fig. 3, including the forward flight process from position 0 to position 1, the side flight process from position 1 to position 2, the docking process from position 2 to position 3, the retreat process from position 3 to position 4, and the side flight process from position 4 to position 5. During the whole flight process, the receiver may collide with the tanker or the refueling equipment. In order to facili-

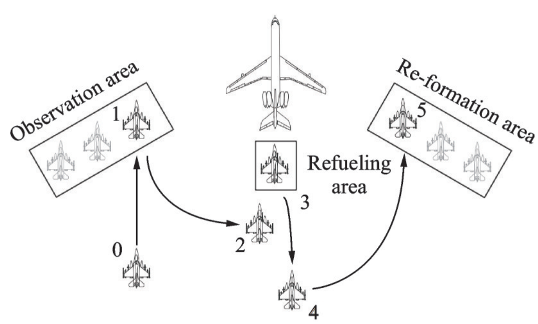

Fig.3 Aerial refueling process

tate the evaluation of collision probability, the following assumption is assumed.

Assumption 1 The collision range of the receiver and the tanker is regarded as a circle on a two-dimensional plane, the center of mass is the center of the circle, and the radius is r_1 and r_2, as shown in Fig.4.

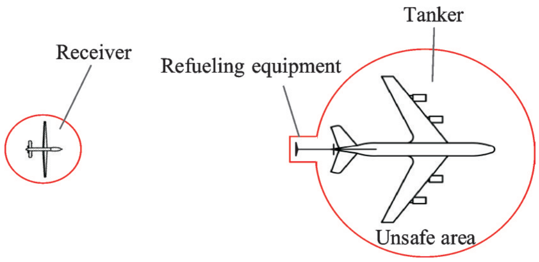

Fig.4 Collision range of the receiver and the tanker

Assume that the position of the center of mass of the receiver is $\boldsymbol{p}_{\mathrm{r}}=[x_{\mathrm{r}} \quad y_{\mathrm{r}}]^{\mathrm{T}}$ in the tanker coordinate system at time t, the collision range of the receiver is

$$R_{\mathrm{r}}=\left\{(x,y)\middle|(x-x_{\mathrm{r}})^2+(y-y_{\mathrm{r}})^2\leqslant r_1^2\right\} \tag{11}$$

The collision range of the refueling equipment can be regarded as a rectangle with the width of w_0. Suppose that the longitudinal maximum distance ($o_{\mathrm{t}}x_{\mathrm{t}}$ direction) from the drogue to the center of mass of the tanker is l_0, then the collision range of the refueling equipment is

$$R_{\mathrm{t1}}=\left\{(x,y)\middle|x+l_0\geqslant 0,|y|\leqslant 0.5w_0\right\} \tag{12}$$

In the tanker coordinate system, the center of mass of the tanker is the origin, so the collision range of the tanker is

$$R_{\mathrm{t2}}=\left\{(x,y)\middle|x^2+y^2\leqslant r_2^2\right\} \tag{13}$$

Based on the above analysis, the collision range of the tanker system is

$$R_{\mathrm{t}}=R_{\mathrm{t1}}\cup R_{\mathrm{t2}}=\left\{(x,y)\middle|x^2+y^2\leqslant r_2^2\right\}\cup\left\{(x,y)\middle|x+l_0\geqslant 0,|y|\leqslant w_0\right\} \tag{14}$$

Trajectory space of the receiver is the set of all paths satisfying the closed-loop model of the receiver in Eq.(10), namely

$$\Omega=\left\{\tilde{\boldsymbol{y}}_{\mathrm{r}}\middle|\tilde{\boldsymbol{y}}_{\mathrm{r}}=\boldsymbol{C}\tilde{\boldsymbol{x}}_{\mathrm{r}},\dot{\tilde{\boldsymbol{x}}}_{\mathrm{r}}=(\boldsymbol{A}-\boldsymbol{B}\boldsymbol{K}_x)\tilde{\boldsymbol{x}}_{\mathrm{r}}-\boldsymbol{B}\boldsymbol{K}_{\mathrm{e}}\int_0^t[\tilde{\boldsymbol{y}}_{\mathrm{r}}(\tau)-\tilde{\boldsymbol{y}}_{\mathrm{r}}^{\mathrm{d}}(\tau)]\mathrm{d}\tau+\bar{\boldsymbol{d}}\right\} \tag{15}$$

Define collision event A. Within time $t\in[0,T]$, the receiver moves along the nominal trajectory $\tilde{\boldsymbol{y}}_{\mathrm{r}}^{\mathrm{d}}$ (i. e., reference trajectory), and the collision range of the receiver intersects with that of the tanker system, which is written in the mathematical form as

$$A:=R_{\mathrm{r}}\cap R_{\mathrm{t}}\neq\varnothing \quad (x(t),y(t))\in\tilde{\boldsymbol{y}}_{\mathrm{r}},t\in[0,T] \tag{16}$$

It should be noted that the relationship between the actual trajectory of the receiver and its nominal trajectory meets the closed-loop model of the receiver in Eq.(10).

