Review: Recent Developments in Dynamic Load Identification for Aerospace Vehicles Considering Multi-source Uncertainties

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Abstract: The determination of the dynamic load is one of the indispensable technologies for structure design and health monitoring for aerospace vehicles. However, it is a significant challenge to measure the external excitation directly. By contrast, the technique of dynamic load identification based on the dynamic model and the response information is a feasible access to obtain the dynamic load indirectly. Furthermore, there are multi-source uncertainties which cannot be neglected for complex systems in the load identification process, especially for aerospace vehicles. In this paper, recent developments in the dynamic load identification field for aerospace vehicles considering multi-source uncertainties are reviewed, including the deterministic dynamic load identification and uncertain dynamic load identification. The inversion methods with different principles of concentrated and distributed loads, and the quantification and propagation analysis for multi-source uncertainties are discussed. Eventually, several possibilities remaining to be explored are illustrated in brief.

Key words: dynamic load identification; concentrated dynamic load; distributed dynamic load; stochastic load; probabilistic uncertainties; non-probabilistic uncertainties

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0 Introduction

It is common knowledge that the development of aerospace vehicles is closely associated with the national economy and defense strategy. Aircraft structures are often burdened with various dynamic loads in the flight process, for instance, the thrust load in the launch phase, the pulse pressure in the transonic phase and the strong aerodynamic load in the return phase. With the rapid enhancement of flight speed, flight distance and maneuverability of aerospace vehicles, the dynamic loads have become increasingly severe and intricate. The acquirement of external exciting forces is a prerequisite for the delicacy management such as flight control and health monitoring for aerospace vehicles. In practical engineering, it is difficult to measure external loads straightforwardly through force sensors, while structural responses under the load effect, such as the displacement, acceleration and strain, may be achieved effortlessly[1]. Therefore, it is a feasible approach to calculate external loads indirectly via measured dynamic responses and structural characteristics in combination with remarkable inversion approaches. The concept of load identification originated in the aviation field in the 1970s. It is proposed to acquire the actual load to enhance the performance of aircrafts[2].

In general, multi-source uncertainties are ineluctable for the dynamic load identification of aerospace vehicles, which signifies the identified load may also be indeterminate owing to the transitivity of multi-source uncertainties. On the one hand, the dynamic load may be stochastic with the speedy change of service environment[3]. Under this circum-

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stance, the random external load and structural responses must be quantified as stochastic processes. On the other hand, the intrinsic characteristics and measured responses may also be uncertain, caused by either static factors (e. g., material dispersion, machining tolerance and modeling error) or time-varying parameters (e. g., disturbance of boundary conditions and deviation of instrument measurement). In addition, these aforementioned uncertainties may be accumulated during the service process[4] of aerospace vehicles, and their cross-coupling effects will lead to numerous noise independent of the real loads, which hinders the precise identification of the external dynamic load.

The schematic diagram of dynamic load identification for aerospace vehicles considering multi-source uncertainties is demonstrated in Fig.1. In brief, it can be subdivided into two major categories: (1) The establishment of the deterministic load identification model[5]; (2) The quantification and propagation analysis of multi-source uncertainties. How to handle the influence of uncertainties on the inversion model and how to identify the uncertain dynamic load efficiently[6] have become hot-spot issues for many scholars[7-10]. Herein, the developed identification methods for different dynamic loads and the uncertainty analysis methods for uncertain load are reviewed in this paper, which can be seen in Fig.2.

