# Adaptive Backstepping Control for Uncertain Systems with Compound Nonlinear Characteristics

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**Abstract:** An adaptive backstepping multi-sliding mode approximation variable structure control scheme is proposed for a class of uncertain nonlinear systems. An actuator model with compound nonlinear characteristics is established based on the model decomposition method. The unmodeled dynamic term of the radial basis function neural network approximation system is presented. The Nussbaum gain design technique is utilized to overcome the problem that the control gain is unknown. The adaptive law estimation is used to estimate the upper boundary of neural network approximation and uncertain interference. The adaptive approximate variable structure control effectively weakens the control signal chattering while enhancing the robustness of the controller. Based on the Lyapunov stability theory, the stability of the entire control system is proved. The main advantage of the designed controller is that the compound nonlinear characteristics are considered and solved. Finally, simulation results are given to show the validity of the control scheme.

**Key words:** compound nonlinearities; saturation; hysteresis; adaptive backstepping control; radial basis function (RBF) neural network

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## 0 Introduction

The inherent characteristics of physical devices, mechanical design and manufacturing deviations make nonlinear characteristics, like saturation and hysteresis, inevitably exist in the actual control systems, including mechanical systems, servo systems, and piezoelectric systems, affecting the overall performance. It may even cause instability in the system, like divergence and shock. With the development and application of information technology, the new material technology and the continuous improvement of system control performance requirements, it is necessary to adopt certain methods to eliminate or reduce the influence of nonlinear characteristics during the design and analysis of the control system.

In recent years, the problems of uncertain system control with nonlinear characteristics of actuators have received considerable attentions and become an active research area[1-10]. But there are also some limitations, e. q., requiring nonlinear model parameter information to be partially known. The method of model decomposition requires that the nonlinear types are clear, and most works study specific nonlinear input-output constraints in control design procedure[2]. In engineering practice, the nonlinear characteristics of the actuator are often difficult to accurately judge, and sometimes they are a mixture of multiple situations[11].

The inputs of the actual systems are limited by uncertain factors, but there are few studies on the

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control problems of uncertain systems considering the inputs being affected by compound nonlinearities. In this paper, a strict feedback nonlinear system with compound nonlinear characteristics and unknown control gain is considered. Its robust controller is constructed, by utilizing the adaptive backstepping sliding mode control method, dynamic surface control technology and radial basis function (RBF) neural network approximation technology.

# 1 Problem Statement

## 1.1 Actuator model with saturated nonlinear characteristics

An actuator model with saturated nonlinear characteristics is described as

$$v(u(t))=\mathrm{sat}(u(t))=\begin{cases}v_{\max} & u\geqslant v_{\max}\\ u & v_{\min}<u<v_{\max}\\ v_{\min} & u\leqslant v_{\min}\end{cases} \tag{1}$$

where $v_{\max}>0$ and $v_{\min}<0$ represent saturation nonlinearities. Its input-output relationship is shown in Fig.1.

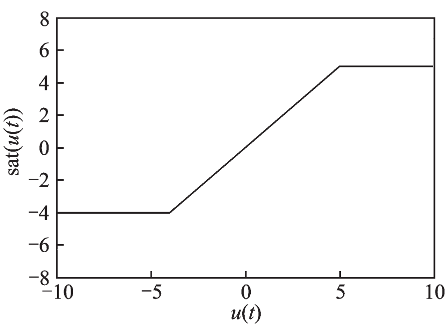


Fig.1 Input-output relationship of saturated nonlinear actuator

A piecewise smoothing function to approximate the saturation function is described as[10]

$$g(u(t))=\begin{cases}v_{\max}\times\tanh(u/v_{\max})\\ v_{\min}\times\tanh(u/v_{\min})\end{cases}=$$

$$\begin{cases}v_{\max}\times\dfrac{\mathrm{e}^{u/v_{\max}}-\mathrm{e}^{-u/v_{\max}}}{\mathrm{e}^{u/v_{\max}}+\mathrm{e}^{-u/v_{\max}}} & u\geqslant 0\\ v_{\min}\times\dfrac{\mathrm{e}^{u/v_{\min}}-\mathrm{e}^{-u/v_{\min}}}{\mathrm{e}^{u/v_{\min}}+\mathrm{e}^{-u/v_{\min}}} & u<0\end{cases} \tag{2}$$

The saturation function in Eq.(1) can be expressed as

$$\mathrm{sat}(u(t))=g(u(t))+d_1(t) \tag{3}$$

where $d_1(t)$ is a bounded function.

$$\left|d_1(t)\right|=\left|\mathrm{sat}(u(t))-g(u(t))\right|\leqslant \max\left\{v_{\max}(1-\tanh(1)),-v_{\min}(1-\tanh(1))\right\} \tag{4}$$

Fig.2 shows the input-output relationship of the approximate smooth saturation function $g(u(t))$.

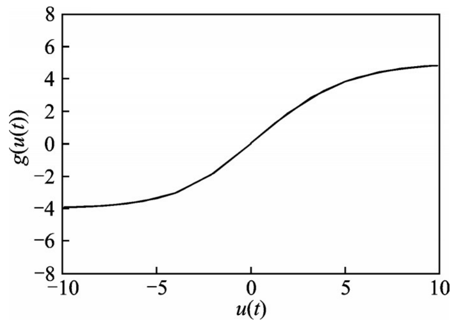


Fig.2 Input-output relationship of smooth saturation function

According to the median theorem, for the constant $\lambda$, we have

$$g(u)=g(u_0)+g_{u_\lambda}(u-u_0)\qquad 0<\lambda<1 \tag{5}$$

where

$$g_{u_\lambda}=\left.\frac{\partial g(u)}{\partial u}\right|_{u=u_\lambda} \tag{6}$$

$$u_\lambda=\lambda u+(1-\lambda)u_0 \tag{7}$$

When $u_0=0$

$$g(u)=g_{u_\lambda}u\quad 0<\lambda<1 \tag{8}$$

Considering Eqs.(3,5), we have

$$\mathrm{sat}(u)=g_{u_\lambda}u+d_1(t) \tag{9}$$

In control engineering, the control input $u(t)$ would be increased indefinitely. The following assumption exists.

**Assumption 1** Coefficient $g_{u_\lambda}$ is unknown but bounded

$$0<g_{\mathrm{m}}\leqslant g_{u_\lambda}\leqslant 1 \tag{10}$$

where $g_{\mathrm{m}}$ is positive.

