Mei Symmetry for Constrained Mechanical System on Time Scales

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Abstract: Mei symmetry on time scales is investigated for Lagrangian system, Hamiltonian system, and Birkhoffian system. The main results are divided into three sections. In each section, the definition and the criterion of Mei symmetry are first presented. Then the conserved quantity deduced from Mei symmetry is obtained, and perturbation to Mei symmetry and adiabatic invariant are studied. Finally, an example is given to illustrate the methods and results in each section. The conserve quantity achieved here is a special case of adiabatic invariant. And the results obtained in this paper are more general because of the definition and property of time scale.

Key words: Mei symmetry; time scale; Lagrangian system; Hamiltonian system; Birkhoffian system

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0 Introduction

Mei symmetry was first introduced by Mei[1] in 2000. Mei symmetry is a kind of invariance that the dynamical functions of system, under infinitesimal transformations of time and coordinates, still satisfy the original differential equations of motion. Conserved quantity, which helps find the solution to the differential equation, can be deduced from Mei symmetry. Therefore, Mei symmetry and conserved quantity are important aspects deserved to be studied in analytical mechanics. And lots of research on Mei symmetry can be found in Refs.[2-5].

Time scale was first introduced by Stefan Hilger in 1988[6]. Time scale means an arbitrary nonempty closed subset of the real numbers. Generally, research can be done on time scales first, then different results will be obtained from specific time scale. The real numbers **R**, the integers **Z**, the natural numbers **N**, the nonnegative integers $\mathbf{N}_0$, the Cantor set, etc. are all specific time scales.

Constrained mechanical system on time scales has been studied recently. For example, calculus of variations on time scales[7-8], Noether symmetry and conserved quantity on time scales[9-13], Lie symmetry and conserved quantity on time scales[14-16], and so on. In this paper, Mei symmetry and conserved quantity on time scales will be presented. The definitions and basic properties of time scale calculus used here can be read in Ref.[17] for details.

1 Mei Symmetry for Lagrangian System on Time Scales

1.1 Mei symmetry and conserved quantity

Lagrange equation on time scales has the form[7]

$$\frac{\Delta}{\Delta t}\frac{\partial L}{\partial q_i^{\Delta}}=\frac{\partial L}{\partial q_i^{\sigma}} \tag{1}$$

where $L=L(t,q_j^{\sigma}(t),q_j^{\Delta}(t))$; $i,j=1,2,\cdots,n$ is the Lagrangian on time scales, q_j the coordinate,

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$q_j^\sigma(t)=q_j(\sigma(t)),\ q_j^\Delta(t)=\frac{\Delta}{\Delta t}q_j(t).$

Taking account of the Lagrangian L after the following infinitesimal transformations

$$t^*=t,\ q_i^*=q_i+\theta_L\xi_{Li}^0 \tag{2}$$

we obtain

$$L^*=L(t,q_j^{*\sigma}(t),q_j^{*\Delta}(t))=L(t,q_j^\sigma+\theta_L\xi_{Lj}^{0\sigma},q_j^\Delta+\theta_L\xi_{Lj}^{0\Delta})=$$
$$L(t,q_j^\sigma,q_j^\Delta)+\frac{\partial L}{\partial q_i^\sigma}\cdot\theta_L\xi_{Li}^{0\sigma}+\frac{\partial L}{\partial q_i^\Delta}\cdot\theta_L\xi_{Li}^{0\Delta}+o(\theta_L^2) \tag{3}$$

where θ_L is an infinitesimal parameter and $\xi_{Li}^0=\xi_{Li}^0(t,q_j)$ is the infinitesimal generator.

Definition 1 If the form of Eq.(1) keeps invariant when the original Lagrangian L is replaced by L^*, that is

$$\frac{\Delta}{\Delta t}\frac{\partial L^*}{\partial q_i^\Delta}=\frac{\partial L^*}{\partial q_i^\sigma} \tag{4}$$

holds, then this invariance is called the Mei symmetry of Lagrangian system on time scales.

Substituting Eqs.(1,3) into Eq.(4), and omitting the higher order of θ_L, we obtain

$$\frac{\Delta}{\Delta t}\frac{\partial}{\partial q_i^\Delta}\left(\frac{\partial L}{\partial q_j^\sigma}\cdot\xi_{Lj}^{0\sigma}+\frac{\partial L}{\partial q_j^\Delta}\cdot\xi_{Lj}^{0\Delta}\right)=\frac{\partial}{\partial q_i^\sigma}\left(\frac{\partial L}{\partial q_j^\sigma}\cdot\xi_{Lj}^{0\sigma}+\frac{\partial L}{\partial q_j^\Delta}\cdot\xi_{Lj}^{0\Delta}\right) \tag{5}$$

Criterion 1 If the infinitesimal generator ξ_{Lj}^0 satisfies Eq.(5), the corresponding invariance is the Mei symmetry of the Lagrangian system on time scales.

Eq.(5) is called the criterion equation of the Mei symmetry for the Lagrangian system (Eq.(1)) on time scales.

Generally speaking, additional conditions are necessary when conserved quantity is wanted to be deduced from the Mei symmetry.

Theorem 1 For the Lagrangian system (Eq.(1)), if the infinitesimal generator ξ_{Lj}^0, which meets the requirement of the Mei symmetry (Eq.(5)), and a gauge function $G_L^0=G_L^0(t,q_j^\sigma,q_j^\Delta)$ satisfies

$$\frac{\partial L}{\partial q_j^\sigma}\cdot\xi_{Lj}^{0\sigma}+\frac{\partial L}{\partial q_j^\Delta}\cdot\xi_{Lj}^{0\Delta}+G_L^{0\Delta}=0 \tag{6}$$

then the Mei symmetry can deduce the following conserved quantity

$$I_{L0}=\frac{\partial L}{\partial q_j^\Delta}\cdot\xi_{Lj}^0+G_L^0=\text{const} \tag{7}$$

Proof Using Eqs.(1,6), we have

$$\frac{\Delta}{\Delta t}I_{L0}=\frac{\partial L}{\partial q_j^\Delta}\cdot\xi_{Lj}^{0\Delta}+\frac{\Delta}{\Delta t}\left(\frac{\partial L}{\partial q_j^\Delta}\right)\cdot\xi_{Lj}^{0\sigma}+G_L^{0\Delta}=\frac{\partial L}{\partial q_j^\Delta}\cdot\xi_{Lj}^{0\Delta}+\frac{\partial L}{\partial q_j^\sigma}\cdot\xi_{Lj}^{0\sigma}+G_L^{0\Delta}=0$$

This proof is completed.