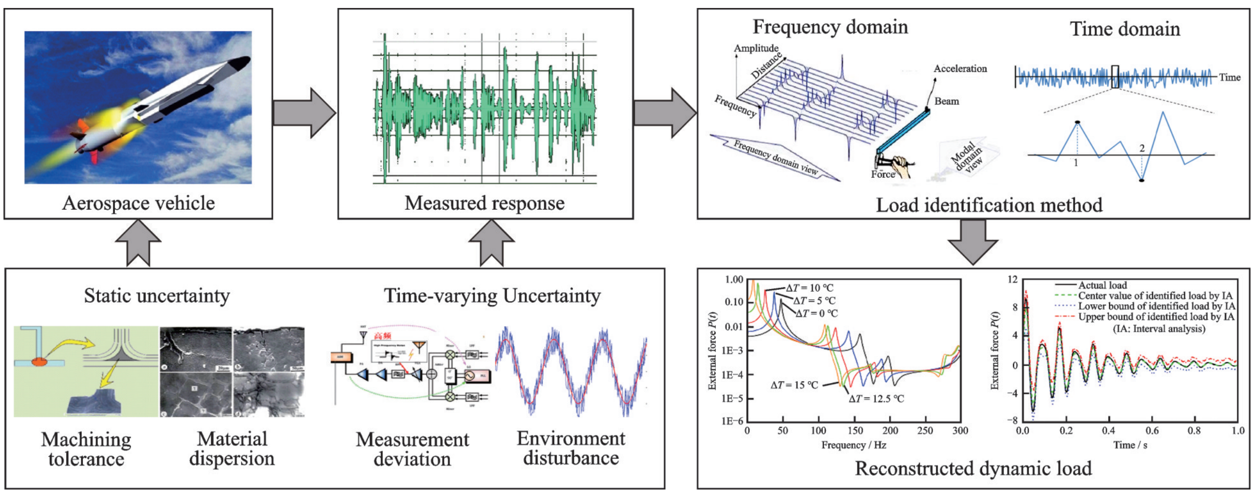

Fig.1 Schematic diagram of dynamic load identification for aerospace vehicles

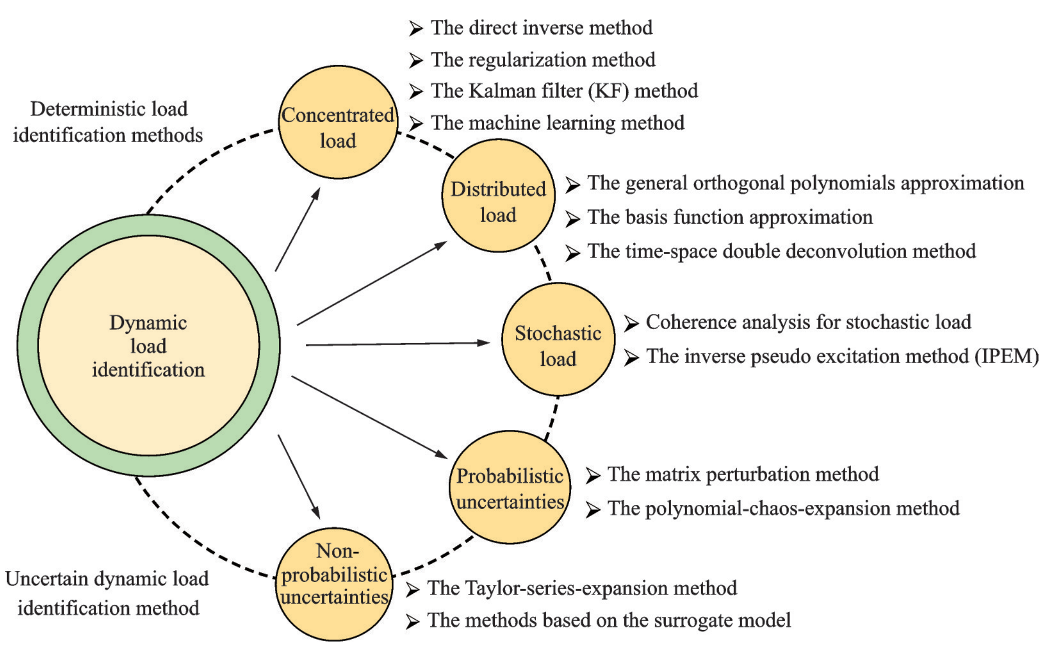

Fig.2 Dynamic load identification method

It is worthwhile mentioning that only the identification of loading history is summarized, without the identification of loading position and loading direction[11].

1 Deterministic Load Identification Methods

Different from the forward problem of structural dynamics, the dynamic load identification, called as the second inverse problem, is more complicated than solving the quadratic differential equation. Given the mathematical model, it can be divided into the frequency-domain-based method and the time-domain-based method[12]. It is noted that the time-domain-based method has attracted more attention, since it can reconstruct the dynamic time history forthrightly. In general, the dynamic load identification is implemented under the framework of the finite element method (FEM). The governing dynamic equation of aerospace vehicles can be commonly depicted as

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t)+\boldsymbol{C}\dot{\boldsymbol{u}}(t)+\boldsymbol{K}\boldsymbol{u}(t)=\boldsymbol{F}(t) \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$, stand for the mass, the damping and the stiffness matrices, respectively; $\boldsymbol{u}(t)$, $\dot{\boldsymbol{u}}(t)$ and $\ddot{\boldsymbol{u}}(t)$ are the displacement, the velocity and the acceleration responses, respectively; $\boldsymbol{F}(t)$ denotes the external force. Enlightened by the modal transformation, Eq.(1) can be rewritten as

$$\boldsymbol{M}_p\ddot{\boldsymbol{q}}(t)+\boldsymbol{C}_p\dot{\boldsymbol{q}}(t)+\boldsymbol{K}_p\boldsymbol{q}(t)=\boldsymbol{P}(t) \tag{2}$$

where $\boldsymbol{q}(t)$, $\dot{\boldsymbol{q}}(t)$ and $\ddot{\boldsymbol{q}}(t)$ are corresponding modal responses; $\boldsymbol{M}_p$, $\boldsymbol{C}_p$ and $\boldsymbol{K}_p$ represent the related modal characteristic matrices; $\boldsymbol{P}(t)$ means the modal force. Decoupling Eq.(2), we can get some linear differential equations, i.e.

$$m_r\ddot{q}_r(t)+c_r\dot{q}_r(t)+k_rq_r(t)=p_r(t)\qquad r=1,2,\cdots,m \tag{3}$$

where r means the r-th order equation, and m the number of truncated modes.

Actually, for simple structures, the dynamics calculation may be analyzed in physical space as Eq.(1), while for large-scale structures, it should be performed in modal space to reduce the computing effort. There are two categories to be discussed in this section: The concentrated load (single-point or multi-point) and the distributed load, among which the latter is completed by modifying the identification algorithm of the concentrated load.

1.1 Concentrated dynamic load identification

There are many excellent methods dealing with the identification of dynamic load, including the direct inverse method, the regularization method, the Kalman filter method and the machine learning method[13].

1.1.1 The direct inverse method

The direct inverse method aims to deconvolve this relationship between external load and system response, which is the most common approach in the early stage owing to its intuitionistic advantage[14-15]. In frequency domain, carrying out the Fourier transform on both sides of Eq.(2), yields

$$-\omega^2\boldsymbol{M}_p\boldsymbol{Q}(\omega)+\mathrm{i}\omega\boldsymbol{C}_p\boldsymbol{Q}(\omega)+\boldsymbol{K}_p\boldsymbol{Q}(\omega)=\boldsymbol{P}(\omega) \tag{4}$$

where ω is the frequency, $\boldsymbol{Q}(\omega)$ and $\boldsymbol{P}(\omega)$ are Fourier spectrums of the external load and the structural response, respectively. Introducing the matrix $\boldsymbol{H}(\omega)=(-\omega^2\boldsymbol{M}_p+\mathrm{i}\omega\boldsymbol{C}_p+\boldsymbol{K}_p)^{-1}$ of frequency response function (FRF), the load spectrum $\boldsymbol{P}(\omega)$ can be obtained by

$$\boldsymbol{P}(\omega)=\boldsymbol{H}^{+}(\omega)\boldsymbol{Q}(\omega) \tag{5}$$

where the superscript $+$ denotes the pseudo-inversion. Bartlett and Flannelly[16] used measured acceleration responses to identify vertical and lateral dynamic loads on the helicopter hub center in frequency domain initially. Hillary et al.[17] reconstructed the dynamic load of a cantilever structure using the FRF of different positions, and proved that strain responses outperform acceleration responses. Hansen and Starkey[18-19] revealed the ill condition of the FRF near the resonance region, and the growing identification error with the increase of the number of loads. Doyle[20-22] investigated considerable researches to reconstruct the location and history of the impact load.

In contrast, the direct inversion method in time domain emerged relatively late. Refs.[23-25] proposed this method for discrete-time systems based on modal coordinate transformation with regard to the flight load of rockets. It is assumed that the load is regarded as a constant during the period $[t_k, t_{k+1}]$, i.e., $p_r(t)=p_{rk}$, $t\in[t_k, t_{k+1}]$. Based on the Durham integral and the vibration equation, the result of Eq.(3) can be obtained as

$$\begin{cases} q(t)=q_r(t_k)C_r+\dfrac{\dot{q}_r(t_k)+\zeta_r\omega_r q_r(t_k)}{\omega_{dr}}S_r+ \\ \qquad \dfrac{p_{rk}}{m_r\omega_r^2}\left[1-C_r-\dfrac{\zeta_r\omega_r}{\omega_{dr}}S_r\right] \\ C_r(t,k)=\mathrm{e}^{-\zeta_r\omega_r(t-t_k)}\cos\omega_{dr}(t-t_k) \\ S_r(t,k)=\mathrm{e}^{-\zeta_r\omega_r(t-t_k)}\sin\omega_{dr}(t-t_k) \end{cases} \tag{6}$$

where $\omega_r=\sqrt{k_r/m_r}$ and $\omega_{dr}=\omega_r\sqrt{1-\zeta_r^2}$ are the angular frequencies; and $\zeta_r=c_r/(2m_r\omega_r)$ is the damping ratio. Differentiating Eq.(6), the modal responses of $\dot{q}_r(t)$ and $\ddot{q}_r(t)$ can be got. The sequence $\boldsymbol{P}(t_k)=[p_1(t_k), p_2(t_k), \cdots, p_m(t_k)]$ of modal loads can be provided utilizing modal responses. The dynamic load can be inversed by modal transformation

$$\boldsymbol{F}(t_k)=[\boldsymbol{\Phi}^{\mathrm{T}}]^{+}\boldsymbol{P}(t_k) \tag{7}$$

where $\boldsymbol{\Phi}$ is the truncated modal matrix.