It should be noted that $g_{u_\lambda}$ is handled as an unknown control gain.

## 1.2 Actuator model with hysteretic nonlinear characteristics

Currently hysteresis nonlinear models are mainly divided into two categories. One is a rate-independent hysteresis model, including Dual model, LuGre model, Backlash-like model, Prandtl-Ishlinskii model, Preisach model, etc. The other is the rate-dependent hysteresis model, which mainly includes the semi-linear Duhem model, and the modified Prandtl-Ishlinskii model[12-17]. The hysteresis nonlinearity of this paper is characterized by the Backlash-like model as follows

$$\frac{\mathrm{d}v}{\mathrm{d}t}=A\left|\frac{\mathrm{d}u}{\mathrm{d}t}\right|(Cu-v)+B\frac{\mathrm{d}u}{\mathrm{d}t} \tag{11}$$

where $A, B, C$ are constants, $C>0$ and $C>B$.

According to the analysis in Ref.[18], it leads to

$$v=Cu+\bar{d}(u) \tag{12}$$

$$\bar{d}(u)=(v(0)-Cu(0))\mathrm{e}^{-A(u-u(0))\mathrm{sign}(\dot{u})}+\mathrm{e}^{-Au\mathrm{sign}(\dot{u})}\int_{u(0)}^{u}(B-C)\mathrm{e}^{A\xi\mathrm{sign}(\dot{u})}\mathrm{d}\xi \tag{13}$$

where $\bar{d}(u)$ is bounded.

The input-output relationship is shown in Fig.3.

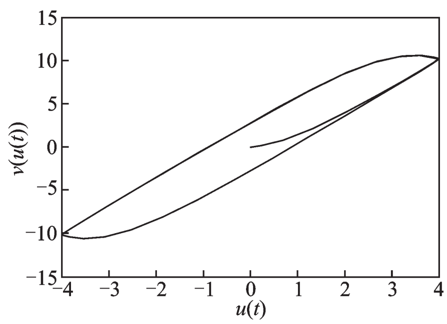


Fig.3 Input-output relationship of hysteresis nonlinear actuator

### 1.3 Actuator model with compound nonlinear characteristics

According to the model decomposition, a unified actuator model with nonlinear characteristics of saturation and hysteresis is established as

$$v_2(v_1(u))=\varphi_2(u,t)\varphi_1(u,t)u(t)+\varphi_2(u,t)d_1(t)+d_2(t)=\varphi(u,t)u(t)+d(t) \tag{14}$$

where $v_1(u)$ is the hysteresis nonlinearity, $v_2(v_1)$ the input saturation nonlinearity, $\varphi(u,t)=\varphi_2(u,t)\varphi_1(u,t)$ the linear coefficient, and $d(t)=\varphi_2(u,t)d_1(t)+d_2(t)$ the nonlinear part of the model.

### 1.4 A class of uncertain systems with compound nonlinear characteristics

Consider the following class of uncertain systems with compound nonlinear characteristics.

$$\begin{cases}\dot{x}_i=x_{i+1}+f_i(\bar{x}_i)+\Delta_i(\bar{x}_n,t) & i=1,\cdots,n-1\\ \dot{x}_n=f_n(\bar{x}_n)+g(\bar{x}_n)v(u(t))+\Delta_n(\bar{x}_n,t)\\ y=x_1\end{cases} \tag{15}$$

where $\bar{\boldsymbol{x}}_i=[x_1,x_2,\cdots,x_i]^{\mathrm{T}}\in\boldsymbol{R}^i$ is system state vector, $u(t)\in\boldsymbol{R}$ control signal to be designed, $v(u(t))\in\boldsymbol{R}$ control signal which actually acts on the control input of the system, $y=x_1\in\boldsymbol{R}$ output signal, and $f_i(\bar{x}_i)(i=1,\cdots,n)$ and $g(\bar{x}_n)$ are unknown nonlinear smooth functions, indicating that the system has uncertainties such as unmodeled dynamics or modeling errors. $\Delta_i(\bar{x}_n,t)(i=1,\cdots,n)$ is unknown uncertain disturbance.

Considering Eqs.(14,15), we can deduce that

$$\begin{cases}\dot{x}_i=x_{i+1}+f_i(\bar{x}_i)+\Delta_i(\bar{x}_n,t) & i=1,\cdots,n-1\\ \dot{x}_n=f_n(\bar{x}_n)+g(\bar{x}_n)\varphi(u,t)u(t)+\Delta_n'(\bar{x}_n,t)\\ y=x_1\end{cases} \tag{16}$$

where $\Delta_n'(\bar{x}_n,t)=g(\bar{x}_n)d(t)+\Delta_n(\bar{x}_n,t)$ is compound uncertain disturbance of the subsystem.

**Assumption 2** Time-varying perturbation $d(t)$ is bounded, and there is unknown positive constant $D_0>0$

$$|d(t)|\leqslant D_0 \tag{17}$$

**Assumption 3** Control gain $g(\bar{x}_n)$ is unknown but bounded

$$0<g_0\leqslant|g(\bar{x}_n)|\leqslant g_1 \tag{18}$$

**Assumption 4** The reference command signal $y_{\mathrm{r}}(t)$ and its derivatives $\dot{y}_{\mathrm{r}}(t)$, $\ddot{y}_{\mathrm{r}}(t)$ exist and are bounded.

**Assumption 5** $\Delta_i(\bar{x}_n,t)$ is bounded, and there is an unknown constant $D_i\ (i=1,\cdots,n)$ that satisfies the following inequality

$$|\Delta_i(\bar{x}_n,t)|\leqslant D_i \quad i=1,\cdots,n \tag{19}$$

According to Eqs. (17—19), we have

$$|\Delta_n'(\bar{x}_n,t)|=|g(\bar{x}_n)d(t)+\Delta_n(\bar{x}_n,t)|\leqslant g_1D_0+D_n\leqslant D \tag{20}$$

where $D>0$ is an unknown positive constant.