1.2 Perturbation to Mei symmetry and adiabatic invariant

When the Lagrangian system (Eq.(1)) is disturbed, the conserved quantity may also change.

Assuming the Lagrangian system on time scales is disturbed as

$$\frac{\Delta}{\Delta t}\frac{\partial L}{\partial q_i^\Delta}=\frac{\partial L}{\partial q_i^\sigma}-\varepsilon_L Q_{Li}(t,q_j^\sigma,q_j^\Delta) \tag{8}$$

If the disturbed infinitesimal generator ξ_{Li} and the disturbed gauge function G_L are

$$\begin{cases}\xi_{Li}=\xi_{Li}^0+\varepsilon_L\xi_{Li}^1+\varepsilon_L^2\xi_{Li}^2+\cdots=\xi_{Li}^0+\varepsilon_L^m\xi_{Li}^m\\ G_L=G_L^0+\varepsilon_L G_L^1+\varepsilon_L^2 G_L^2+\cdots=G_L^0+\varepsilon_L^m G_L^m\end{cases}$$
$$m=1,2,\cdots \tag{9}$$

then the infinitesimal transformations can be expressed as

$$t^*=t,\ q_i^*=q_i+\theta_L\xi_{Li} \tag{10}$$

From the Mei symmetry of the disturbed Lagrangian system (Eq.(8)), that is

$$\frac{\Delta}{\Delta t}\frac{\partial L^*}{\partial q_i^\Delta}=\frac{\partial L^*}{\partial q_i^\sigma}-\varepsilon_L Q_{Li}^* \tag{11}$$

we obtain

$$\frac{\Delta}{\Delta t}\frac{\partial}{\partial q_i^\Delta}\left(\frac{\partial L}{\partial q_j^\sigma}\cdot\xi_{Lj}^{m\sigma}+\frac{\partial L}{\partial q_j^\Delta}\cdot\xi_{Lj}^{m\Delta}\right)=\frac{\partial}{\partial q_i^\sigma}\left(\frac{\partial L}{\partial q_j^\sigma}\cdot\xi_{Lj}^{m\sigma}+\frac{\partial L}{\partial q_j^\Delta}\cdot\xi_{Lj}^{m\Delta}\right)-\varepsilon_L\left(\frac{\partial Q_{Li}}{\partial q_j^\sigma}\cdot\xi_{Lj}^{m\sigma}+\frac{\partial Q_{Li}}{\partial q_j^\Delta}\cdot\xi_{Lj}^{m\Delta}\right) \tag{12}$$

Eq.(12) is called the criterion equation of the Mei symmetry for the disturbed Lagrangian system (Eq.(8)) on time scales.

Definition 2 If a quantity I_z, with ε one of its elements, satisfies that the highest power of ε is z and $\Delta I_z/\Delta t$ is in direct proportion to ε^{z+1}, then I_z is called the zth order adiabatic invariant on time scales. And we have the following theorem.

Theorem 2 For the disturbed Lagrangian system (Eq. (8)), if the infinitesimal generator ξ_{Lj}^{m}, which meets the requirement of the Mei symmetry (Eq.(12)), and the gauge function G_L^m satisfies

$$\frac{\partial L}{\partial q_j^\sigma}\cdot\xi_{Lj}^{m\sigma}+\frac{\partial L}{\partial q_j^\Delta}\cdot\xi_{Lj}^{m\Delta}+G_L^{m\Delta}-Q_{Li}\xi_{Li}^{(m-1)\sigma}=0 \quad (13)$$

where $\xi_{Li}^{(m-1)\sigma}=0$ when $m=0$, then there exists an adiabatic invariant

$$I_{Lz}=\sum_{m=0}^{z}\varepsilon_L^m\left(\frac{\partial L}{\partial q_j^\Delta}\cdot\xi_{Lj}^{m}+G_L^m\right) \quad (14)$$

Proof Using Eqs.(8,13), we have

$$\frac{\Delta}{\Delta t}I_{Lz}=$$

$$\sum_{m=0}^{z}\varepsilon_L^m\left(\frac{\partial L}{\partial q_j^\Delta}\cdot\xi_{Lj}^{m\Delta}+\frac{\Delta}{\Delta t}\left(\frac{\partial L}{\partial q_j^\Delta}\right)\cdot\xi_{Lj}^{m\sigma}+G_L^{m\Delta}\right)=$$

$$\sum_{m=0}^{z}\varepsilon_L^m\left(\frac{\partial L}{\partial q_j^\Delta}\cdot\xi_{Lj}^{m\Delta}+\left(\frac{\partial L}{\partial q_i^\sigma}-\varepsilon_L Q_{Li}\right)\cdot\xi_{Lj}^{m\sigma}+G_L^{m\Delta}\right)=$$

$$\sum_{m=0}^{z}\varepsilon_L^m\left(-\varepsilon_L Q_{Lj}\cdot\xi_{Lj}^{m\sigma}+Q_{Li}\xi_{Li}^{(m-1)\sigma}\right)=-\varepsilon_L^{z+1}Q_{Lj}\cdot\xi_{Lj}^{z\sigma}$$

This proof is completed.

Remark 1 When $z=0$, the adiabatic invariant obtained from Theorem 2 has a special name, i.e., exact invariant. Besides, Theorem 2 reduces to Theorem 1 when $z=0$. Therefore, a conserved quantity is actually an exact invariant.

1.3 An example

The Lagrangian is

$$L=\frac{1}{2}\left[(q_1^\Delta)^2+(q_2^\Delta)^2\right]q_2^\sigma \quad (15)$$

We try to find out its conserved quantity and adiabatic invariant deduced from the Mei symmetry on the time scale $T=h\mathbf{Z}=\{hk: k\in\mathbf{Z}\}$, $h>0$.