Suppose that the number of measured responses is n_r, and the number of external concentrated load is n_f. The following relationship should be satisfied: $n_r\geqslant m\geqslant n_f$. Ref.[26] presented five methods to obtain the transfer function between force and strain response to reduce noise interference. Sandesh et al.[27] identified external excitations and interface forces with the iterative time-domain identification. Liu et al.[28] proposed a time-domain Galerkin method for dynamic load identification, which can overcome the influence of noise

1.1.2 The regularization method

The direct inverse method can be summarized as $\boldsymbol{Ax}=\boldsymbol{y}$, $\boldsymbol{x}\in\boldsymbol{X}$, $\boldsymbol{y}\in\boldsymbol{Y}$, where $\boldsymbol{A}$ is the operator, $\boldsymbol{X}$ the solution space, and $\boldsymbol{Y}$ the data space. Unfortunately, the inversion of operator $\boldsymbol{A}$ is often ill-conditioned leading to unstable results. There is no denying that the regularization method aims to find a stable approximate solution to replace the exact solution of the inverse problem, in which two issues should be discussed: (1) The construction of regularization operator; (2) The selection of regularization parameters. Tihonov regularization[29] is an outstanding method for dynamic load identification, whose basic idea can be described as follows. For a bounded linear operator $\boldsymbol{A}$, solve $\boldsymbol{x}^\alpha\in\boldsymbol{X}$ to minimize the Tikhonov functional $J_\alpha(\boldsymbol{x})$, namely

$$J_\alpha(\boldsymbol{x})=\left\|\boldsymbol{Ax}-\boldsymbol{y}\right\|_Y^2+\alpha\boldsymbol{\Omega}(\boldsymbol{z})=\left\|\boldsymbol{Ax}-\boldsymbol{y}\right\|_Y^2+\alpha\left\|\boldsymbol{x}\right\|_X^2 \tag{8}$$

where $\|\times\|$ represents the norm, α the regularization parameter and $\boldsymbol{\Omega}(\boldsymbol{z})$ is set as the stabilization functional. Minimizing the functional $J_\alpha(\boldsymbol{x})$, the result of the Tikhonov regularization method can be expressed as $\boldsymbol{x}^\alpha=(\boldsymbol{A}^*\boldsymbol{A}+\alpha\boldsymbol{I})^{-1}\boldsymbol{A}^*\boldsymbol{y}$. To obtain the numerical solution, take the singular value decomposition (SVD) for $\boldsymbol{A}$, i.e., $\boldsymbol{A}=\boldsymbol{U}_{m\times n}\boldsymbol{\Sigma}_{n\times n}\boldsymbol{V}_{n\times n}^{\mathrm{T}}=\sum_{i=1}^{n}\boldsymbol{u}_i\sigma_i\boldsymbol{v}_i^{\mathrm{T}}$. The solution $\boldsymbol{x}^\alpha$ may be further rewritten as

$$\boldsymbol{x}^\alpha=\sum_{i=1}^{n}f_\alpha(\sigma_i^2)\frac{\boldsymbol{u}_i^{\mathrm{T}}\boldsymbol{y}}{\sigma_i}\boldsymbol{v}_i \tag{9}$$

where $\boldsymbol{A}^*$ is the adjoint matrix, and $f_\alpha(\sigma_i^2)$ the Tikhonov filtration factor determined by regularization parameter α and the singular value σ_i^2.

Other regularization methods, like the truncated singular value decomposition (TSVD)[30] and the iterative regularization method[31], have also been extensively employed. Given that improper selection of regularization parameters will lead to unacceptable results, some methods, including generalized deviation criterion[32], generalized cross-validation (GCV) method[33] and L-curve criterion[34] have been established to determine the regularization parameter. Jacquelin et al.[35] introduced the regularization algorithm to the impact load identification process, and discussed the influence of different regularization methods on the identification results. Wang et al.[36] studied the load identification on composite laminated cylindrical shells through Tikhonov regularization with a new regularized filter operator. Numerous researches[37-39] have confirmed the benefits of the regularization method.

1.1.3 The Kalman filter (KF) method

Kalman filter is a recursive algorithm originated from the control field[40], which is modeled on the basis of the state-space equation. Different from the traditional Kalman filter, this method for dynamic load identification can estimate the system state and the unknown input simultaneously. The discrete state-space equation of Eq.(1) can be expressed as

$$\begin{cases} \boldsymbol{X}(k+1)=\boldsymbol{A}\boldsymbol{X}(k)+\boldsymbol{B}\boldsymbol{F}(k)+\boldsymbol{w}(k) \\ \boldsymbol{Z}(k)=\boldsymbol{C}\boldsymbol{X}(k)+\boldsymbol{v}(k) \end{cases} \tag{10}$$

where $\boldsymbol{X}(k)$ embodies the state vector, and $\boldsymbol{Z}(k)$ the observation vector. $\boldsymbol{w}(k)$ and $\boldsymbol{v}(k)$ are the process noise vector and measurement noise vector. $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ denote the state transition matrix, the input matrix and the identity matrix, respectively.

The two-stage and two-step recursion method[41-42], combining the Kalman filter and the least square algorithm, is most widespread for dynamic load identification. The dynamic load may be estimated recursively by virtue of the gain matrix, update state and covariance matrix generated, which was described in Ref.[13]. In terms of the weighting coefficient of the recursive least-squares algorithm, the conventional weighting input estimation (WIE)[43], the adaptive WIE[44] and the intelligent fuzzy WIE[45] have been presented successively. In addition, Gillijns et al.[46-47] proposed an unbiased minimum-variance input and state estimation for linear discrete-time systems with acceleration and displacement responses. Hsieh et al.[48] extended the input and state estimation from one-step delay to multi-step delay. As for the nonlinear system, Ma et al.[49] contributed the extended Kalman filter (EKF) for nonlinear estimation, in which the first-order Taylor expansion is used to linearize the nonlinear model, and the standard Kalman filter algorithm is used to estimate the state and unknown load. Ref.[50] used the unscented Kalman filter (UKF) to avoid the derivatives, Jacobians calculation and linearization approximations of EKF.

