A NURBS Fitting Optimization Method for High-Speed Five-Axis NC Machining Path Based on Curvature Smoothing Preset Point Constraint

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Abstract: Existing curve fitting algorithms of NC machining path mainly focus on the control of fitting error, but ignore the problem that the original discrete cutter position points are not enough in the high curvature area of the tool path. It may cause a sudden change in the drive force of the feed axis, resulting in a large fluctuation in the feed speed. This paper proposes a new non-uniform rational B-spline (NURBS) curve fitting optimization method based on curvature smoothing preset point constraints. First, the short line segments generated by the CAM software are optimally divided into different segment regions, and then the curvature of the short line segments in each region is adjusted to make it smoother. Secondly, a set of characteristic points reflecting the change of the curvature of the fitted curve is constructed as the control apex of the fitted curve, and the curve is fitted using the NURBS curve fitting optimization method based on the curvature smoothing preset point constraint. Finally, the curve fitting error and curve volatility are analyzed with an example, which verifies that the method can significantly improve the curvature smoothness of the high-curvature tool path, reduce the fitting error, and improve the feed speed.

Key words: curvature smoothing; NC machining path; NURBS curve fitting; weighted constraint

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0 Introduction

High-speed five-axis NC machining has become an important technology for efficient machining of complex surface parts in aerospace. However, the smoothness of tool paths is still a major factor affecting five-axis high-speed and high-precision machining. At present, most CAM software generates tool paths in accordance with the ISO 6983 standard by approximating short line segments to the original design curve. In the high-speed cutting process, this method may cause frequent fluctuations in the feed speed, which seriously affects the cutting efficiency and machining surface quality.

Tool path smoothing optimization is the key to solve such problems. At present, there are mainly two methods to realize smoothing optimization of tool path: Polynomial curve interpolation and spline curve fitting. The polynomial curve interpolation method requires all data points on the tool path to pass through, resulting in a large amount of calculation and poor real-time performance. The spline curve fitting method replaces the traditional short line segment with a spline curve, and the tool path is more superior in terms of geometric accuracy, continuity and code length. Spline curve fitting mainly includes Bezier curve fitting, PH curve fitting, B-spline curve fitting, non-uniform rational B-spline

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(NURBS) curve fitting and so on[1]. Among them, NURBS curve fitting provides a common mathematical form, which is used to accurately represent standard analysis shapes, free curves and curved surfaces. It is the mainstream method of current tool path smoothing optimization research.

Many scholars at home and abroad have used NURBS curves to study tool path smoothing. Ref.[2] divided the curve into several small line segments by identifying sharp corners, and used the feed rate modification equation to adjust the feed speed for interpolation, and the effectiveness of the algorithm were verified through experiments. Ref.[3] used the least square progressive and iterative approximation (LSPIA) algorithm to iteratively optimize the NURBS curve, which improved the fitting accuracy of the NURBS curve. Refs.[4-6] used different iterative methods for the parameter increment of the sampling period of the NURBS parameter curve, which improved the NURBS fitting accuracy and calculation efficiency. Refs.[7-8] obtained a good path curve smoothness by correcting the acceleration and jerk of the drive axis in the sensitive area of the NURBS tool path feed contour. Based on work coordinate system (WCS), synchronization parameters between the bicubic NURBS curve of the tool tip translation trajectory and the tool axis rotation trajectory and the corresponding line segments were constructed, and a five-axis angular smooth transition algorithm is proposed[9]. Aiming at the change of machining speed caused by contour error, a NURBS curve interpolation algorithm with adaptive acceleration and deceleration control in the sensitive region of feed speed mutation was proposed[10]. Ref.[11] proposed a NURBS curve interpolation method with real-time flexible acceleration/deceleration control, which verified the smoothness and effectiveness of the interpolation. The above methods mainly focus on the control of fitting errors, but ignore the curvature fluctuations caused by the accuracy of the original nodes.

In the research on node division, part of the research adopted the interval equal division method, which divided the entire parameter space into $n-p+1$ parts equally. The method can achieve a better fitting effect when the curvature changes little. Ref.[12] searched for a set of characteristic points that can characterize the unevenness of the curve by setting the threshold of the turning angle of adjacent short line segments. Ref.[13] analyzed the corners of adjacent short line segments and the double-chord error based on the data point model of the corner feature to conduct node division. However, the above-mentioned research methods have poor fitting effects in high-curvature curve fitting situations, which easily cause discontinuous and rapid rotation of the rotating shaft, affecting the smoothness control of the feed speed.

This paper proposes a new NURBS curve fitting optimization method based on curvature smoothing preset point constraints. First, the short line segments generated by the CAM software are optimally divided into different segment regions, and then the curvature of the short line segments in each region is adjusted to make it smoother. Then, a point set reflecting the characteristics of curve curvature change is constructed as the control vertex of the fitted curve. Finally, based on these control vertices and the constraint conditions that require the fitting curve to strictly pass the curvature smoothing preset point, the new NURBS curve fitting method mentioned is used for fitting.