From Eq.(5) and Eq.(6), we have

$$\frac{\Delta}{\Delta t}\frac{\partial}{\partial q_1^\Delta}(-\xi_{L2}^{0\sigma}+q_1^\Delta\cdot\xi_{L1}^{0\Delta}+q_2^\Delta\cdot\xi_{L2}^{0\Delta})=\frac{\partial}{\partial q_1^\sigma}(-\xi_{L2}^{0\sigma}+q_1^\Delta\cdot\xi_{L1}^{0\Delta}+q_2^\Delta\cdot\xi_{L2}^{0\Delta}) \quad (16)$$

$$\frac{\Delta}{\Delta t}\frac{\partial}{\partial q_2^\Delta}(-\xi_{L2}^{0\sigma}+q_1^\Delta\cdot\xi_{L1}^{0\Delta}+q_2^\Delta\cdot\xi_{L2}^{0\Delta})=\frac{\partial}{\partial q_2^\sigma}(-\xi_{L2}^{0\sigma}+q_1^\Delta\cdot\xi_{L1}^{0\Delta}+q_2^\Delta\cdot\xi_{L2}^{0\Delta}) \quad (17)$$

$$-\xi_{L2}^{0\sigma}+q_1^\Delta\cdot\xi_{L1}^{0\Delta}+q_2^\Delta\cdot\xi_{L2}^{0\Delta}+G_L^{0\Delta}=0 \quad (18)$$

It is easy to verify that

$$\xi_{L1}^0=1,\ \xi_{L2}^0=0,\ G_L^0=0 \quad (19)$$

satisfy Eqs. (16—18). Then from Theorem 1, a conserved quantity can be obtained, namely

$$I_{L0}=q_1^\Delta=\text{const} \quad (20)$$

When the system is disturbed by $Q_{L1}=0$, $Q_{L2}=t^3-2t$, from Eq.(12) and Eq.(13), we have

$$\frac{\Delta}{\Delta t}\frac{\partial}{\partial q_1^\Delta}(-\xi_{L2}^{1\sigma}+q_1^\Delta\cdot\xi_{L1}^{1\Delta}+q_2^\Delta\cdot\xi_{L2}^{1\Delta})=\frac{\partial}{\partial q_1^\sigma}(-\xi_{L2}^{1\sigma}+q_1^\Delta\cdot\xi_{L1}^{1\Delta}+q_2^\Delta\cdot\xi_{L2}^{1\Delta}) \quad (21)$$

$$\frac{\Delta}{\Delta t}\frac{\partial}{\partial q_2^\Delta}(-\xi_{L2}^{1\sigma}+q_1^\Delta\cdot\xi_{L1}^{1\Delta}+q_2^\Delta\cdot\xi_{L2}^{1\Delta})=\frac{\partial}{\partial q_2^\sigma}(-\xi_{L2}^{1\sigma}+q_1^\Delta\cdot\xi_{L1}^{1\Delta}+q_2^\Delta\cdot\xi_{L2}^{1\Delta}) \quad (22)$$

$$-\xi_{L2}^{1\sigma}+q_1^\Delta\cdot\xi_{L1}^{1\Delta}+q_2^\Delta\cdot\xi_{L2}^{1\Delta}+G_L^{1\Delta}=0 \quad (23)$$

Taking calculation, we obtain

$$\xi_{L1}^1=0,\ \xi_{L2}^1=1,\ G_L^1=t \quad (24)$$

Then

$$I_{L1}=q_1^\Delta+\varepsilon_L(q_2^\Delta+t) \quad (25)$$

can be achieved as the first order adiabatic invariant from Theorem 2. Higher order adiabatic invariants can certainly be deduced.

2 Mei Symmetry for Hamiltonian System

2.1 Mei symmetry and conserved quantity

Hamilton equation on time scales has the form[12-13]

$$q_i^\Delta=\frac{\partial H}{\partial p_i},\ p_i^\Delta=-\frac{\partial H}{\partial q_i^\sigma} \quad (26)$$

where $H=H(t,q_j^\sigma,p_j)$ is the Hamiltonian on time scales and p_j the generalized momentum, $i,j=1,2,\cdots,n$.

Taking account of the Hamiltonian H after the following infinitesimal transformations

$$t^*=t,\ q_i^*=q_i+\theta_H\xi_{Hi}^0,\ p_i^*=p_i+\theta_H\eta_{Hi}^0 \quad (27)$$

we obtain

$$H^*=H(t,q_j^{*\sigma}(t),p_j^*(t))=H(t,q_j^\sigma+\theta_H\xi_{Hj}^{0\sigma},p_j+\theta_H\eta_{Hj}^0)=H(t,q_j^\sigma,p_j)+\frac{\partial H}{\partial q_i^\sigma}\cdot\theta_H\xi_{Hi}^{0\sigma}+\frac{\partial H}{\partial p_i}\cdot\theta_H\eta_{Hi}^0+o(\theta_H^2) \quad (28)$$

where θ_H is an infinitesimal parameter, $\xi_{Hi}^0=\xi_{Hi}^0(t,q_j,p_j)$ and $\eta_{Hi}^0=\eta_{Hi}^0(t,q_j,p_j)$ are called the infinitesimal generators.

Definition 3 If the form of Eq.(26) keeps invariant when the original Hamiltonian H is replaced by H^*, that is

$$q_i^\Delta=\frac{\partial H^*}{\partial p_i},\ p_i^\Delta=-\frac{\partial H^*}{\partial q_i^\sigma} \quad (29)$$

holds, this invariance is called the Mei symmetry of Hamiltonian system on time scales.

Substituting Eqs.(26, 28) into Eq.(29), and omitting the higher order of θ_H, we obtain

$$\frac{\partial}{\partial p_i}\left(\frac{\partial H}{\partial q_j^\sigma}\cdot\xi_{Hj}^{0\sigma}+\frac{\partial H}{\partial p_j}\cdot\eta_{Hj}^0\right)=0$$

$$\frac{\partial}{\partial q_i^\sigma}\left(\frac{\partial H}{\partial q_j^\sigma}\cdot\xi_{Hj}^{0\sigma}+\frac{\partial H}{\partial p_j}\cdot\eta_{Hj}^0\right)=0 \tag{30}$$

Criterion 2 If the infinitesimal generators ξ_{Hi}^0, η_{Hi}^0 satisfy Eq.(30), the corresponding invariance is the Mei symmetry of the Hamiltonian system on time scales.