Another feasible approach is to extend the unknown input vector to the state vector, then use the standard Kalman filter to estimate the extended state vector. Lourens et al.[51] proposed an augmented Kalman filter (AKF) technique for joint input-state estimation based on reduced-order models and vibration data from a limited number of sensors. Ref.[52] investigated a multi-metric approach to enhance the stability and accuracy of the force estimation by the AKF method.

1.1.4 The machine learning method

Substantially, the dynamic load identification can be regarded as an optimization problem. With the development of computer technology, some intelligent optimization algorithms based on machine learning have been proposed, and been gradually integrated into the field of dynamic load identification. The characteristics of a specific structure have been concealed in input-output samples, so the complex nonlinear relationship between the dynamic response and load may be reasoned and learned by machine learning models.

As a classical intelligent optimization algorithm, the neural network was used for load identification initially, whose processor can be summarized as follows: (1) Determine the topological structure of the neural network; (2) Adjust the network parameters in the training and learning process; (3) Obtain the load sequence by inputting the measured response. Cao et al.[53] simulated the strain-load relationship of aircraft wings by the artificial neural network, and analyzed the influence of network structure, training algorithm and learning speed. Trivailo et al.[54] predicted both high-frequency buffet and low-frequency manoeuvre loading through the Elman network to improve the fatigue monitoring capability of F/A-18 Empennage. Zhou et al.[55] reconstructed the impact load of nonlinear structures using the deep recurrent neural network, whose effectiveness was verified through an experiment of a composite plate. Compared with the traditional methods, the neural-network-based method may have a wider prospect owing to the higher identification accuracy and stronger anti-interference ability. However, there is no universal method to determine the network structure. Under such circumstances, the support vector machine has been introduced into the load identification, and its validity is also clarified in Refs.[56-57] Furthermore, the optimization algorithms like genetic algorithm have also been applied to the load identification. Yan et al.[58] established an objective function for load identification based on the minimum difference between the calculated response and the measured response. Thus, the inverse problem can be transformed into a forward problem of parameter optimization.

Much work so far has focused on the identification of concentrated dynamic load. In addition to the aforementioned methods, there have also been many remarkable methods with distinguished advantages, which are not be expound them in detail herein.

1.2 Distributed dynamic load identification

The distributed dynamic load acting on continuous structures is more complicated than the concentrated load. There are relatively few studies devoted to distributed dynamic load identification. Time variable and space variable are both involved for the distributed dynamic load, which may be independent or coupled with each other. For the continuous structure, the governing equation can be commonly depicted as

$$\rho\alpha\frac{\partial^2 u(\boldsymbol{x},t)}{\partial t^2}+c\frac{\delta u(\boldsymbol{x},t)}{\partial t}+E\beta\frac{\partial^4 u(\boldsymbol{x},t)}{\partial \boldsymbol{x}^4}=f(\boldsymbol{x},t) \tag{11}$$

where $\boldsymbol{x}(x,y,z)$ represents the space variable. ρ, c and E signify the material density, damping coefficient and elastic modulus; α and β are the structural parameters; and $u(\boldsymbol{x},t)$ and $f(\boldsymbol{x},t)$ are the displacement response the distributed dynamic load. It is known that $u(\boldsymbol{x},t)$ and $f(\boldsymbol{x},t)$ are both continuous functions in time-space dimension. In general, only discrete responses of limited measuring points may be obtained, which increases the difficulty to reconstruct the distributed function. Thus, it is necessary to transform the infinite-dimensional function into a finite-dimensional subspace. Approximating the distributed load by a set of linearly independent basis functions is a promising choice. Several identification methods by functional approximation will be discussed below.

1.2.1 The generalized orthogonal polynomials approximation

The generalized orthogonal polynomial is a common function approximation method in a specific interval, such as Legendre orthogonal polynomials, Chebyshev orthogonal polynomials, Laguerre orthogonal polynomials and Hermite orthogonal polynomials. Due to the title of "the most economical expansion", Chebyshev orthogonal polynomials are usually used in distributed load identification. The coordinates of structures need to be projected to the standard interval $[-1,1]$ firstly. The expression of Chebyshev orthogonal polynomial[59] with weight function $\rho(x)=1/\sqrt{1-x^2}$ is

$$T_n(x)=\cos(n\arccos x)\qquad -1\leqslant x\leqslant 1,\ n=0,1,2,\cdots \tag{12}$$

By discretizing the dynamic response in time dimension, the distributed load at time t_i can be fitted by one-dimensional orthogonal polynomials taking one-dimensional structures for example, namely

$$f(x,t_i)=\sum_{j=1}^{J}T_j(x)a_j(t_i) \tag{13}$$

where J is the order of a polynomial, and $a_j(t_i)$ the coefficient of the j-th polynomial.

By virtue of the FEM, the distributed load will be considered as a series of discretized loads on each node of the structure, which can be expanded as

$$f(\boldsymbol{x},t)\rightarrow \boldsymbol{F}(\boldsymbol{x},t_i)=\boldsymbol{T}(\boldsymbol{x})\boldsymbol{A}(t_i) \tag{14}$$

where $\boldsymbol{T}(\boldsymbol{x})$ denotes the polynomial matrix corresponding to the node position, and $\boldsymbol{A}(t_i)$ the coefficient vector. On the basis of the concentrated load identification method reviewed in Section 1.1, the coefficients of orthogonal polynomials will be calculated. Dessi[60] identified the distribution of a wave load acting on a slender floating body using the proper orthogonal decomposition and integral spline approximation technique. Wang et al.[4,61] proposed the distributed dynamic load acting on continuous structures including the cantilever beam and cantilever plate, in which the spatial load is approximated by Chebyshev orthogonal polynomials in time history under the load assumption of piecewise format.

1.2.2 The basis function approximation

For the linear system, the distributed dynamic load and the structural response can be expanded by the given basis function[62], namely

$$\begin{cases} f(\boldsymbol{x},t_i)=\sum_{i=1}^{N}W_i(t_i)\chi_i(\boldsymbol{x}) \\ u(\boldsymbol{x},t_i)=\sum_{i=1}^{N}W_i(t_i)\varphi_i(\boldsymbol{x}) \end{cases} \tag{15}$$

where $\chi_i(\boldsymbol{x})$ denotes the i-th basis function only spanning in the forcing space, $\varphi_i(\boldsymbol{x})$ the structural responses generated by the load basis function $\chi_i(\boldsymbol{x})$, and $W_i(t_i)$ the common weighting coefficient for dynamic load $F(\boldsymbol{x}, t_i)$ and structural response $u(\boldsymbol{x}, t_i)$. That is, the dynamic response $\varphi_i(\boldsymbol{x})$ is produced by the force $\chi_i(\boldsymbol{x})$ purely. The dynamic load may be a single harmonic or multi-harmonic cases. Thus, the determination of weighting coefficient $W_i(t_i)$ is the key to the reconstruction of distributed load, which can be numerically obtained by conventional modal analysis. As the curve-fitting method

$$\boldsymbol{GW}=\boldsymbol{B} \tag{16}$$

where $G_{ij}=\int\varphi_i(\boldsymbol{x})\varphi_j(\boldsymbol{x})\mathrm{d}\Omega$ and $B_{ij}=\int\varphi_i(\boldsymbol{x})u(\boldsymbol{x}, t_i)\mathrm{d}\Omega$. The $\varphi_i(\boldsymbol{x})$ can be obtained by either numerical or experimental studies. Thus, the weighting coefficient $W_i(t_i)$ may be inversed by Eq.(16), and the distributed load may be expanded by Eq.(15).