2.3 Collision detection of the AAR system

The relative position and related parameters of the receiver system and the tanker system are shown in Fig.5. The center of mass of the tanker is the origin o_{t} of the tanker coordinate system, and the collision range of the tanker is r_2. The center of mass of the receiver is o_{r}, and the two-dimensional coordinates in the tanker coordinate system are $(x_{\mathrm{r}},y_{\mathrm{r}})$. The tanker collision range is r_1. The collision detection can be divided into two parts: One is the collision detection between the receiver and the tanker, and the other is the collision detection between the receiver and the refueling equipment.

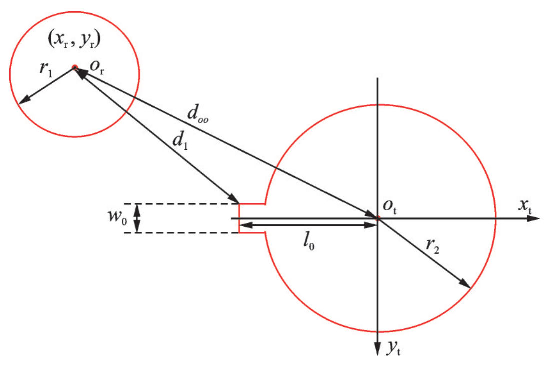

Fig.5 Collision detection of the aerial refueling system

The distance between the center of mass of the receiver and the center of mass of the tanker is d_{oo}. The collision will occur if and only if

$$d_{oo}=\sqrt{x_{\mathrm{r}}^2+y_{\mathrm{r}}^2}\leqslant r_1+r_2 \tag{17}$$

Then, the collision detection between the receiver and the refueling equipment is analyzed. Before docking, the receiver is located at the left side of the tanker, namely the forward flight process from position 0 to position 1 and the side flight process from position 1 to position 2 in Fig.3. The collision range of the receiver is generally much larger than the longitudinal length of the refueling equipment, i.e., there is a relationship $r_1 \gg l_0 - r_2$. Under the circumstance, only the distance from the center of mass of the receiver to the edge of the collision range of the refueling equipment needs to be considered, that is d_1 in Fig.5. The collision will occur if and only if

$$\begin{cases} d_1 = \sqrt{(|x_r| - l_0)^2 + (|y_r| - 0.5w_0)^2} \leqslant r_1 \\ |x_r| - r_1 \leqslant l_0 \end{cases} \tag{18}$$

2.4 Algorithm design of collision probability estimation

According to the idea of the Monte-Carlo simulation, the random variable $\bar{\boldsymbol{y}}_r$ represents the trajectory of the receiver, A denotes a collision event, and f is an indicator function to indicate that a collision has occurred, namely $f(A)=1$. On the contrary, $f(\bar{A})=0$ with $\bar{A}$ being the mutually exclusive event of A, which means no collision occurs. Therefore, the collision probability can be expressed as $E[f(A|\bar{\boldsymbol{y}}_r)] = P(A)$. Based on m Monte-Carlo simulation experiments, the collision probability can be estimated by

$$P(A) = \frac{1}{m}\sum_{i=1}^{m} f(A|\bar{\boldsymbol{y}}_r^{(i)}) \tag{19}$$

Algorithm

Step 1 Give a set of system uncertainty parameters $\boldsymbol{d} \sim N(\boldsymbol{\mu}, \Sigma)$, or the level of wind disturbance. Initialize the nominal trajectory $\bar{\boldsymbol{y}}_r^d$ and the number of collision count = 0.

Step 2 Carry out m Monte-Carlo simulation experiments, and $i = 1, 2, \cdots, m$ for each simulation. The obtained trajectory corresponds to an actual trajectory $\bar{\boldsymbol{y}}_r^{(i)}$.

Determine whether there is a collision according to the collision detection model:

(1) If there is a collision, $f(A|\bar{\boldsymbol{y}}_r^{(i)}) = 1$, count = count + 1.

(2) If there is no collision, the next simulation experiment needs be carried out.

Sclect n waypoints as samples equidistantly on the nominal trajectory $\boldsymbol{p}_r^{(j)}, j = 1, 2, \cdots, n$, and mark the waypoints before the collision as 0 and the waypoints after the collision as 1.

Step 3 Calculate the collision probability of the whole trajectory by $P(A)$ = count$/m$.