Eq. (30) is called the criterion equation of the Mei symmetry for the Hamiltonian system (Eq. (26)) on time scales. Therefore, we have

Theorem 3 For the Hamiltonian system (Eq. (26)), if the infinitesimal generators ξ_{Hi}^0, η_{Hi}^0, which meet the requirement of the Mei symmetry (Eq. (30)), and a gauge function $G_H^0=G_H^0(t,q_j^\sigma,p_j)$ satisfies

$$p_j\cdot\xi_{Hj}^{0\Delta}-\frac{\partial H}{\partial q_j^\sigma}\cdot\xi_{Hj}^{0\sigma}+G_H^{0\Delta}=0 \tag{31}$$

the Mei symmetry can deduce the following conserved quantity

$$I_{H0}=p_j\cdot\xi_{Hj}^0+G_H^0=\text{const} \tag{32}$$

Proof Using Eqs. (26,31), we have

$$\frac{\Delta}{\Delta t}I_{H0}=p_j\cdot\xi_{Hj}^{0\Delta}+p_j^\Delta\cdot\xi_{Hj}^{0\sigma}+G_H^{0\Delta}=$$
$$\frac{\partial H}{\partial q_j^\sigma}\cdot\xi_{Hj}^{0\sigma}-G_H^{0\Delta}+p_j^\Delta\cdot\xi_{Hj}^{0\sigma}+G_H^{0\Delta}=0$$

This proof is completed.

2.2 Perturbation to Mei symmetry and adiabatic invariant

When the Hamiltonian system (Eq. (26)) is disturbed, the conserved quantity may also change.

Assuming the Hamiltonian system on time scales is disturbed as

$$q_i^\Delta=\frac{\partial H}{\partial p_i},\ p_i^\Delta=-\frac{\partial H}{\partial q_i^\sigma}-\varepsilon_H Q_{Hi}(t,q_j^\sigma,p_j) \tag{33}$$

If the disturbed infinitesimal generators ξ_{Hi}, η_{Hi} and the disturbed gauge function G_H are

$$\begin{cases}\xi_{Hi}=\xi_{Hi}^0+\varepsilon_H\xi_{Hi}^1+\varepsilon_H^2\xi_{Hi}^2+\cdots=\xi_{Hi}^0+\varepsilon_H^m\xi_{Hi}^m\\ \eta_{Hi}=\eta_{Hi}^0+\varepsilon_H\eta_{Hi}^1+\varepsilon_H^2\eta_{Hi}^2+\cdots=\eta_{Hi}^0+\varepsilon_H^m\eta_{Hi}^m\\ G_H=G_H^0+\varepsilon_H G_H^1+\varepsilon_H^2G_H^2+\cdots=G_H^0+\varepsilon_H^m G_H^m\end{cases}$$

$$m=1,2,\cdots \tag{34}$$

the infinitesimal transformations can be expressed as

$$t^*=t,\ q_i^*=q_i+\theta_H\xi_{Hi},\ p_i^*=p_i+\theta_H\eta_{Hi} \tag{35}$$

From the Mei symmetry of the disturbed Hamiltonian system (Eq. (33)), that is

$$q_i^\Delta=\frac{\partial H^*}{\partial p_i},\ p_i^\Delta=-\frac{\partial H^*}{\partial q_i^\sigma}-\varepsilon_H Q_{Hi}^* \tag{36}$$

we obtain

$$\begin{cases}\frac{\partial}{\partial q_i^\sigma}\left(\frac{\partial H}{\partial q_j^\sigma}\cdot\xi_{Hj}^{m\sigma}+\frac{\partial H}{\partial p_j}\cdot\eta_{Hj}^m\right)-\varepsilon_H\left(\frac{\partial Q_{Hi}}{\partial q_j^\sigma}\cdot\xi_{Hj}^{m\sigma}+\frac{\partial Q_{Hi}}{\partial p_j}\cdot\eta_{Hj}^m\right)=0\\ \frac{\partial}{\partial p_i}\left(\frac{\partial H}{\partial q_j^\sigma}\cdot\xi_{Hj}^{m\sigma}+\frac{\partial H}{\partial p_j}\cdot\eta_{Hj}^m\right)=0\end{cases}$$

$$m=0,1,2,\cdots \tag{37}$$

Eq.(37) is called the criterion equation of the Mei symmetry for the disturbed Hamiltonian system (Eq. (33)) on time scales. Then we have

Theorem 4 For the disturbed Hamiltonian system (Eq.(33)), if the infinitesimal generators ξ_{Hj}^m, η_{Hj}^m meet the requirement of the Mei symmetry (Eq. (37)), and the gauge function G_H^m satisfies

$$p_j\cdot\xi_{Hj}^{m\Delta}-\frac{\partial H}{\partial q_j^\sigma}\cdot\xi_{Hj}^{m\sigma}+G_H^{m\Delta}-Q_{Hi}\xi_{Hi}^{(m-1)\sigma}=0 \tag{38}$$

where $\xi_{Hi}^{(m-1)\sigma}=0$ when $m=0$, then there exists an adiabatic invariant

$$I_{Hz}=\varepsilon_H^m(p_j\cdot\xi_{Hj}^m+G_H^m) \tag{39}$$

Proof Using Eqs. (33,38), we have

$$\frac{\Delta}{\Delta t}I_{Hz}=\sum_{m=0}^{z}\varepsilon_H^m(p_j\cdot\xi_{Hj}^{m\Delta}+p_j^\Delta\cdot\xi_{Hj}^{m\sigma}+G_H^{m\Delta})=$$
$$\sum_{m=0}^{z}\varepsilon_H^m\left(\frac{\partial H}{\partial q_j^\sigma}\cdot\xi_{Hj}^{m\sigma}-G_H^{m\Delta}+Q_{Hi}\xi_{Hi}^{(m-1)\sigma}+p_j^\Delta\cdot\xi_{Hj}^{m\sigma}+G_H^{m\Delta}\right)=$$
$$\sum_{m=0}^{z}\varepsilon_H^m(-\varepsilon_H Q_{Hj}\cdot\xi_{Hj}^{m\sigma}+Q_{Hi}\xi_{Hi}^{(m-1)\sigma})=-\varepsilon_H^{z+1}Q_{Hj}\cdot\xi_{Hj}^{z\sigma}$$

This proof is completed.

Remark 2 Theorem 4 reduces to Theorem 3 when $z=0$.