This paper uses the Nussbaum gain design technique to overcome the problem that the control gain $g(\bar{x}_n)\varphi(u,t)$ is unknown. Define continuous functions $N(\zeta):\boldsymbol{R}\rightarrow\boldsymbol{R}$, if the following conditions are satisfied.

$$\lim_{k\to+\infty}\sup\frac{1}{s}\int_0^s N(\zeta)\mathrm{d}\zeta=+\infty \tag{21}$$

$$\lim_{k\to+\infty}\inf\frac{1}{s}\int_0^s N(\zeta)\mathrm{d}\zeta=-\infty \tag{22}$$

where $N(\zeta)$ is the Nussbaum function

$$N(\zeta)=\mathrm{e}^{\zeta^2}\cos\left(\frac{\zeta\pi}{2}\right) \tag{23}$$

For the Nussbaum function $N(\zeta)$, the following lemma exists.

**Lemma 1**[19] If both $V(\cdot)$ and $\zeta(\cdot)$ are smooth functions on $[0,t_f)$, $V(t)\geqslant 0, t\in[0,t_f)$, and the following inequality exists.

$$0\leqslant V(t)\leqslant c_0+\mathrm{e}^{-c_1 t}\int_0^t(h(x(\tau))N(\zeta)+1)\dot{\zeta}\mathrm{e}^{-c_1\tau}\mathrm{d}\tau \tag{24}$$

Then $V(t)$, $\zeta(t)$ and $\int_0^t h(x(\tau))N(\zeta)\dot{\zeta}\mathrm{d}\tau$ are bounded on the interval $[0,t_f)$. Here $c_0$ is constant, $c_1>0$, and $h(x(\tau))$ is an arbitrary function whose value is bounded in the interval $[l^-,l^+]$, $0\notin[l^-,l^+]$.

## 2 Design of Adaptive Backstepping Control Scheme and Stability Analysis

### 2.1 Design of adaptive backstepping control scheme

Define tracking error

$$\begin{cases} e_1=x_1-y_r \\ e_i=x_i-\beta_{i-1} \quad i=2,\cdots,n \end{cases} \tag{25}$$

where $e_1$ is the system tracking error, and $\beta_{i-1}$ the virtual control signal of the $i-1$ order subsystem.

**Step 1** Differentiating $e_1$ yields

$$\dot{e}_1=f_1(x_1)+x_2+\Delta_1(\bar{x}_n,t)-\dot{y}_r \tag{26}$$

The adaptive RBF neural network is used to approximate the unknown nonlinear function $f_1(x_1)$. For the compact set $\Omega\subset\mathrm{R}$, there exists an optimal weight vector $\boldsymbol{W}_1^*$

$$f_1(x_1)=\boldsymbol{W}_1^{*\mathrm{T}}\xi_1(x_1)+\varepsilon_1 \tag{27}$$

where $\varepsilon_1$ is the approach error and $|\varepsilon_1|\leqslant\varepsilon_1^*$. Define

$$\tilde{\boldsymbol{W}}_1=\boldsymbol{W}_1^*-\hat{\boldsymbol{W}}_1 \tag{28}$$

where $\hat{\boldsymbol{W}}_1$ is the estimate of the optimal weight of the neural network and $\tilde{\boldsymbol{W}}_1$ the estimation error. The adaptive law of neural network weight vector is taken as

$$\dot{\hat{\boldsymbol{W}}}_1=\boldsymbol{\Gamma}_1(e_1\xi_1(\bar{x}_1)-\sigma_{10}\hat{\boldsymbol{W}}_1) \tag{29}$$

where $\sigma_{10}>0$ is the parameter to be designed, $\boldsymbol{\Gamma}_1=\boldsymbol{\Gamma}_1^{\mathrm{T}}$ is the gain matrix to be designed and elements of the matrix are all positive.

Define the boundary value $D_1'=D_1+\varepsilon_1^*$ and choose the adaptive law as

$$\dot{\hat{D}}_1=\gamma_1|e_1|-\sigma_{11}\gamma_1\hat{D}_1 \tag{30}$$

where $\sigma_{11}>0$ is the parameter to be designed and $\gamma_1$ the adaptive gain coefficient to be designed. The estimated error is $\tilde{D}_1=D_1'-\hat{D}_1$.

The first error surface is defined as $s_1=e_1$. Then the virtual control law is chosen

$$\alpha_1=-k_1e_1-\hat{\boldsymbol{W}}_1^{\mathrm{T}}\xi_1(x_1)-\hat{D}_1\frac{1-\exp(-\upsilon_1\hat{D}_1e_1)}{1+\exp(-\upsilon_1\hat{D}_1e_1)}+\dot{y}_r \tag{31}$$

where $\upsilon_1>0$ and $k_1>0$ are parameters to be designed.

To avoid repeatedly differentiating virtual controllers, which will lead to the so-called "explosion of complexity", we employ the dynamic surface control technique to eliminate this problem. We introduce a first-order filter $\beta_1$, and let $\alpha_1$ pass through it with the time constant $\tau_1$

$$\tau_1\dot{\beta}_1+\beta_1=\alpha_1 \qquad \beta_1(0)=\alpha_1(0) \tag{32}$$

By defining the output error of this filter as $\omega_1=\beta_1-\alpha_1$, we have

$$\dot{\beta}_1=\frac{\alpha_1-\beta_1}{\tau_1}=-\frac{\omega_1}{\tau_1} \tag{33}$$

Define the Lyapunov function

$$V_1=\frac{1}{2}e_1^2+\frac{1}{2}\tilde{\boldsymbol{W}}_1^{\mathrm{T}}\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{W}}_1+\frac{1}{2\gamma_1}\tilde{D}_1^2+\frac{1}{2}\omega_1^2 \tag{34}$$

Differentiating $V_1$ yields

$$\dot{V}_1=e_1\dot{e}_1-\tilde{\boldsymbol{W}}_1^{\mathrm{T}}\boldsymbol{\Gamma}_1^{-1}\dot{\hat{\boldsymbol{W}}}_1-\frac{1}{\gamma_1}\tilde{D}_1\dot{\hat{D}}_1+\omega_1\dot{\omega}_1 \tag{35}$$

According to Eqs.(26,27,31), we obtain

$$\dot{e}_1=e_2+\omega_1+\alpha_1+f_1(x_1)+\Delta_1(\bar{x}_n,t)-\dot{y}_r=-k_1e_1+e_2+\omega_1+\tilde{\boldsymbol{W}}_1^{\mathrm{T}}\xi_1(x_1)+\varepsilon_1+\Delta_1(\bar{x}_n,t)-\hat{D}_1\frac{1-\exp(-\upsilon_1\hat{D}_1e_1)}{1+\exp(-\upsilon_1\hat{D}_1e_1)} \tag{36}$$

Differentiating $\omega_1$ yields

$$\dot{\omega}_1=\dot{\beta}_1-\dot{\alpha}_1=-\frac{\omega_1}{\tau_1}+\phi_1(e_1,e_2,\omega_1,\hat{\boldsymbol{W}}_1,\hat{D}_1,y_d,\dot{y}_d,\ddot{y}_d) \tag{37}$$

where $\phi_1(e_1, e_2, \omega_1, \hat{\boldsymbol{W}}_1, \hat{D}_1, y_d, \dot{y}_d, \ddot{y}_d)$ is a continuous function abbreviated as $\phi_1(\bullet)$.