Li et al.[63] assumed that the time history and the distribution function of the load are independent. The spatial function of the distributed load and response are fitted by finite basis functions using polynomial the selection technique, and then the time history can be reconstructed based on the shape function method of moving least-square-fitting. Cameron et al.[64-65] conducted the identification of the distributed flight load acting along the span and chord direction of aircraft through a least-squares minimization of Fourier coefficients with database Fourier coefficients.

1.2.3 The time-space double deconvolution method

For the distributed load $f(\boldsymbol{x}, t)$ with time-space decoupled characteristics, it can be expressed by the product of the distribution function $\psi(\boldsymbol{x})$ and the time history function $s(t)$, namely $f(\boldsymbol{x}, t)=\psi(\boldsymbol{x})s(t)$. The displacement responses can be analyzed by

$$u(\boldsymbol{x}, t)=\int_0^t s(\tau)\mathrm{d}\tau\int_{\varsigma_0}^{\varsigma_1}\psi(\boldsymbol{x}')g(\boldsymbol{x}|\boldsymbol{x}', t-\tau)\mathrm{d}\boldsymbol{x}' \tag{17}$$

where $g(\boldsymbol{x}|\boldsymbol{x}', t-\tau)$ is the Green's kernel function. $[\varsigma_0, \varsigma_1]$ is the loading area. In general, the arbitrary response can be regarded as the superposition of the responses caused by all loads. By discretizing them in time and space dimension, Eq.(17) can be transformed as

$$u(\boldsymbol{x}, t_i)=\sum_{k=1}^{i}s(k\Delta t)\Delta t\sum_{n=1}^{N}\psi(\boldsymbol{x}^n)\Delta\boldsymbol{x}'g(\boldsymbol{x}|\boldsymbol{x}^n,(i-k)\Delta t) \tag{18}$$

In the following, three situations will be discussed for Eq.(18). For the identification of the time history function $s(t)$, the distribution function $\psi(\boldsymbol{x})$ is assumed to be known in prior. Eq.(18) can be transformed as $\boldsymbol{u}=\boldsymbol{\Psi}_1\boldsymbol{S}_1$, in which $\boldsymbol{u}=[u(t_1)\ u(t_2)\ \cdots\ u(t_m)]^{\mathrm{T}}$. $\boldsymbol{\Psi}_1$ is composed of $\boldsymbol{\Psi}_1(t_i)=\sum_{n=1}^{N}\psi(\boldsymbol{x}^n)\Delta\boldsymbol{x}'g(\boldsymbol{x}|\boldsymbol{x}^n,(i-k)\Delta t)$, and $\boldsymbol{S}=[s(t_1)s(t_2)\cdots s(t_m)]^{\mathrm{T}}$ is the time sequence to be identified. For the identification of the distribution function $\psi(\boldsymbol{x})$, the time history function $s(t)$ is assumed to be known in advance. Eq.(18) can be transformed as $\boldsymbol{u}=\boldsymbol{S}_2\boldsymbol{\Psi}_2$, in which $\boldsymbol{S}_2$ is composed of $\boldsymbol{S}_2=\sum_{k=1}^{m}s(k\Delta t)\Delta tg(\boldsymbol{x}|\boldsymbol{x}^n,(i-k)\Delta t)$, $\boldsymbol{\Psi}_2=[\psi(\Delta\boldsymbol{x}')\ \psi(2\Delta\boldsymbol{x}')\cdots\psi(N\Delta\boldsymbol{x}')]^{\mathrm{T}}$ is the space sequence to be identified. For the identification of both $\psi(\boldsymbol{x})$ and $s(t)$, an initialization assumption should be made in advance, then the two aforementioned steps should be repeatedly. This process is named as double iterative optimization.

Under the guidance of this method, Liu et al.[66-67] studied the iterative identification method of line distributed load on a composite plate using the displacement response, which assumed that the time-domain and spatial-domain of the load may be separated. Jiang et al.[68] identified the distributed dynamic load of a vibrating Euler-Bernoulli beam based on the mode-selection method using the consistent spatial expression. Li et al.[69] proposed a decoupling strategy based on the Green's function method and the orthogonal polynomial approximation to identify the time history and special distribution separately.

To achieve a better understanding of the identification methods of deterministic concentrated/distributed load, their merits and demerits are summarized in Table 1.

Table 1 The advantages and disadvantages of deterministic load identification

Classification	Method	Advantage	Disadvantage
Concentrated dynamic load identification	The direct inverse method	Its principle is simple, and it is easy to apply to practical engineering.	The numerical calculation is unstable and the error accumulates in time domain.
	The regularization method	It has good anti-noise and robust performance.	The selection of optimal regularization parameters is not universal.
	The KF method	It has little dependence on boundary conditions and initial values.	It struggles with the issue of low-frequency-drift only using acceleration responses.
	The machine learning method	It has high accuracy, strong noise resistance	It is difficult to determine the super parameters of intelligent algorithms.
Distributed dynamic load identification	The generalized orthogonal polynomials approximation	Its principle is simple and more perceivable.	It is only applicable to the distributed loads with a combination of polynomials.
	The basis function approximation	This method is simple to understand.	Common weighting coefficient needs to be determined in advance.
	The time-space double deconvolution method	The time dimension and the space dimension of dynamic distributed load are separated.	It converges slowly if the spatiotemporal variables are identified simultaneously.

2 Uncertain Dynamic Load Identification Methods

The dynamic load identification method described in Section 2 is carried out under the deterministic assumption of structural performance, measured response and dynamic load. However, uncertain factors exist widely in all processes of load identification, which leads to the deviation between the reconstructed results and the actual load. Thus, exploring the influence of multi-source uncertainties on the identification of dynamic load is of great significance for guiding the design and analysis of aerospace vehicles. The dynamic equation considering multi-source uncertainties can be transformed as

$$\boldsymbol{M}(\boldsymbol{\alpha})\ddot{\boldsymbol{u}}(\boldsymbol{\alpha},t)+\boldsymbol{C}(\boldsymbol{\alpha})\dot{\boldsymbol{u}}(\boldsymbol{\alpha},t)+\boldsymbol{K}(\boldsymbol{\alpha})\boldsymbol{u}(\boldsymbol{\alpha},t)=\boldsymbol{F}(\boldsymbol{\alpha},t) \tag{19}$$

where $\boldsymbol{\alpha}=[\alpha_1,\alpha_2,\cdots,\alpha_q]$ denotes the q-dimensional uncertain parameters.