2.3 An example

The Hamiltonian is

$$H=\frac{1}{2}(p_1^2+p_2^2)+q_1^\sigma \tag{40}$$

We try to find out its conserved quantity and adiabatic invariant deduced from the Mei symmetry on the time scale $T=\{2^n: n\in N\cup\{0\}\}$.

From Eqs. (30,31), we have

$$\frac{\partial}{\partial p_1}(\xi_{H1}^{0\sigma}+p_1\cdot\eta_{H1}^0+p_2\cdot\eta_{H2}^0)=0,\ \frac{\partial}{\partial p_2}(\xi_{H1}^{0\sigma}+p_1\cdot\eta_{H1}^0+p_2\cdot\eta_{H2}^0)=0 \quad (41)$$

$$\frac{\partial}{\partial q_1^\sigma}(\xi_{H1}^{0\sigma}+p_1\cdot\eta_{H1}^0+p_2\cdot\eta_{H2}^0)=0,\ \frac{\partial}{\partial q_2^\sigma}(\xi_{H1}^{0\sigma}+p_1\cdot\eta_{H1}^0+p_2\cdot\eta_{H2}^0)=0 \quad (42)$$

$$p_1\xi_{H1}^{0\Delta}+p_2\xi_{H2}^{0\Delta}-\xi_{H1}^{0\sigma}+G_H^{0\Delta}=0 \quad (43)$$

It is easy to verify that

$$\xi_{H1}^0=\xi_{H2}^0=1,\ \eta_{H1}^0=\eta_{H2}^0=0,\ G_H^0=t \quad (44)$$

satisfy Eqs. (41—43). Then from Theorem 3, a conserved quantity can be obtained

$$I_{H0}=p_1+p_2+t=\text{const} \quad (45)$$

When the system is disturbed by $Q_{H1}=3t$, $Q_{H2}=0$, from Eqs.(37,38), we have

$$\begin{cases}\frac{\partial}{\partial p_1}(\xi_{H1}^{1\sigma}+p_1\cdot\eta_{H1}^1+p_2\cdot\eta_{H2}^1)=0\\ \frac{\partial}{\partial p_2}(\xi_{H1}^{1\sigma}+p_1\cdot\eta_{H1}^1+p_2\cdot\eta_{H2}^1)=0\\ \frac{\partial}{\partial q_1^\sigma}(\xi_{H1}^{1\sigma}+p_1\cdot\eta_{H1}^1+p_2\cdot\eta_{H2}^1)=0\end{cases} \quad (46)$$

$$\frac{\partial}{\partial q_2^\sigma}(\xi_{H1}^{1\sigma}+p_1\cdot\eta_{H1}^1+p_2\cdot\eta_{H2}^1)=0 \quad (47)$$

$$p_1\xi_{H1}^{1\Delta}+p_2\xi_{H2}^{1\Delta}-\xi_{H1}^{1\sigma}+G_H^{1\Delta}-3t=0 \quad (48)$$

Taking calculation, we obtain

$$\xi_{H1}^1=1,\ \xi_{H2}^1=0,\ \eta_{H1}^1=\eta_{H2}^1=0,\ G_H^1=t^2+t \quad (49)$$

Then

$$I_{H1}=p_1+p_2+t+\varepsilon_H(p_1+t^2+t) \quad (50)$$

can be obtained as the first order adiabatic invariant from Theorem 4. Higher order adiabatic invariants can certainly be deduced.

3 Mei Symmetry for Birkhoffian System

3.1 Mei symmetry and conserved quantity

The Birkhoff equation on time scales has the form[11]

$$\frac{\partial R_\nu}{\partial a_\rho^\sigma}\cdot a_\nu^\Delta-\frac{\partial B}{\partial a_\rho^\sigma}-R_\rho^\Delta=0 \quad (51)$$

where $B=B(t,a_\mu^\sigma)$ is the Birkhoffian on time scales, $R_\nu=R_\nu(t,a_\mu^\sigma)$ is the Birkhoff's function on time scales, $a_\mu^\sigma(t)=(a_\mu\circ\sigma)(t)$, $a_\nu^\Delta(t)=\frac{\Delta}{\Delta t}a_\nu(t)$, $\mu,\nu,\rho=1,2,\cdots,2n$.

Taking account of the Birkhoffian B and the Birkhoff's function R_ν after the following infinitesimal transformations

$$t^*=t,\ a_\mu^*=a_\mu+\theta_B\xi_{B\mu}^0 \quad (52)$$

we have

$$R_\nu^*=R_\nu(t,a_\mu^{*\sigma})=R_\nu(t,a_\mu^\sigma+\theta_B\xi_{B\mu}^{0\sigma})=R_\nu(t,a_\mu^\sigma)+\frac{\partial R_\nu}{\partial a_\rho^\sigma}\cdot\theta_B\xi_{B\rho}^{0\sigma}+o(\theta_B^2)$$

$$B^*=B(t,a_\mu^{*\sigma})=B(t,a_\mu^\sigma+\theta_B\xi_{B\mu}^{0\sigma})=B(t,a_\mu^\sigma)+\frac{\partial B}{\partial a_\rho^\sigma}\cdot\theta_B\xi_{B\rho}^{0\sigma}+o(\theta_B^2) \quad (53)$$

where θ_B is an infinitesimal parameter, $\xi_{B\mu}^0=\xi_{B\mu}^0(t,a_\nu)$ called the infinitesimal generator.

Definition 4 If the form of Eq.(51) keeps invariant when the original Birkhoffian B and the Birkhoff's function R_ν are replaced by B^* and R_ν^*, that is,

$$\frac{\partial R_\nu^*}{\partial a_\rho^\sigma}\cdot a_\nu^\Delta-\frac{\partial B^*}{\partial a_\rho^\sigma}-R_\rho^{*\Delta}=0 \quad (54)$$

holds, this invariance is called the Mei symmetry of Birkhoffian system on time scales.