1 NURBS Curve Fitting Based on Curvature Smooth Pre-adjustment

1.1 Optimally dividing discrete cutter location point into different segment regions

Because there are sharp points in the original design of the processing path, direct fitting the cutter location points will cause a smooth global transition, which violates the design intention. Therefore, before the curvature smoothing pre-adjustment, it is necessary to optimally divide the short line segments generated by the CAM software into different segment regions, and then perform curve fitting for each region. This paper adopts a method based on the length ratio and the turning angle of adjacent short line segments. The length ratio and the turning angle between the short line segments are shown in Fig.1, where P_i, P_{i+1}, $\cdots$, P_{i+7} are continuous data points, and L_i, L_{i+1}, $\cdots$, L_{i+6} are the lengths between adjacent data points.

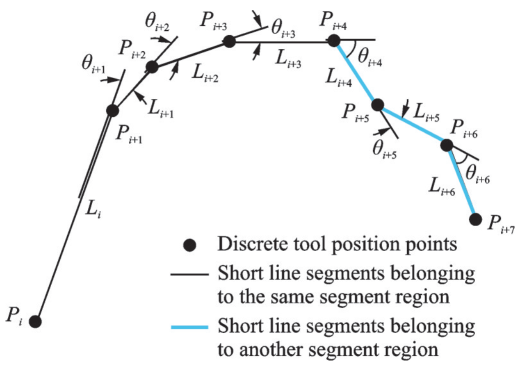

Fig.1 Length ratio and turning angle of adjacent short line segments

The arc curve is regarded as the fitting curve as the analysis object in order to simplify the description in this section. The relationship between the fitting error and the length ratio of adjacent short line segments can be expressed as

$$\xi_{lr}=\frac{1}{2\sin\theta}\left(\sqrt{L_i^2-2L_iL_{i+1}\cos\theta+L_{i+1}^2}-\sqrt{L_i^2-2L_iL_{i+1}\cos\theta+\cos^2\theta L_{i+1}^2}\right) \tag{1}$$

where L_i and L_{i+1} are the lengths of two adjacent short line segments, θ is the turning angle from L_i to L_{i+1} and ξ_{lr} the fitting error of the curve. The length ratio of adjacent short line segments $\varepsilon_{L_iL_{i+1}}$ can be expressed as

$$\varepsilon_{L_iL_{i+1}}=\frac{L_{i+1}}{L_i} \tag{2}$$

When θ is fixed, it can be seen that ξ_{lr} increases with the increase of $\varepsilon_{L_iL_{i+1}}$.

Assuming that two adjacent short line segments have the same length, the relationship between the fitting error and the turning angle of the adjacent short line segment can be expressed as

$$\xi_{ta}=\frac{L}{2\tan\left(90-\dfrac{\theta}{4}\right)} \tag{3}$$

where L is the length of the short line segment, θ the turning angle, and ξ_{ta} the fitting error of the curve. When θ increases, ξ_{ta} also increases.

The process of optimally dividing short line segments into different segment regions is shown in Fig.2 where the length ratio threshold ε_{rl} and the rotation angle threshold ε_θ depend on the maximum allowable chord height error. At point P_{i+4}, according to the length ratio threshold and the corner threshold of the adjacent short line segments, the continuous short line segment chain is divided into two segment regions, which are represented by black and blue in Fig.1, respectively.

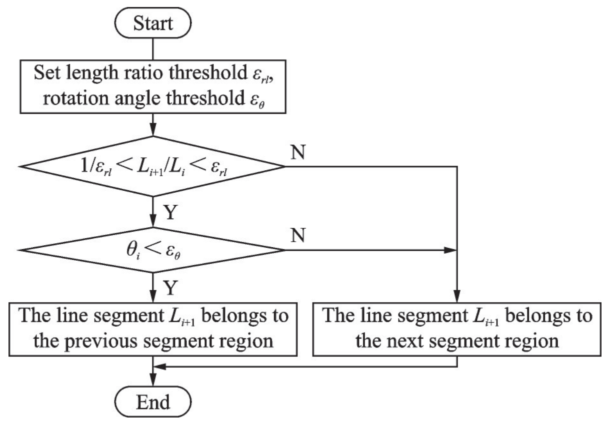

Fig.2 Process of optimally dividing short line segments into different segment regions

1.2 Curvature smooth pre-adjustment method of original point

From the viewpoint of CNC machining, the smaller the curvature or the smaller the curvature fluctuation is, the smoother the feed speed and the easier the processing speed will be, which is beneficial to the vibration suppression of the machine tool. Considering that the curvature of the fitted curve by the conventional method fluctuates greatly, fitting the curve first and then smoothly, the curvature will produce a large amount of computation. This section uses a curvature smoothing pre-adjustment method to pre-adjust the original cutter position point, which is used to obtain the smallest fluctuation of curvature. Since it is meaningless to directly discuss the curvature of the broken line tool path composed of short line segments, this method uses three non-collinear points as a circle to estimate the curvature of the fitted curve. When the radius of the circle made is smaller than a certain threshold, the fitting error is used as a constraint condition to adjust the position of the intermediate point, thereby adjusting the radius of the circle, and the adjustment range can be used as a parameter of curvature smoothing. The adjusted point is referred to herein as the curvature smoothing preset point.