Substituting Eqs.(36,37) into Eq.(35) yields

$$\dot{V}_1 = e_1\dot{e}_1 - \tilde{\boldsymbol{W}}_1^{\mathrm{T}}\boldsymbol{\Gamma}_1^{-1}\dot{\hat{\boldsymbol{W}}}_1 - \frac{1}{\gamma_1}\tilde{D}_1\dot{\hat{D}}_1 + \omega_1\dot{\omega}_1 \leqslant$$
$$-k_1e_1^2 + e_1e_2 + e_1\omega_1 + \tilde{\boldsymbol{W}}_1^{\mathrm{T}}\boldsymbol{\Gamma}_1^{-1}(\boldsymbol{\Gamma}_1e_1\xi_1(x_1) - \dot{\hat{\boldsymbol{W}}}) +$$
$$|e_1|\hat{D}_1 - e_1\hat{D}_1\frac{1-\exp(-\upsilon_1\hat{D}_1e_1)}{1+\exp(-\upsilon_1\hat{D}_1e_1)} +$$
$$\frac{1}{\gamma_1}\tilde{D}_1(\gamma_1|e_1| - \dot{\hat{D}}_1) - \frac{\omega_1^2}{\tau_1} + \omega_1\phi_1(\bullet) \tag{38}$$

According to Eqs.(29, 30), and boundary inequalities, we obtain

$$\dot{V}_1 \leqslant -k_1e_1^2 - \frac{\omega_1^2}{\tau_1} + e_1e_2 + e_1\omega_1 +$$
$$\omega_1\phi_1(\bullet) + \sigma_{10}\tilde{\boldsymbol{W}}_1^{\mathrm{T}}\hat{\boldsymbol{W}}_1 + \sigma_{11}\tilde{D}_1\hat{D}_1 + \frac{1}{\upsilon_1} \tag{39}$$

**Step *i*** The derivative of $e_i$ $(i=2,\cdots,n-1)$ is

$$\dot{e}_i = x_{i+1} + f_i(\bar{\boldsymbol{x}}_i) + \Delta_i(\bar{\boldsymbol{x}}_n, t) - \dot{\beta}_{i-1} \tag{40}$$

where $\beta_{i-1}$ is the output of the first-order filter

$$\begin{cases}\tau_{i-1}\dot{\beta}_{i-1} + \beta_{i-1} = \alpha_{i-1} \quad \beta_{i-1}(0) = \alpha_{i-1}(0) \\ \dot{\beta}_{i-1} = -\dfrac{\omega_{i-1}}{\tau_{i-1}}\end{cases} \tag{41}$$

where $\omega_{i-1} = \beta_{i-1} - \alpha_{i-1}$, $\alpha_{i-1}$ is the input of the first-order filter and $\tau_{i-1}$ the time constant. Substituting Eq.(41) into Eq.(40) yields

$$\dot{e}_i = x_{i+1} + f_i(\bar{\boldsymbol{x}}_i) + \Delta_i(\bar{\boldsymbol{x}}_n, t) + \frac{\omega_{i-1}}{\tau_{i-1}} \tag{42}$$

The adaptive RBF neural network is used to approximate the unknown nonlinear function $f_i(\bar{\boldsymbol{x}}_i)$. For the compact set $\Omega_{\bar{x}_i} \subset \mathbf{R}^i$, there exists an optimal weight vector $\boldsymbol{W}_i^*$

$$f_i(\bar{\boldsymbol{x}}_i) = \boldsymbol{W}_i^{*\mathrm{T}}\xi_i(\bar{\boldsymbol{x}}_i) + \varepsilon_i \tag{43}$$

where $\varepsilon_i$ is the approach error, $|\varepsilon_i| \leqslant \varepsilon_i^*$, $\hat{\boldsymbol{W}}_i$ the estimate of the optimal weight of the neural network and $\tilde{\boldsymbol{W}}_i = \boldsymbol{W}_i^* - \hat{\boldsymbol{W}}_i$. The adaptive law of neural network weight vector is chosen as

$$\dot{\hat{\boldsymbol{W}}}_i = \boldsymbol{\Gamma}_i(e_i\xi_i(\bar{\boldsymbol{x}}_i) - \sigma_{i0}\hat{\boldsymbol{W}}_i) \tag{44}$$

where $\sigma_{i0} > 0$ is the parameter to be designed, $\boldsymbol{\Gamma}_i = \boldsymbol{\Gamma}_i^{\mathrm{T}}$ is the gain matrix to be designed and elements of the matrix are all positive.

Define the boundary value $D_i' = D_i + \varepsilon_i^*$, and choose the adaptive law to estimate $D_i'$.

$$\dot{\hat{D}}_i = \gamma_i|e_i| - \sigma_{i1}\gamma_i\hat{D}_i \tag{45}$$

where $\sigma_{i1} > 0$ is the parameter to be designed, and $\gamma_i$ the adaptive gain coefficient to be designed. The estimated error is $\tilde{D}_i = D_i' - \hat{D}_i$.

The virtual control law of the $i$th order subsystem is chosen

$$\alpha_i = -k_ie_i - \hat{\boldsymbol{W}}_i^{\mathrm{T}}\xi_i(\bar{\boldsymbol{x}}_i) - \hat{D}_i\frac{1-\exp(-\upsilon_i\hat{D}_ie_i)}{1+\exp(-\upsilon_i\hat{D}_ie_i)} -$$
$$\frac{\omega_{i-1}}{\tau_{i-1}} \tag{46}$$

where $\upsilon_i > 0$ and $k_i > 0$ are the parameters to be designed.