Substituting Eqs. (51, 53) into Eq. (54), and omitting the higher order of θ_B, we obtain

$$\frac{\partial}{\partial a_\rho^\sigma}\left(\frac{\partial R_\nu}{\partial a_\mu^\sigma}\cdot\xi_{B\mu}^{0\sigma}\right)\cdot a_\nu^\Delta-\frac{\partial}{\partial a_\rho^\sigma}\left(\frac{\partial B}{\partial a_\mu^\sigma}\cdot\xi_{B\mu}^{0\sigma}\right)-\frac{\Delta}{\Delta t}\left(\frac{\partial R_\rho}{\partial a_\mu^\sigma}\cdot\xi_{B\mu}^{0\sigma}\right)=0 \quad (55)$$

Criterion 3 If the infinitesimal generator $\xi_{B\mu}^0$ satisfies Eq. (55), the corresponding invariance is the Mei symmetry of the Birkhoffian system on time scales.

Eq.(55) is called the criterion equation of Mei symmetry for the Birkhoffian system (Eq. (51)) on time scales. Therefore, we have

Theorem 5 For the Birkhoffian system (Eq. (51)), if the infinitesimal generator $\xi_{B\mu}^0$, which meets the requirement of the Mei symmetry (Eq. (55)), and a gauge function $G_B^0=G_B^0(t,a_\mu^\sigma)$ satisfies

$$\frac{\partial R_\nu}{\partial a_\rho^\sigma}\cdot\xi_{B\rho}^{0\sigma}\cdot a_\nu^\Delta+R_\nu\xi_{B\nu}^{0\Delta}-\frac{\partial B}{\partial a_\rho^\sigma}\cdot\xi_{B\rho}^{0\sigma}+G_B^{0\Delta}=0 \quad (56)$$

the Mei symmetry can deduce the following conserved quantity

$$I_{B0}=R_\mu\cdot\xi_{B\mu}^0+G_B^0=\text{const} \quad (57)$$

Proof Using Eqs. (51,56), we have

$$\frac{\Delta}{\Delta t}I_{B0}=R_\mu\cdot\xi_{B\mu}^{0\Delta}+R_\mu^\Delta\cdot\xi_{B\mu}^{0\sigma}+G_B^{0\Delta}=-\frac{\partial R_\nu}{\partial a_\rho^\sigma}\cdot\xi_{B\rho}^{0\sigma}\cdot a_\nu^\Delta+\frac{\partial B}{\partial a_\rho^\sigma}\cdot\xi_{B\rho}^{0\sigma}-G_B^{0\Delta}+R_\mu^\Delta\cdot\xi_{B\mu}^{0\sigma}+G_B^{0\Delta}=0$$

This proof is completed.

3.2 Perturbation to Mei symmetry and adiabatic invariant

When the Birkhoffian system (Eq. (51)) is disturbed, the conserved quantity may also change.

Assuming the Birkhoffian system on time scales is disturbed as

$$\frac{\partial R_\nu}{\partial a_\rho^\sigma}\cdot a_\nu^\Delta-\frac{\partial B}{\partial a_\rho^\sigma}-R_\rho^\Delta=\varepsilon_B Q_{B\rho}(t,a_\mu^\sigma) \tag{58}$$

If the disturbed infinitesimal generator $\xi_{B\mu}$ and the disturbed gauge function G_B are

$$\begin{cases}\xi_{B\mu}=\xi_{B\mu}^0+\varepsilon_B\xi_{B\mu}^1+\varepsilon_B^2\xi_{B\mu}^2+\cdots=\xi_{B\mu}^0+\varepsilon_B^m\xi_{B\mu}^m\\ G_B=G_B^0+\varepsilon_B G_B^1+\varepsilon_B^2 G_B^2+\cdots=G_B^0+\varepsilon_B^m G_B^m\end{cases}\quad m=1,2,\cdots \tag{59}$$

the infinitesimal transformations can be expressed as

$$t^*=t,\ a_\mu^*=a_\mu+\theta_B\xi_{B\mu} \tag{60}$$

From the Mei symmetry of the disturbed Birkhoffian system (Eq. (58)), that is

$$\frac{\partial R_\nu^*}{\partial a_\rho^\sigma}\cdot a_\nu^\Delta-\frac{\partial B^*}{\partial a_\rho^\sigma}-R_\rho^{*\Delta}=\varepsilon_B Q_{B\rho}^* \tag{61}$$

we obtain

$$\frac{\partial}{\partial a_\rho^\sigma}\left(\frac{\partial R_\nu}{\partial a_\mu^\sigma}\cdot\xi_{B\mu}^{m\sigma}\right)\cdot a_\nu^\Delta-\frac{\partial}{\partial a_\rho^\sigma}\left(\frac{\partial B}{\partial a_\mu^\sigma}\cdot\xi_{B\mu}^{m\sigma}\right)-\frac{\Delta}{\Delta t}\left(\frac{\partial R_\rho}{\partial a_\mu^\sigma}\cdot\xi_{B\mu}^{m\sigma}\right)=\varepsilon_B\frac{\partial Q_{B\rho}}{\partial a_\mu^\sigma}\cdot\xi_{B\mu}^{m\sigma}\quad m=0,1,2,\cdots \tag{62}$$

Eq.(62) is called the criterion equation of the Mei symmetry for the disturbed Birkhoffian system (Eq. (58)) on time scales. Then we have the following theorem.

Theorem 6 For the disturbed Birkhoffian system (Eq. (58)), if the infinitesimal generator $\xi_{B\mu}^m$, which meets the requirement of the Mei symmetry (Eq. (62)), and the gauge function G_B^m satisfies

$$\frac{\partial R_\nu}{\partial a_\rho^\sigma}\cdot\xi_{B\rho}^{m\sigma}\cdot a_\nu^\Delta+R_\nu\xi_{B\nu}^{m\Delta}-\frac{\partial B}{\partial a_\rho^\sigma}\cdot\xi_{B\rho}^{m\sigma}+G_B^{m\Delta}-Q_{B\rho}\xi_{B\rho}^{(m-1)\sigma}=0 \tag{63}$$

where $\xi_{B\rho}^{(m-1)\sigma}=0$ when $m=0$, there exists an adiabatic invariant