2.1 The identification of the stochastic dynamic load

The difference between the stochastic dynamic load and deterministic dynamic load lies in the uncertainties in time history and the correlation between each load. Since the stochastic dynamic load cannot be expressed by an exact time function, their power spectrum (PS) characteristics in frequency domain based on the theory of probability statistics are always considered as the variables to be identified.

2.1.1 Coherence analysis for stochastic loads

Multi-point stochastic dynamic load will host the basis in this section. The coherence can be divided into three categories: Complete coherence, partial coherence and complete incoherence, which reflects the degree of linearity between two loads and used for the mean value analysis. Complete coherence means that the stochastic dynamic loads are homologous, while complete incoherence signifies the cross-PS between any two loads is zero. The PS matrix of n-point stochastic excitation is defined as $\boldsymbol{S}_F(\omega)=[S_{f_i f_j}]_{n\times n}$, $i,j=1,2,\cdots,n$, whose properties vary with their coherence. Through the coherence analysis of three kinds of stochastic excitation[70], it can be concluded that the nonnegative definite PS has a unified spectral decomposition formula, namely

$$\boldsymbol{S}_F(\omega)=\sum_{i=1}^{r}\boldsymbol{l}_i\boldsymbol{l}_i^{\mathrm{H}} \tag{20}$$

where r indicates the rank of $\boldsymbol{S}_F(\omega)$. When the multi-point stochastic dynamic loads are completely coherent, $r=1$. When they are completely incoherent, $r=n$. When they are partial coherent, $1<r<n$. In addition, the PS matrix $\boldsymbol{S}_F(\omega)$ and its spectral vector $\boldsymbol{l}_i$ is one-to-one corresponding due to the uniqueness of the spectral decomposition formula, eigenvalues and eigenvectors, namely, the $\boldsymbol{S}_F(\omega)$

can be represented exclusively by $\boldsymbol{l}_i$. This is the theoretical source of the identification of the stochastic dynamic load.

2.1.2 The inverse pseudo excitation method (IPEM)

Based on the theory of stationary stochastic vibration, the PS $\boldsymbol{S}_F(\omega)$ can be got by the PS $\boldsymbol{S}_U(\omega)$ of response and the FRF matrix $\boldsymbol{H}(\omega)$, i.e.

$$\boldsymbol{S}_F(\omega)=[\boldsymbol{H}^+(\omega)]^*\boldsymbol{S}_U(\omega)[\boldsymbol{H}^+(\omega)]^{\mathrm{T}} \quad (21)$$

However, direct inverse as Eq.(21) is faced with the ill-posed problem of FRF matrix. The IPEM is proposed by Lin[71-72], which is simple and efficient for stochastic vibration. It decomposes the PS $\boldsymbol{S}_U(\omega)$ and constructs a virtual response y_i as follows

$$\boldsymbol{S}_U(\omega)=\sum_{i=1}^{r}\boldsymbol{b}_i\boldsymbol{b}_i^{\mathrm{H}} \qquad y_i=\boldsymbol{b}_i\mathrm{e}^{\mathrm{j}\omega t} \quad (22)$$

where the virtual response y_i may be assumed to be generated by the virtual excitation $f_i=\boldsymbol{l}_i\mathrm{e}^{\mathrm{j}\omega t}$. Combining $f_i=\boldsymbol{H}^+(\omega)y_i$, the PS $\boldsymbol{S}_F(\omega)$ can be obtained as Eq.(20).

Guo et al.[73] identified the power spectral density matrix of uncorrelated or partially correlated random excitation experimentally, and confirmed the efficiency of the IPEM. It is noted that the inverse operation of the FRF matrix still exists in the traditional IPEM. Thus, some weighted techniques are introduced to change the condition number of the FRF matrix. Leclere et al.[74] reconstructed the internal loads exciting the engine block via weighted pseudo-inverse of the transfer matrix during operation to alleviate its ill-conditioning. Jia et al.[75-76] proposed a Tikhonov regularization approach based on error analysis and weighted total least squares method, and provided a selection method and a concrete form of weighting matrix. For the distributed stochastic excitation, Granger et al.[77] adopted the Tikhonov regularization method with Newton iteration to reconstruct the distributed load on nonlinear structures. Jiang et al.[78] identified the one-dimension distributed stochastic load by the IPEM combined with generalized Fourier expansion and projection technique.

2.2 The load identification under structural probabilistic uncertainties

This section aims at the identification of conventional loads with exact time functions for uncertain aerospace vehicles with random fluctuation. When the uncertainties with sufficient sample information, the quantification and propagation analysis based on the probabilistic model has been developed completely. The Monte Carlo simulation (MCS)[79] can obtain the statistical properties of uncertain load by generating enough samples according to the probability density function (PDF) of uncertain parameters. However, it is not suitable for engineering application, and often be treated as a verification method. Subsequently, some methods have been proposed as follows.

2.2.1 The matrix perturbation method

The probabilistic uncertain parameter $\boldsymbol{\alpha}$ can be defined as its mean $\boldsymbol{\alpha}_{\mathrm{m}}$ plus a small random perturbation $\Delta\boldsymbol{\alpha}_{\mathrm{r}}$, namely $\boldsymbol{\alpha}=\boldsymbol{\alpha}_{\mathrm{m}}+\Delta\boldsymbol{\alpha}_{\mathrm{r}}$. Based on the perturbation theory[80], the relationship between external dynamic load and responses considering probabilistic uncertainties be written as

$$\boldsymbol{U}=[\boldsymbol{G}_{\mathrm{m}}(\boldsymbol{\alpha})+\Delta\boldsymbol{G}_{\mathrm{r}}(\boldsymbol{\alpha})][\boldsymbol{F}_{\mathrm{m}}(\boldsymbol{\alpha})+\Delta\boldsymbol{F}_{\mathrm{r}}(\boldsymbol{\alpha})] \quad (23)$$

By comparing the coefficients on both sides of Eq.(23), it can be transformed into two kinds of deterministic issues on the bases of Taylor series expansion, namely

$$\boldsymbol{U}=\boldsymbol{G}_{\mathrm{m}}(\boldsymbol{\alpha})\boldsymbol{F}_{\mathrm{m}}(\boldsymbol{\alpha})-\frac{\partial \boldsymbol{G}(\boldsymbol{\alpha})}{\partial \alpha_i}\boldsymbol{F}_{\mathrm{m}}(\boldsymbol{\alpha})= \boldsymbol{G}_{\mathrm{m}}(\boldsymbol{\alpha})\frac{\partial \boldsymbol{F}(\boldsymbol{\alpha})}{\partial \alpha_i} \qquad i=1,2,\cdots,q \quad (24)$$