Using the similar way, we introduce a first-order filter $\beta_i$, and let $\alpha_i$ pass through it with the time constant $\tau_i$

$$\tau_i\dot{\beta}_i + \beta_i = \alpha_i \quad \beta_i(0) = \alpha_i(0) \tag{47}$$

Define the Lyapunov function

$$V_{n-1} = \frac{1}{2}\sum_{i=1}^{n-1}e_i^2 + \frac{1}{2}\sum_{i=1}^{n-1}\tilde{\boldsymbol{W}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\tilde{\boldsymbol{W}}_i + \frac{1}{2}\sum_{i=1}^{n-1}\frac{\tilde{D}_i^2}{\gamma_i} +$$
$$\frac{1}{2}\sum_{i=1}^{n-1}\omega_i^2 \tag{48}$$

Similarly, it can be obtained as

$$\dot{V}_{n-1} \leqslant \sum_{i=1}^{n-1}\Bigg(-k_ie_i^2 - \frac{\omega_i^2}{\tau_i} + e_ie_{i+1} + e_i\omega_i + \omega_i\phi_i(\bullet) +$$
$$\sigma_{i0}\tilde{\boldsymbol{W}}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_i + \sigma_{i1}\tilde{D}_i\hat{D}_i + \frac{1}{\upsilon_i}\Bigg) \tag{49}$$

where $\phi_i(\bullet)$ is a continuous function.

**Step *n*** The $n$th order subsystem's error of the system is $e_n = x_n - \beta_{n-1}$. The derivative of $e_n$ is

$$\dot{e}_n = f_n(\bar{\boldsymbol{x}}_n) + g(\bar{\boldsymbol{x}}_n)\varphi(u,t)u(t) + \Delta_n'(\bar{\boldsymbol{x}}_n, t) + \frac{\omega_{n-1}}{\tau_{n-1}} \tag{50}$$

where $\omega_{n-1} = \beta_{n-1} - \alpha_{n-1}$ and $\tau_{n-1}$ is the time constant.

The adaptive RBF neural network is used to approximate the unknown nonlinear function term $f_n(\bar{\boldsymbol{x}}_n)$ of the system. For the compact set $\Omega_{\bar{x}_n} \subset \boldsymbol{R}^n$, there exists an optimal weight vector $\boldsymbol{W}_n^*$

$$f_n(\bar{\boldsymbol{x}}_n) = \boldsymbol{W}_n^{*\mathrm{T}}\xi_n(\bar{\boldsymbol{x}}_n) + \varepsilon_n \tag{51}$$

where $\varepsilon_n$ is the approach error, $|\varepsilon_n| \leqslant \varepsilon_n^*$; $\hat{\boldsymbol{W}}_n$ the estimate of the optimal weight of the neural network and $\tilde{\boldsymbol{W}}_n = \boldsymbol{W}_n^* - \hat{\boldsymbol{W}}_n$. The adaptive law of neural network weight vector is chosen as

$$\dot{\hat{\boldsymbol{W}}}_n = \boldsymbol{\Gamma}_n (s\xi_n(\bar{\boldsymbol{x}}_n) - \sigma_{n0}\hat{\boldsymbol{W}}_n) \tag{52}$$

where $\sigma_{n0} > 0$ is the parameter to be designed and $\boldsymbol{\Gamma}_n = \boldsymbol{\Gamma}_n^{\mathrm{T}}$ is the gain matrix to be designed and elements of the matrix are all positive.

The boundary value is defined as $D'_n = D + \varepsilon_n^*$, and the adaptive law to estimate $D'_n$ is chosen.

$$\dot{\hat{D}}_n = \gamma_n |e_n| - \sigma_{n1}\gamma_n \hat{D}_n \tag{53}$$

where $\sigma_{n1} > 0$ is the parameter to be designed, and $\gamma_n$ the adaptive gain coefficient to be designed. The estimated error is $\tilde{D}_n = D'_n - \hat{D}_n$.

Define the sliding surface $s = e_n$. The control law is designed as

$$u = [g(\bar{\boldsymbol{x}}_n)\varphi(u,t)]^{-1}\left(-k_n e_n - \hat{\boldsymbol{W}}_n^{\mathrm{T}}\xi_n(\bar{\boldsymbol{x}}_n) - \hat{D}\frac{1-\exp(-\upsilon_n \hat{D}_n e_n)}{1+\exp(-\upsilon_n \hat{D}_n e_n)} - \frac{\omega_{n-1}}{\tau_{n-1}}\right) \tag{54}$$

Design the control law using Nussbaum

$$\begin{cases} u = N(\zeta)\left(k_n e_n + \hat{\boldsymbol{W}}_n^{\mathrm{T}}\xi_n(\bar{\boldsymbol{x}}_n) + \hat{D}\dfrac{1-\exp(-\upsilon_n \hat{D}_n e_n)}{1+\exp(-\upsilon_n \hat{D}_n e_n)} + \dfrac{\omega_{n-1}}{\tau_{n-1}}\right) \\ \dot{\zeta} = k_n e_n^2 + e_n \hat{\boldsymbol{W}}_n^{\mathrm{T}}\xi_n(\bar{\boldsymbol{x}}_n) + e_n\hat{D}\dfrac{1-\exp(-\upsilon_n \hat{D}_n e_n)}{1+\exp(-\upsilon_n \hat{D}_n e_n)} + e_n\dfrac{\omega_{n-1}}{\tau_{n-1}} \end{cases} \tag{55}$$

where $\upsilon_n > 0$ and $k_n > 0$ are parameters to be designed.

## 2.2 Stability analysis

**Theorem 1** Consider the uncertain nonlinear systems (16), the controller (55), and the corresponding adaptive law. If the proposed control system satisfies Assumptions 1—5, for the system with any bounded initial state, all signals of the closed-loop system are semiglobally uniformly bounded. And, by tuning the designed parameters, the system tracking error $e_1$ converges to a small neighborhood near the origin.