Step 4 The reachability probability of each waypoint is $P(\boldsymbol{p}_r^{(j)}) = 1 - \frac{1}{m}\sum_{i=1}^{m} f(A|\boldsymbol{p}_r^{(j)})$.

The corresponding algorithm flow chart is shown in Fig.6.

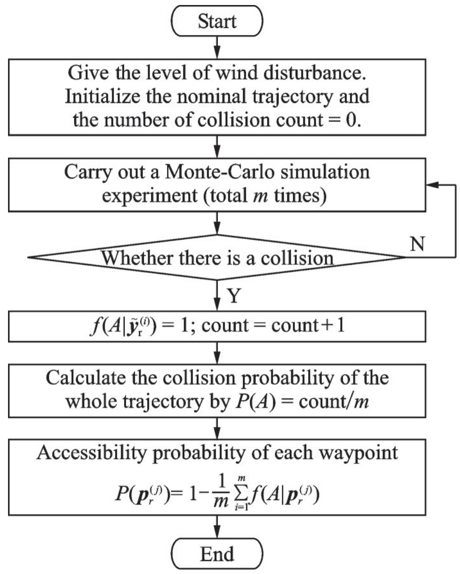

Fig.6 Algorithm flow chart

3 Simulation

In the simulation, the influence of wind disturbances and the tail vortex of the tanker are considered. The simulation conditions are set as follows: The initial position of the receiver is $\boldsymbol{p}_{r0} = [-30 \quad -30]^{\mathrm{T}}$ as shown in Fig.7. There are three levels of wind disturbances: (1) The probability of exceedance intensity is 10^{-2}, which is called as "Class II turbulence"; (2) the probability of exceedance intensity is 10^{-3}, which is called as "Class III turbulence"; (3) the probability of exceedance intensity is 10^{-5}, which is called as "Level IV turbulence". The trajectory of the receiver is along the $o_t x_t$ direction, flying straight and level for 20 m at a speed of 2 m/s. Other parameters are set as $r_1 = 8$ m, $r_2 = 21$ m, $l_0 = 25$ m, $w_0 = 5$ m, simulation times $m = 100$.

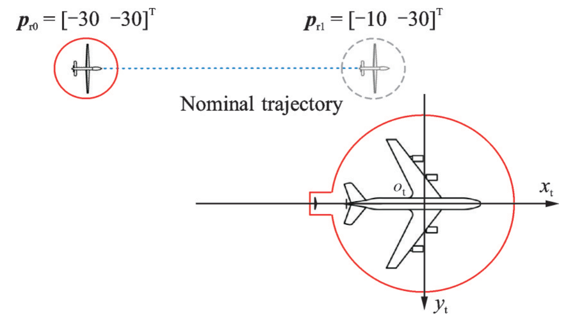

Fig.7 Nominal trajectory of the receiver

(1) Simulation results under level II turbulence

Fig.8 is the statistical histogram of the frequency of collisions under the level II turbulence. A total of three collisions have occurred. Based on the collision time of these three collisions, the achievable probability distribution diagram is shown in Fig.9. If the safety threshold is set as 0.9, under the level II turbulence, the receiver is in the safe area all the time.

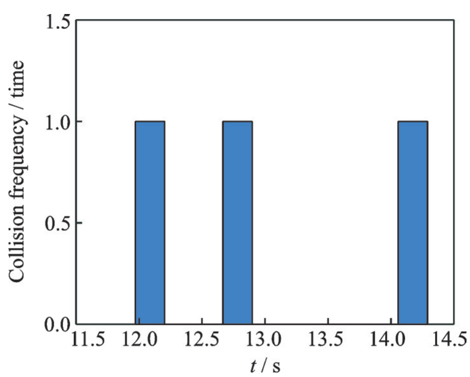

Fig.8 Statistical histogram of the frequency of collisions under the level II turbulence

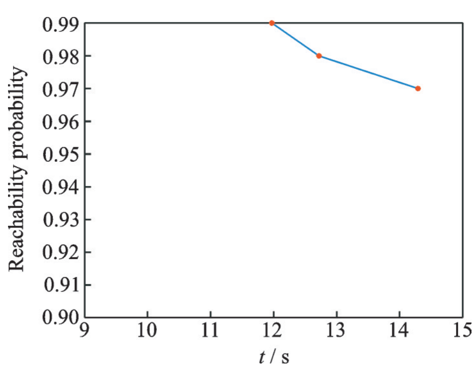

Fig.9 Distribution diagram of reachability probability under the level II turbulence

(2) Simulation results under level III turbulence

Fig.10 is the statistical histogram of the frequency of collisions under the level III turbulence. A total of 14 collisions have occurred. Based on the collision time of these 14 collisions, the achievable probability distribution diagram is shown in Fig.11. Under the level III turbulence, the receiver is in the safe area for the beginning 14.3 s, after which the reachability probability of the receiver drops below the safety threshold.