$$I_{Bz}=\sum_{m=0}^{z}\varepsilon_B^m(R_\mu\cdot\xi_{B\mu}^m+G_B^m) \tag{64}$$

Proof Using Eqs. (58,63), we have

$$\frac{\Delta}{\Delta t}I_{Bz}=\sum_{m=0}^{z}\varepsilon_B^m(R_\mu\cdot\xi_{B\mu}^{m\Delta}+R_\mu^\Delta\cdot\xi_{B\mu}^{m\sigma}+G_B^{m\Delta})=\sum_{m=0}^{z}\varepsilon_B^m\left(-\frac{\partial R_\nu}{\partial a_\rho^\sigma}\cdot\xi_{B\rho}^{m\sigma}\cdot a_\nu^\Delta+\frac{\partial B}{\partial a_\rho^\sigma}\cdot\xi_{B\rho}^{m\sigma}-G_B^{m\Delta}+Q_{B\rho}\xi_{B\rho}^{(m-1)\sigma}+R_\mu^\Delta\cdot\xi_{B\mu}^{m\sigma}+G_B^{m\Delta}\right)=\sum_{m=0}^{z}\varepsilon_B^m(-\varepsilon_B Q_{B\rho}\cdot\xi_{B\rho}^{m\sigma}+Q_{B\rho}\xi_{B\rho}^{(m-1)\sigma})=-\varepsilon_B^{z+1}Q_{B\rho}\cdot\xi_{B\rho}^{z\sigma}$$

This proof is completed.

Remark 3 Theorem 6 reduces to Theorem 5 when $z=0$.

3.3 An example

The Birkhoffian and Birkhoff's functions are

$$B=\frac{1}{2}\left[(a_2^\sigma)^2+2a_2^\sigma a_3^\sigma+(a_3^\sigma)^2\right]$$

$$R_1=a_2^\sigma+a_3^\sigma,R_2=0,\ R_3=a_4^\sigma,\ R_4=0 \tag{65}$$

try to find out its conserved quantity and adiabatic invariant deduced from the Mei symmetry on the time scale $T=\left\{\frac{1}{n}:n\in\mathbf{N}\right\}\cup\{0\}$.

From Eqs. (55,56), we have

$$a_1^\Delta\frac{\partial}{\partial a_1^\sigma}(\xi_{B2}^{0\sigma}+\xi_{B3}^{0\sigma})+a_3^\Delta\frac{\partial\xi_{B4}^{0\sigma}}{\partial a_1^\sigma}-\frac{\partial}{\partial a_1^\sigma}[(a_2^\sigma+a_3^\sigma)(\xi_{B2}^{0\sigma}+\xi_{B3}^{0\sigma})]-\frac{\Delta}{\Delta t}(\xi_{B2}^{0\sigma}+\xi_{B3}^{0\sigma})=0 \tag{66}$$

$$a_1^\Delta\frac{\partial}{\partial a_2^\sigma}(\xi_{B2}^{0\sigma}+\xi_{B3}^{0\sigma})+a_3^\Delta\frac{\partial\xi_{B4}^{0\sigma}}{\partial a_2^\sigma}-\frac{\partial}{\partial a_2^\sigma}[(a_2^\sigma+a_3^\sigma)(\xi_{B2}^{0\sigma}+\xi_{B3}^{0\sigma})]=0 \tag{67}$$

$$a_1^\Delta\frac{\partial}{\partial a_3^\sigma}(\xi_{B2}^{0\sigma}+\xi_{B3}^{0\sigma})+a_3^\Delta\frac{\partial\xi_{B4}^{0\sigma}}{\partial a_3^\sigma}-\frac{\partial}{\partial a_3^\sigma}[(a_2^\sigma+a_3^\sigma)(\xi_{B2}^{0\sigma}+\xi_{B3}^{0\sigma})]-\frac{\Delta}{\Delta t}\xi_{B4}^{0\sigma}=0 \tag{68}$$

$$a_1^\Delta\frac{\partial}{\partial a_4^\sigma}(\xi_{B2}^{0\sigma}+\xi_{B3}^{0\sigma})+a_3^\Delta\frac{\partial\xi_{B4}^{0\sigma}}{\partial a_4^\sigma}-\frac{\partial}{\partial a_4^\sigma}[(a_2^\sigma+a_3^\sigma)(\xi_{B2}^{0\sigma}+\xi_{B3}^{0\sigma})]=0 \tag{69}$$

$$\xi_{B2}^{0\sigma}a_1^\Delta+\xi_{B3}^{0\sigma}a_1^\Delta+\xi_{B4}^{0\sigma}a_3^\Delta+(a_2^\sigma+a_3^\sigma)\xi_{B1}^{0\Delta}+a_4^\sigma\xi_{B3}^{0\Delta}-(a_2^\sigma+a_3^\sigma)(\xi_{B2}^{0\sigma}+\xi_{B3}^{0\sigma})+G_B^{0\Delta}=0 \tag{70}$$

It is easy to verify that

$$\xi_{B1}^0=1,\ \xi_{B2}^0=\xi_{B3}^0=\xi_{B4}^0=0,\ G_B^0=0 \tag{71}$$

satisfy Eqs. (66,70). Then from Theorem 5, a conserved quantity can be obtained

$$I_{B0}=a_2^\sigma+a_3^\sigma=\text{const} \tag{72}$$

When the system is disturbed by $Q_{B2}=t^2+1$,

$Q_{B1}=Q_{B3}=Q_{B4}=0$, from Eqs. (62,63), we have

$$a_1^\Delta \frac{\partial}{\partial a_1^\sigma}(\xi_{B2}^{1\sigma}+\xi_{B3}^{1\sigma})+a_3^\Delta \frac{\partial \xi_{B4}^{1\sigma}}{\partial a_1^\sigma}-\frac{\partial}{\partial a_1^\sigma}[(a_2^\sigma+a_3^\sigma)(\xi_{B2}^{1\sigma}+\xi_{B3}^{1\sigma})]-\frac{\Delta}{\Delta t}(\xi_{B2}^{1\sigma}+\xi_{B3}^{1\sigma})=0 \tag{73}$$

$$a_1^\Delta \frac{\partial}{\partial a_2^\sigma}(\xi_{B2}^{1\sigma}+\xi_{B3}^{1\sigma})+a_3^\Delta \frac{\partial \xi_{B4}^{1\sigma}}{\partial a_2^\sigma}-\frac{\partial}{\partial a_2^\sigma}[(a_2^\sigma+a_3^\sigma)(\xi_{B2}^{1\sigma}+\xi_{B3}^{1\sigma})]=0 \tag{74}$$