In other words, the dynamic load can be identified by the calculation of the mean value of external load and its sensitivity with respect to each random parameter. The stochastic characteristics of identified dynamic load can be further obtained as

$$\begin{cases} E[\boldsymbol{F}(\boldsymbol{\alpha})]=E[\boldsymbol{F}_{\mathrm{m}}(\boldsymbol{\alpha})]+E[\Delta\boldsymbol{F}_{\mathrm{r}}(\boldsymbol{\alpha})]=\boldsymbol{F}_{\mathrm{m}}(\boldsymbol{\alpha}) \\ \mathrm{var}[\boldsymbol{F}(\boldsymbol{\alpha})]=\sum_{i=1}^{q}\left[\frac{\partial \boldsymbol{F}(\boldsymbol{\alpha})}{\partial \alpha_i}\sigma(\alpha_i)\right]^2 \end{cases} \quad (25)$$

By far the most works have been devoted to the response analysis for stochastic structures, yet a few examples have been applied in the dynamic load identification. Considering the randomness of geometrical, physical and boundary property, Sun et al.[81], He et al.[82] and Wang et al.[83] combined the perturbation theory and regularization method to evaluate the dynamic load of random structures using noisy responses, whose accuracy and efficiency are explained by several numerical examples compared with the MCS. Particularly, the first-order matrix perturbation method is restricted to the situation where the coefficient of variation of random parameters is quite small. Then, the Neumann expansion[84] is usually used to obtain the higher-order statistics information for the matrix perturbation method.

2.2.2 The polynomial-chaos-expansion method

As mentioned above, the first-order perturbation method is not a perfect way for stochastic structures[85] with large fluctuation. Moreover, it disregards the distribution form of random parameters, so the same results will be obtained no matter what the PDF is. In view of this, Liu et al.[86-88] proposed a novel uncertain load identification method for stochastic structures with unimodal and bounded PDF based on polynomial chaos expansion. Similar to the matrix perturbation method, it also converts the complex stochastic analysis to several deterministic problems. If the uncertain parameter α_i is unimodal, it can be expressed as the function of a stochastic parameter β_i with λ-PDF, i.e.

$$\alpha_i \approx b_{0i} + b_{1i}\beta_i + b_{2i}\beta_i^2 \qquad i = 1, 2, \cdots, q \tag{26}$$

Then, the transfer matrix $\boldsymbol{G}(\boldsymbol{\alpha})$ can be depicted by the function of stochastic parameter β_i, as

$$\boldsymbol{G}(\boldsymbol{\alpha}) = \boldsymbol{G}(\boldsymbol{\beta}) = \boldsymbol{G}_0 + \sum_{i=1}^{q} (\boldsymbol{G}_i'\beta_i + \boldsymbol{G}_i''\beta_i^2) \tag{27}$$

The external load $\boldsymbol{F}(\boldsymbol{\alpha})$ can be expanded as the sum of a series of polynomial chaos $\boldsymbol{\varphi}$

$$\boldsymbol{F}(\boldsymbol{\alpha}) = \boldsymbol{F}(\boldsymbol{\beta}) = \sum_{i_1=0}^{N_1} \cdots \sum_{i_q=0}^{N_q} z_{i_1 \cdots i_q} \varphi_{i_1}^{\lambda_1}(\beta_i) \cdots \varphi_{i_q}^{\lambda_q}(\beta_q) \tag{28}$$

Inspired by the orthogonality of polynomials, the coefficients $z_{i_1 \cdots i_q}$ can be solved by the deterministic load identification technology. Eventually, the statistical properties of uncertain loads can be derived from the definitions of mean value and covariance. Schoefs et al.[89] estimated the system characteristics of offshore platforms by polynomial chaos expansion, and then identified the periodic tidal loads. Wu et al.[90] reconstructed the responses and forces of a stochastic system using polynomial chaos expansion and the Karhunen-Loève expansion, and investigated the influence of different correlation lengths of random system parameters on identified results. In addition, several other excellent methods have been successively used to deal with load identification for stochastic structures. For example, Batou et al.[91] reconstructed stochastic loads of a nonlinear dynamical system considering model uncertainties and data uncertainties, in which the mean value and dispersion parameter of PS function are calculated using the computational stochastic model and experimental responses. Zhang et al.[92] presented a Bayesian approach for force reconstruction considering measurement noise and model uncertainty, in which uncertain FRFs are settled by Monte Carlo Markov chain methods.

2.3 The load identification under structural non-probabilistic uncertainties

Subjected to measurement cost and technology, it is impossible to gain enough samples of uncertain parameters to determine their PDF. Given this, the non-probabilistic model is introduced to quantize uncertain parameters. The interval model based on interval mathematics is a general method, in which only the upper and lower boundaries are necessary. The uncertain interval parameter vector[93] can be defined as

$$\boldsymbol{\alpha} \in \boldsymbol{\alpha}^I = [\underline{\boldsymbol{\alpha}}, \bar{\boldsymbol{\alpha}}] \qquad \alpha_i \in \alpha_i^I = [\underline{\alpha}_i, \bar{\alpha}_i] \tag{29}$$

where $\underline{\times}$ and $\overline{\times}$ denote the lower bound and upper bound, respectively. For the issue of dynamic load identification, the uncertain load can be expressed as

$$\boldsymbol{F}(\boldsymbol{\alpha}) \in \boldsymbol{F}^I(\boldsymbol{\alpha}) = [\underline{\boldsymbol{F}}(\boldsymbol{\alpha}), \bar{\boldsymbol{F}}(\boldsymbol{\alpha})] \tag{30}$$

where $\underline{\boldsymbol{F}}(\boldsymbol{\alpha}) = \min[\boldsymbol{F}(\boldsymbol{\alpha})]$ and $\bar{\boldsymbol{F}}(\boldsymbol{\alpha}) = \max[\boldsymbol{F}(\boldsymbol{\alpha})]$ are the variables that should be calculated. The vertex combination method is the most effortless method for interval uncertainty analysis, but it is only suitable for monotonic problems. The uncertainty propagation methods based on the Taylor-series-expansion method and surrogate model will be reviewed in the following.