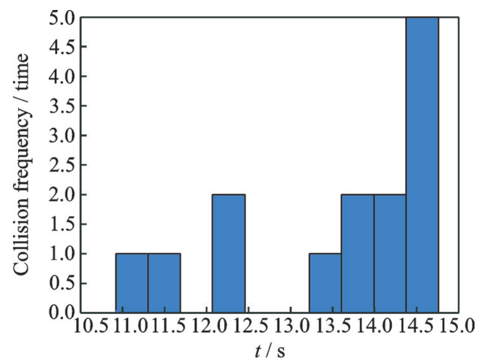

Fig.10 Statistical histogram of the frequency of collisions under the level III turbulence

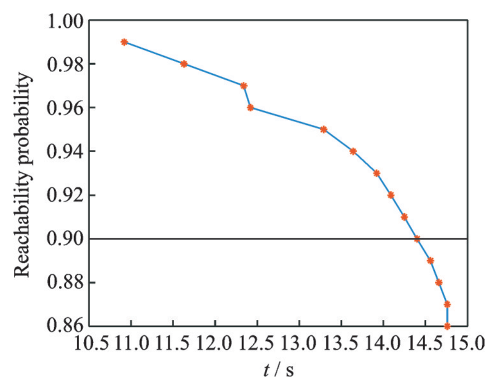

Fig.11 Distribution diagram of reachability probability under the level III turbulence

(3) Simulation results under level IV turbulence

Fig.12 shows the statistical diagram of the collision number under the level IV turbulence. A total of 68 collisions have occurred, and the reachable probability distribution diagram is shown in Fig.13. The receiver is in the safe area for the beginning 9 s, and in the dangerous area after 9 s.

In summary, as the intensity of turbulence increases, the collision probability between the receiver and the tanker increases.

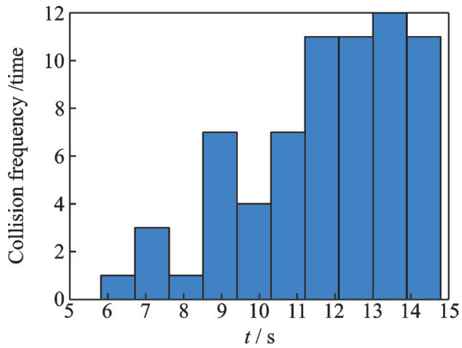

Fig.12 Statistical histogram of the frequency of collisions under the level IV turbulence

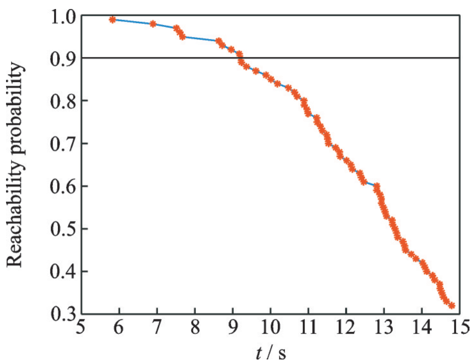

Fig.13 Distribution diagram of turbulent reachability probability under the level IV turbulence

4 Conclusions

The safety assessment problem for the autonomous aerial refueling system is solved by the reachability analysis method. The main contribution of this paper is that the reachability probability and the collision probability can be obtained by using the Monte-Carlo experiment method. The analysis results can provide guidance in advance to decide whether autonomous aerial refueling should be carried out. The proposed method is simple and effective. Simulation results show that the method can assess system safety properly. We will try more computational efficient methods to calculate the reachable sets without the need for computationally intensive Monte Carlo simulations in the context of aerial refueling operation in future work.

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Acknowledgement This work was supported by the National Natural Science Foundation of China (No. 61933010).

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Competing interests The authors declare no competing interests.

(Production Editor:ZHANG Bei)

基于可达性分析的自主空中加油安全评估

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摘要:自主空中加油(Autonomous aerial refueling, AAR)可以扩展飞机的航程和续航能力,为航空业带来巨大效益,对自主空中加油系统进行安全评估具有重要意义。本文采用可达性分析方法来评估系统安全性。由于系统不确定性,空中加油系统可视为一个随机系统,因此考虑概率可达性。由于可达性概率与碰撞概率有着密切的关系,本文利用可达性分析技术对自主空中加油系统的碰撞概率进行分析。然后,利用蒙特卡罗实验方法来获取碰撞概率。最后,通过仿真展示了所提出的安全评估方法的有效性。

关键词:空中加油;安全评估;碰撞概率;概率可达性;蒙特卡洛方法