$$a_1^\Delta \frac{\partial}{\partial a_3^\sigma}(\xi_{B2}^{1\sigma}+\xi_{B3}^{1\sigma})+a_3^\Delta \frac{\partial \xi_{B4}^{1\sigma}}{\partial a_3^\sigma}-\frac{\partial}{\partial a_3^\sigma}[(a_2^\sigma+a_3^\sigma)(\xi_{B2}^{1\sigma}+\xi_{B3}^{1\sigma})]-\frac{\Delta}{\Delta t}\xi_{B4}^{1\sigma}=0 \tag{75}$$

$$a_1^\Delta \frac{\partial}{\partial a_4^\sigma}(\xi_{B2}^{1\sigma}+\xi_{B3}^{1\sigma})+a_3^\Delta \frac{\partial \xi_{B4}^{1\sigma}}{\partial a_4^\sigma}-\frac{\partial}{\partial a_4^\sigma}[(a_2^\sigma+a_3^\sigma)(\xi_{B2}^{1\sigma}+\xi_{B3}^{1\sigma})]=0 \tag{76}$$

$$\xi_{B2}^{1\sigma}a_1^\Delta+\xi_{B3}^{1\sigma}a_1^\Delta+\xi_{B4}^{1\sigma}a_3^\Delta+(a_2^\sigma+a_3^\sigma)\xi_{B1}^{1\Delta}+a_4^\sigma\xi_{B3}^{1\Delta}-(a_2^\sigma+a_3^\sigma)(\xi_{B2}^{1\sigma}+\xi_{B3}^{1\sigma})+G_B^{1\Delta}=0 \tag{77}$$

Taking calculation, we get

$$\xi_{B1}^1=\xi_{B2}^1=1,\ \xi_{B3}^1=-1,\ \xi_{B4}^1=0,\ G_B^1=0 \tag{78}$$

Then

$$I_{B1}=a_2^\sigma+a_3^\sigma+\varepsilon_B[(a_2^\sigma+a_3^\sigma)-a_4^\sigma] \tag{79}$$

can be obtained as the first order adiabatic invariant from Theorem 6. Higher order adiabatic invariants can certainly be deduced.

Remark 4 When the time scale is the real numbers **R**, all the results obtained in this paper are consistent with those in Ref.[18].

4 Conclusions

The Mei symmetry and perturbation to Mei symmetry are studied under special infinitesimal transformations in this paper. Theorems 1—6 are new work. However, further research on Mei symmetry on time scales, for example, Mei symmetry under general infinitesimal transformations on time scales are to be further investigated.

References

[1] MEI Fengxiang. Form invariance of Lagrange system[J]. Journal of Beijing Institute of Technology, 2000, 9(2): 120-124.

[2] WANG Shuyong, MEI Fengxiang. Form invariance and Lie symmetry of equations of nonholonomic systems[J]. Chinese Physics, 2002, 11(1): 5-8.

[3] FU Jingli, CHEN Liqun. Form invariance, Noether symmetry and Lie symmetry of Hamiltonian systems in phase space[J]. Mechanics Research Communications, 2004, 31(1): 9-19.

[4] MEI Fengxiang, WU Huibin. Dynamics of constrained mechanical systems[M]. Beijing: Beijing Institute of Technology, 2009.

[5] MEI Fengxiang, WU Huibin, ZHANG Yongfa. Symmetries and conserved quantities of constrained mechanical systems[J]. International Journal of Dynamics and Control, 2014, 2(3): 285-303.

[6] HILGER S. Ein maßkettenkalkiil mit anwendung auf zentrumsmannigfaltigkeiten[D]. Würzburg: Universität Würzburg, 1988. (in German)

[7] BOHNER M. Calculus of variations on time scales[J]. Dynamic Systems & Applications, 2004, 13(3): 339-349.

[8] HILSCHER R, ZEIDAN V. Calculus of variations on time scales: Weaklocal piecewise solutions with variable endpoints[J]. Journal of Mathematical Analysis & Applications, 2004, 289(1): 143-166.

[9] BARTOSIEWICZ Z, TORRES D F M. Noether's theorem on time scales[J]. Journal of Mathematical Analysis & Applications, 2008, 342(2): 1220-1226.

[10] CAI Pingping, FU Jingli, GUO Yongxin. Noether symmetries of the nonconservative and nonholonomic systems on time scales[J]. Science China Physics, Mechanics & Astronomy, 2013, 56(5): 1017-1028.

[11] SONG Chuanjing, ZHANG Yi. Noether theorem for Birkhoffian systems on time scales[J]. Journal of Mathematical Physics, 2015, 56(10): 102701.

[12] ZHANG Yi. Noether theory for Hamiltonian system on time scales[J]. Chinese Quarterly of Mechanics, 2016, 37(2): 214-224. (in Chinese)

[13] SONG Chuanjing, ZHANG Yi. Conserved quantities for Hamiltonian systems on time scales[J]. Applied Mathematics and Computation, 2017, 313: 24-36.

[14] ZHAI Xianghua, ZHANG Yi. Lie symmetry analysis on time scales and its application on mechanical systems[J]. Journal of Vibration and Control, 2019, 25(3): 581-592.

[15] ZHANG Yi, ZHAI Xianghua. Perturbation to Lie symmetry and adiabatic invariants for Birkhoffian systems on time scales[J]. Communications in Nonlinear Science and Numerical Simulation, 2019, 75: 251-261.

[16] ZHANG Y. Lie symmetry and invariants for a generalized Birkhoffian system on time scales[J]. Chaos, Solitons and Fractals, 2019, 128: 306-312.

[17] BOHNER M, PETERSON A. Dynamic equations

on time scales: An introduction with applications[M]. Boston: Birkhäuser, 2001.

[18] MEI Fengxiang. Analytical mechanics (II)[M]. Beijing: Beijing Institute of Technology Press, 2013. (in Chinese)

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时间尺度上约束力学系统Mei对称性

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摘要:研究了时间尺度上Lagrange系统、Hamilton系统和Birkhoff系统的Mei对称性。首先分别给出每一类系统的Mei对称性定义及判据,然后得到由Mei对称性导出的守恒量,再进一步研究Mei对称性的摄动和绝热不变量,最后再举例说明文中所用方法及所得结果。研究所得的守恒量是绝热不变量的特殊形式,且由于时间尺度的定义和性质,本文研究结果具有普适性。

关键词: Mei对称性;时间尺度;Lagrange系统;Hamilton系统;Birkhoff系统