Define the Lyapunov function

$$V = V_{n-1} + \frac{1}{2}e_n^2 + \frac{1}{2}\tilde{\boldsymbol{W}}_n^{\mathrm{T}}\boldsymbol{\Gamma}_n^{-1}\tilde{\boldsymbol{W}}_n + \frac{1}{2\gamma_n}\tilde{D}_n^2 \tag{56}$$

The derivative of $V$ is

$$\dot{V} = \dot{V}_{n-1} + e_n\dot{e}_n - \tilde{\boldsymbol{W}}_n^{\mathrm{T}}\boldsymbol{\Gamma}_n^{-1}\dot{\hat{\boldsymbol{W}}}_n - \frac{1}{\gamma_n}\tilde{D}_n\dot{\hat{D}}_n \tag{57}$$

According to Eq.(50), we obtain

$$e_n\dot{e}_n = e_n f_n(\bar{\boldsymbol{x}}_n) + e_n g(\bar{\boldsymbol{x}}_n)\varphi(u,t)u(t) + e_n\Delta'_n(\bar{\boldsymbol{x}}_n,t) + e_n\frac{\omega_{n-1}}{\tau_{n-1}} \tag{58}$$

According to the control law (55), we have

$$e_n u(t) = N(\zeta)\dot{\zeta} \tag{59}$$

Substituting Eq.(59) into Eq.(58) yields

$$e_n\dot{e}_n = g(\bar{\boldsymbol{x}}_n)\varphi(u,t)N(\zeta)\dot{\zeta} + \dot{\zeta} - k_n e_n^2 + e_n f_n(\bar{\boldsymbol{x}}_n) - e_n\hat{\boldsymbol{W}}_n^{\mathrm{T}}\xi_n(\bar{\boldsymbol{x}}_n) + e_n\Delta'_n(\bar{\boldsymbol{x}}_n,t) - e_n\hat{D}\frac{1-\exp(-\upsilon_n \hat{D}_n e_n)}{1+\exp(-\upsilon_n \hat{D}_n e_n)} \tag{60}$$

Substituting Eqs.(52, 53, 60) into Eq.(57) yields

$$\dot{V} \leqslant \dot{V}_{n-1} + g(\bar{\boldsymbol{x}}_n)\varphi(u,t)N(\zeta)\dot{\zeta} + \dot{\zeta} - k_n e_n^2 + |e_n|\hat{D}_n - e_n\hat{D}\frac{1-\exp(-\upsilon_n \hat{D}_n e_n)}{1+\exp(-\upsilon_n \hat{D}_n e_n)} + \sigma_{n0}\tilde{\boldsymbol{W}}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_n + \sigma_{n1}\tilde{D}_n\hat{D}_n \tag{61}$$

Invoking the boundary inequality yields

$$|e_n|\hat{D}_n - e_n\hat{D}\frac{1-\exp(-\upsilon_n \hat{D}_n e_n)}{1+\exp(-\upsilon_n \hat{D}_n e_n)} \leqslant \frac{1}{\upsilon_n} \tag{62}$$

Therefore

$$\dot{V} \leqslant \dot{V}_{n-1} + g(\bar{\boldsymbol{x}}_n)\varphi(u,t)N(\zeta)\dot{\zeta} + \dot{\zeta} - k_n e_n^2 + \frac{1}{\upsilon_n} + \sigma_{n0}\tilde{\boldsymbol{W}}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_n + \sigma_{n1}\tilde{D}_n\hat{D}_n \tag{63}$$

Substituting Eq.(49) into Eq.(63) yields

$$\dot{V} \leqslant \left(-\frac{\omega_i^2}{\tau_i} + e_i e_{i+1} + e_i\omega_i + \omega_i\phi_i(\bullet)\right) + \sum_{i=1}^{n}(\sigma_{i0}\tilde{\boldsymbol{W}}_i^{\mathrm{T}}\boldsymbol{W}_i^* - \sigma_{i0}\tilde{\boldsymbol{W}}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_i + \sigma_{i1}\tilde{D}_i D'_i - \sigma_{i1}\tilde{D}_i^2) + \sum_{i=1}^{n}(-k_i e_i^2) + \sum_{i=1}^{n}\frac{1}{\upsilon_i} + g(\bar{\boldsymbol{x}}_n)\varphi(u,t)N(\zeta)\dot{\zeta} + \dot{\zeta} \tag{64}$$

By Assumption 4 and the initial state of the system bounded, we have

$$|\phi_i(\bullet)| < \Phi_i \quad i = 1, \cdots, n-1 \tag{65}$$

According to Young's inequality

$$e_i e_{i+1} \leqslant e_i^2 + \frac{1}{4} e_{i+1}^2 \tag{66}$$

$$e_i \omega_i \leqslant e_i^2 + \frac{1}{4} \omega_i^2 \tag{67}$$

$$\omega_i \phi_i(\cdot) \leqslant \frac{\Phi_i^2}{\mu_i} \omega_i^2 + \frac{\mu_i}{4} \quad \mu_i > 0 \tag{68}$$

$$\sigma_{i0} \tilde{\boldsymbol{W}}_i^{\mathrm{T}} \boldsymbol{W}_i^* \leqslant \frac{\sigma_{i0}}{2} \tilde{\boldsymbol{W}}_i^{\mathrm{T}} \tilde{\boldsymbol{W}}_i + \frac{\sigma_{i0}}{2} \boldsymbol{W}_i^{*\mathrm{T}} \boldsymbol{W}_i^* \tag{69}$$

$$\sigma_{i1} \tilde{D}_i D_i' \leqslant \frac{\sigma_{i1}}{2} \tilde{D}_i^2 + \frac{\sigma_{i1}}{2} D_i'^2 \tag{70}$$

Substituting Eqs.(66—70) into Eq.(64) yields

$$\dot{V} \leqslant (-k_1 + 2) e_1^2 + \sum_{i=2}^{n-1} (-k_i + \frac{9}{4}) e_i^2 + (-k_n + \frac{1}{4}) e_n^2 +$$
$$\sum_{i=1}^{n-1} \left( -\frac{1}{\tau_i} + \frac{1}{4} + \frac{\Phi_i^2}{\mu_i} \right) \omega_i^2 + \sum_{i=1}^{n-1} \frac{\mu_i}{4} + \sum_{i=1}^{n} \frac{1}{\upsilon_i} + \sum_{i=1}^{n} \left( -\frac{\sigma_{i0}}{2} \tilde{\boldsymbol{W}}_i^{\mathrm{T}} \tilde{\boldsymbol{W}}_i - \frac{\sigma_{i1}}{2} \tilde{D}_i^2 \right) + \sum_{i=1}^{n} \left( \frac{\sigma_{i0}}{2} \boldsymbol{W}_i^{*\mathrm{T}} \boldsymbol{W}_i^* + \frac{\sigma_{i1}}{2} D_i'^2 \right) +$$
$$g(\bar{\boldsymbol{x}}_n) \varphi(u,t) N(\zeta) \dot{\zeta} + \dot{\zeta} \tag{71}$$