2.3.1 The Taylor-series-expansion method

To facilitate the analysis and discussion, two variables are defined firstly, namely the interval median $\boldsymbol{\alpha}_c=(\underline{\boldsymbol{\alpha}}+\bar{\boldsymbol{\alpha}})/2$, and the interval radius $\boldsymbol{\alpha}_r=(\bar{\boldsymbol{\alpha}}-\underline{\boldsymbol{\alpha}})/2$. When the uncertain level of all parameters α_i is small, the uncertain load can be expanded by the first-order Taylor-series at the interval median, namely

$$F(\boldsymbol{\alpha})\approx F(\boldsymbol{\alpha}_c)+\sum_{i=1}^{q}\frac{\partial F(\boldsymbol{\alpha}_c)}{\partial \alpha_i}\delta\alpha_i \tag{31}$$

Thus, the load boundaries can be approximated by

$$\begin{cases}\underline{F}(\boldsymbol{\alpha})=F(\boldsymbol{\alpha}_c)-\sum_{i=1}^{q}\left|\frac{\partial F(\boldsymbol{\alpha}_c)}{\partial \alpha_i}\right|\alpha_{ir}\\ \bar{F}(\boldsymbol{\alpha})=F(\boldsymbol{\alpha}_c)+\sum_{i=1}^{q}\left|\frac{\partial F(\boldsymbol{\alpha}_c)}{\partial \alpha_i}\right|\alpha_{ir}\end{cases} \tag{32}$$

Similar to the matrix perturbation method in probabilistic model, the Taylor-series-expansion method only need $q+1$ times deterministic inversing calculation to determine the load boundary, including the load identification at the interval median and the gradient calculation with respect to each parameter.

The dynamic load identification of structures with interval uncertainties has been intensively investigated. Liu et al.[94-96] explored a series of researches of load identification with regularization methods considering measurement noises and model uncertainties, to reconstruct the time history of the load interval. Ahmari et al.[97] established an inverse analysis scheme, in which the result of the impact location is in a rectangle, and the result of time history is bounding sinusoidal curves with deviation. However, the Taylor-series-expansion method has significant advantages only when the uncertainty problem is linear or the uncertain level is small. Therefore, Wang et al.[61] applied the subinterval technique in dynamic load identification to avoid interval extension. But it is important to determine the subinterval number for each variable to make a tradeoff between efficiency and convergence.

2.3.2 The methods based on the surrogate model

The key point of dynamic load identification under non-probabilistic uncertainties is to get its maximum and minimum value in the interval domain of uncertain variables. In order to further reduce the overestimation or underestimation of load interval caused by uncertainty propagation analysis, some surrogate models, such as the Kriging model, polynomial response surface method and artificial neural network, are used to approximate the relationship between uncertain loads and uncertain parameters. The detailed construction ways have been summarized in Ref.[98]. Generally, the construction of the surrogate model is regarded as the uncertainty propagation in the inner layer, and the optimization algorithm is needed in the outer layer to find the extreme point of uncertain load at each sampling instant.

Compared with the numerical simulation of the original FEM, the uncertainty propagation analysis based on surrogate model can effectively reduce the computational cost and filter out the unwished noise. But its accuracy and efficiency are highly dependent on the selection of the surrogate model and its hyper-parameter, which do not have a universal solution for all problems so far. This method is mainly used in the forward analysis of dynamics, and is in its initial stage for inverse issues. Ref.[99] utilized the Chebyshev orthogonal polynomials to fit the relationship between uncertain load and interval parameters at zero-cut of fuzzy interval. The maximum and minimum points of uncertain variables are searched in a dimension-wise manner, and the corresponding loads can be identified via calling inverse methods. However, it ignores the coupling effect between uncertain variables.

In order to fascinate the understanding of uncertainty propagation methods, some prominent features are listed in Table 2.

Table 2 The advantages and disadvantages of different uncertainty propagation methods

Classification	Method	Advantage	Disadvantage
The probabilistic model	The MCS	It is of high accuracy for uncertainty propagation analysis.	It requires enough sample information and expensive calculation.
	The matrix perturbation method	It is simple and computationally efficient for stochastic uncertainties.	It is not suitable for stochastic uncertainties with large fluctuation.
	The polynomial chaos expansion method	It has high statistical reliability and wide applicability.	It requires an explicit expression of PDF.
The non-probabilistic model	The vertex combination method	Its principle is simple to understand.	It is only suitable for monotonic problems with regard to interval parameters.
	The Taylor-series-expansion method	It is simple and computationally efficient for interval uncertainties.	It is not suitable for interval uncertainties with nonlinearity or large uncertainty.
	The methods based on the surrogate model	It can reduce the overestimation or underestimation of load interval.	Its accuracy and efficiency are dependent on surrogate models and their hyper-parameters.

3 Discussion of Development

There are many excellent reviews in the literature dealing with the basic concepts of dynamic load identification in recent decades. However, further effort is required to better deal with the uncertain dynamic load identification of aerospace vehicles.

Firstly, the structural dynamic responses, which are measured by strain gauges, accelerometers or displacement detectors, are the basis of the load identification. Different types and locations of response signals correspond to different identified results. The sensor deployment optimization strategy including their type, number and position should be involved to improve the load identification accuracy. Secondly, the intelligent composite with self-diagnose and self-healing function has been widely applied to aerospace vehicles. The piezoelectric element is a novel component for monitoring the intelligent composite. The future looks bright to investigate the relationship between the external load and piezoelectric response and develop additional identification methods for smart structures. Eventually, aerospace vehicles are always operated in hyperthermal environments, and the change of temperature will cause the fluctuation of the structural dynamic characteristics. Under the circumstance of the temperature effect, the responses caused by the dynamic load may be annihilated. Therefore, it is necessary to propose an effective method to separate the temperature effect from the sensor monitoring data, so as to realize the dynamic load identification of aerospace vehicles in the thermal-mechanical coupling environment.

4 Conclusions

It is an urgent need but still a significant challenge for uncertain dynamic load identification of aerospace vehicles, which can be considered as an interdisciplinary subject between the inverse problem of structural dynamics and the uncertainty analysis. This paper provides a taxonomy and a review of alternative identification methods for both deterministic and indeterministic dynamic load, following their applicability and specialty. The forthcoming research trend is prospected finally, which aims at providing promising applications in the development of aerospace vehicles.

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多源不确定性下的飞行器动载荷识别研究进展

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摘要:动态载荷的确定是进行空天飞行器结构设计和健康监测的关键技术之一。然而,空天飞行器所受的外部激励往往很难直接测得,相反,根据动力学模型和响应信息来间接获取载荷的动态载荷识别是一种可行的手段。而且,复杂系统的载荷识别过程中普遍存在着不可忽略的多源不确定性,尤其对于空天飞行器来讲。本文对考虑多源不确定性的空天飞行器动态载荷识别方面的理论和成果进行了综述,主要包括确定性动态载荷的识别和不确定性动态载荷识别,分别阐述了不同类型的集中、分布载荷的反演方法和多源不确定性的量化、传播分析,并探讨了各自的优缺点。最后,分析了空天飞行器动态载荷识别今后可能的发展方向。

关键词:动态载荷识别;集中动态载荷;分布动态载荷;随机载荷;概率不确定性;非概率不确定性