When

$$r_2 = \sum_{i=1}^{n-1} \frac{\mu_i}{4} + \sum_{i=1}^{n} \frac{1}{\upsilon_i} + \sum_{i=1}^{n} \left( \frac{\sigma_{i0}}{2} \boldsymbol{W}_i^{*\mathrm{T}} \boldsymbol{W}_i^* + \frac{\sigma_{i1}}{2} D_i'^2 \right) \tag{72}$$

Choose

$$k_1 \geqslant 2 + \frac{r_1}{2} \tag{73}$$

$$k_i \geqslant \frac{9}{4} + \frac{r_1}{2} \quad i = 2, \cdots, n-1 \tag{74}$$

$$k_n \geqslant \frac{1}{4} + \frac{r_1}{2} \tag{75}$$

$$\frac{1}{\tau_i} \geqslant \frac{\Phi_i^2}{\mu_i} + \frac{1}{4} + \frac{r_1}{2} \quad i = 1, \cdots, n-1 \tag{76}$$

$$\frac{\sigma_{i0}}{\lambda_{\max}(\boldsymbol{\Gamma}_i^{-1})} \geqslant r_1 \quad i = 1, \cdots, n \tag{77}$$

$$\frac{\sigma_{i1}}{\max\{\gamma_i^{-1}\}} \geqslant r_1 \quad i = 1, \cdots, n \tag{78}$$

where $r_1 > 0$.

$$\dot{V} \leqslant -r_1 V + r_2 + g(\bar{\boldsymbol{x}}_n) \varphi(u,t) N(\zeta) \dot{\zeta} + \dot{\zeta} \tag{79}$$

Multiply both sides of Eq.(79) by $e^{\mu t}$ and integrate

$$V(t) \leqslant \frac{r_2}{r_1} + \left[ V(0) - \frac{r_2}{r_1} \right] \mathrm{e}^{-r_1 t} +$$
$$\mathrm{e}^{-r_1 t} \int_0^t [g(\bar{\boldsymbol{x}}_n) \varphi(u,t) N(\zeta) + 1] \dot{\zeta} \mathrm{e}^{r_1 \tau} \mathrm{d}\tau \leqslant$$
$$\frac{r_2}{r_1} + V(0) +$$
$$\mathrm{e}^{-r_1 t} \int_0^t [g(\bar{\boldsymbol{x}}_n) \varphi(u,t) N(\zeta) + 1] \dot{\zeta} \mathrm{e}^{r_1 \tau} \mathrm{d}\tau \tag{80}$$

According to Assumption 3, we have $\left| g(\bar{\boldsymbol{x}}_n) \varphi(u,t) \right| \in [g_0 \varphi_0, g_1 \varphi_1]$. It can be proved by Lemma 1 that $V(t)$, $\zeta(t)$, and $\int_0^t g(\bar{\boldsymbol{x}}_n) \varphi(u,t) \cdot N(\zeta) \dot{\zeta} \mathrm{d}\tau$ are bounded on the interval $[0, t_f)$. When

$$\mathrm{e}^{-r_1 t} \int_0^t [g(\bar{\boldsymbol{x}}_n) \varphi(u,t) N(\zeta) + 1] \dot{\zeta} \mathrm{e}^{r_1 \tau} \mathrm{d}\tau \leqslant r_3 \tag{81}$$

Eq.(80) can be rewritten as

$$V(t) \leqslant \frac{r_2}{r_1} + V(0) + r_3 \tag{82}$$

Thus, $V(t)$ is bounded. According to Eq.(56), the closed-loop system signals $e_i$, $\omega_i$, $\tilde{\boldsymbol{W}}_i$, and $\tilde{D}_i$ are bounded by a semi-global uniform termination, and $\hat{\boldsymbol{W}}_i$ and $\hat{D}_i$ are bounded. By Assumption 4, the state of the closed-loop system $x_i$ is bounded.

According to Eqs.(56,82), we obtain

$$e_1 \leqslant \sqrt{2V(t)} \leqslant \sqrt{2 r_2 / r_1 + \left[ V(0) - \frac{r_2}{r_1} \right] \mathrm{e}^{-r_1 t} + r_3} \tag{83}$$

Therefore, the convergence radius $\sqrt{2 r_2 / r_1 + r_3}$ of the steady-state tracking error $e_1$ of the system can be reduced by choosing parameters.

# 3 Simulation

Consider the following uncertain nonlinear system

$$\begin{cases} \dot{x}_1 = 0.1 x_1^2 + x_2 + 0.5 x_1^2 \sin t \\ \dot{x}_2 = 0.1 e^{-x_2} + x_3 + 0.5(x_1^2 + x_2^2) \sin t \\ \dot{x}_3 = x_1 x_2 x_3 + (1 + \sin x_1)[\varphi(u,t) u + d(t)] + \\ \qquad 0.5(x_1^2 + x_2^2 + x_3^2) \cos t \\ y = x_1 \end{cases}$$

The center of the Gaussian radial basis function of the RBF neural network $\hat{\boldsymbol{W}}_1^{\mathrm{T}} \boldsymbol{\xi}_1(\bar{\boldsymbol{x}}_1)$ is $\{-1, -2/3, -1/3, 0, 1/3, 2/3, 1\}$, The width is $\eta_1 = 2$, $\hat{\boldsymbol{W}}_1(0) = \boldsymbol{0}$. The center of the Gaussian radial basis function of the RBF neural network $\hat{\boldsymbol{W}}_2^{\mathrm{T}} \boldsymbol{\xi}_2(\bar{x}_2)$ is $\{-1, -1/2, 0, 1/2, 1\} \times \{-1, -1/2, 0, 1/2, 1\}$. The width is $\eta_2 = 2$, $\hat{\boldsymbol{W}}_2(0) = \boldsymbol{0}$.

The center of the Gaussian radial basis function of the RBF neural network $\hat{\boldsymbol{W}}_3^{\mathrm{T}}\xi_3(\bar{x}_3)$ is $\{-1,0,1\}\times\{-1,0,1\}\times\{-1,0,1\}$, The width is $\eta_3=2$, and $\hat{\boldsymbol{W}}_3(0)=\boldsymbol{0}$.

The reference command signal is $y_{\mathrm{d}}=0.5\sin t+0.5\sin(0.5t)$. The initial values are $[x_1(0),x_2(0),x_3(0)]^{\mathrm{T}}=[0.25,0.25,0.25]^{\mathrm{T}}$, $\hat{D}_i(0)=0$ $(i=1,2,3)$ and $\zeta(0)=1$. Time constants are $\tau_1=\tau_2=0.04$. The parameters to be designed are set to $k_i=2$, $\boldsymbol{\Gamma}_i=\mathrm{diag}[0.5]$, $\upsilon_i=10$, $\sigma_{i0}=\sigma_{i1}=0.2$, and $\gamma_i=0.5$.

The simulation results are shown in Figs.4—6. Adaptive backstepping multi-sliding mode variable control without RBF neural network approximation is conducted as a comparative simulation result. The simulation result in Fig.4 shows that the scheme of this paper has better tracking control effect. It can be seen that the designed controller can stably track the reference command while the actuator has nonlinear compound characteristics of hysteresis and input saturation, and the tracking error remains within a certain range. According to Figs.7—9, variables of the closed-loop system state are bounded.

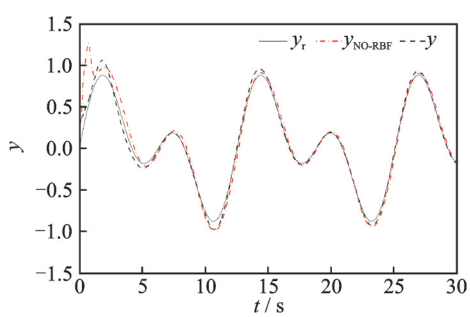


Fig.4 Tracking reference command signal curves

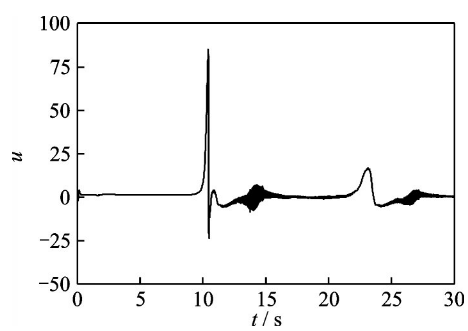


Fig.5 Control signal curve of the system $u$

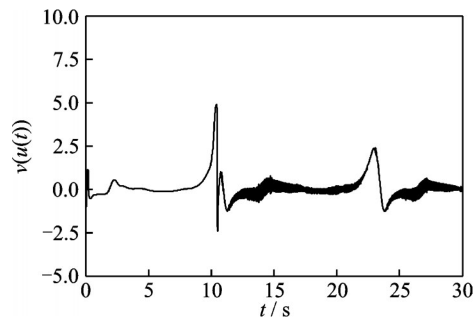


Fig.6 Actual control signal curve of the system $v(u(t))$

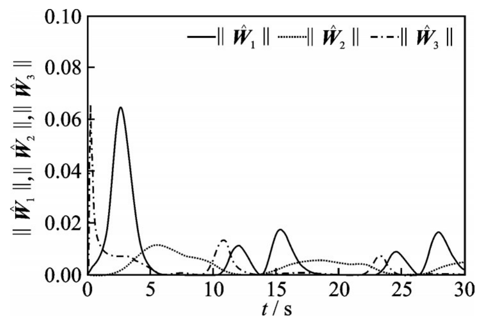


Fig.7 Curves of neural network weight norms

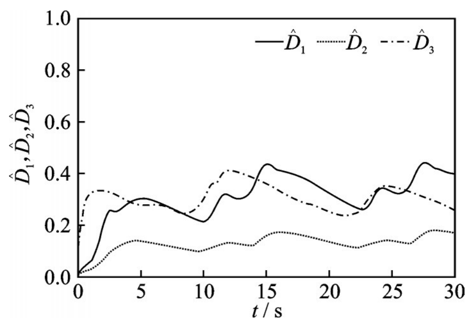


Fig.8 Curves of the adaptive parameters

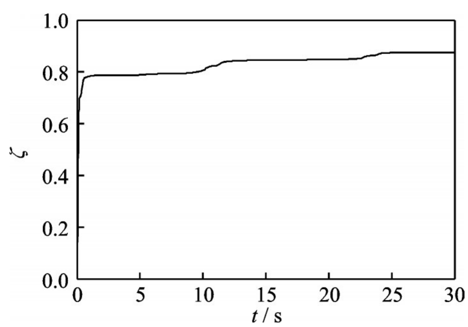


Fig.9 Curve of the adaptive parameter $\zeta$

## 4 Conclusions

A class of uncertain nonlinear systems with compound nonlinear characteristics has been studied. Combined with RBF neural network approximation and adaptive control theory, an adaptive backstepping multi-sliding mode variable structure controller scheme is presented. By utilizing the model decomposition method, a nonlinear actuator model of compound nonlinear characteristics is established, so that the inverse solution of nonlinear features is not needed in the controller design process.

It has been proved that all signals of the closed-loop system are semi-globally uniformly bounded. A simulation example has been conducted to show the validity of the proposed scheme.

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# 具有复合非线性特征不确定系统的自适应反推控制

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**摘要：**针对一类具有复合非线性特征的不确定系统，提出了一种自适应反推多滑模近似变结构控制方案。基于模型分解的方法建立了具有复合非线性特征的执行器模型。论文设计径向基函数神经网络近似系统的未建模项，借鉴Nussbaum增益设计技术解决控制增益未知的问题，使用自适应律估计不确定干扰和神经网络近似误差的上边界。自适应近似变结构控制能够有效削弱控制信号的抖振，同时增强控制器的鲁棒性。基于Lyapunov稳定性理论，证明了整个控制系统的稳定性。该控制方案的主要创新点在于考虑并解决了不确定系统具有复合非线性特征的控制问题。最后，仿真结果证明了控制方案的有效性。

**关键词：**复合非线性；饱和；滞回；自适应反推控制；径向基函数神经网络