Fig.3 shows the pre-adjustment of the radius of curvature of the three-point fitting curve in the order of P_i, P_{i+1} and P_{i+2}, and d_ρ is the pre-adjusted distance. R and O are the radius and center of the circle made by three points. The calculation process of the curvature smoothing preset point is shown in Fig.4. The minimum machining circle radius R_ε depends on the actual machining conditions, and the allowable radial error ξ_p depends on the process requirements. The pre-adjusted distance d_ρ is the distance moved by the intermediate point P_{i+1} toward the center O. This parameter can be used as a processing optimization coefficient for setting according to process requirements during processing. If the machining process requires high machining accuracy, it can be set to a smaller value or zero. If the processing technology requires high stability, it can be set to a larger value until the allowable value ξ_p of the fitted radial error. In this paper, the calculation of d_ρ takes another way, which is shown in Fig.4.

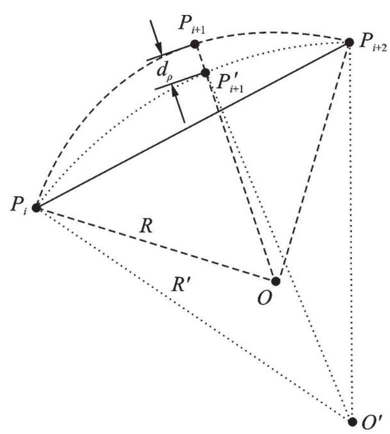

Fig.3 Pre-adjustment of curvature of the fitting curves made by the sequence of three points

1.3 NURBS curve fitting

A NURBS curve can be expressed in rational fractions. Suppose that $R(u)$ represents a NURBS curve and is expressed as

$$R(u)=\frac{\sum_{i=0}^{n} w_i N_{i,p}(u) V_i}{\sum_{i=0}^{n} w_i N_{i,p}(u)} \tag{4}$$

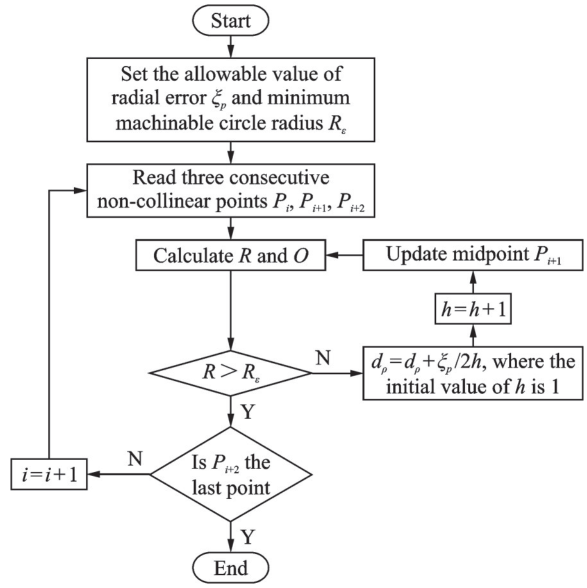

Fig.4 Calculation process of curvature smoothing preset point

where V_i represents the control point, w_i the weight of V_i factor, p the degree of NURBS, $n+1$ the number of the control points, u the interpolation parameter, and $N_{i,p}(u)$ the pth order canonical B-spline basis functions. w_i determines the degree of influence of V_i on the shape of the curve. Since there is no analytical relationship between w_i and the change of the curve, $w_i=1$ ($i=0, 1, \cdots, n$) is generally used to simplify the calculation, after which the NURBS curve degenerates into a B-spline curve. Generally, in order to facilitate engineering applications, the value of p is two or three. This paper will establish a data point set that can characterize the basic shape of the curve and the characteristics of the concave-convex shape, calculate the required node vector, and finally adjust the coordinates and number of the control vertices V_i to change the curve geometry and meet the requirements of accurate fitting.

1.3.1 Method for screening trajectory feature points in segmented regions based on inflection points and curvature extreme points

According to the aforementioned curvature smoothing pre-adjustment method, a set of trajectory characteristic data points composed of curvature smoothing pre-adjustment points, start point and end point can be established. In order to fully reflect the basic shape of the curve and the characteristics of the concave-convex shape, the data point set needs to be further expanded.

(1) Inflection point judgment method

The inflection point can reflect the concave-convex morphological characteristics of the curve, and it is widely used in the field of shape analysis. This section starts from the definition of the inflection point, searches, distinguishes and initially obtains the inflection point in the continuous short line segment by calculating the change trend of the curve near a certain point on the curve. Suppose two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on the plane, specify the direction of the directed line segment as P_1P_2, and the points on the plane that are not on the straight line P_1P_2 are divided into two categories. The inner point about P_1P_2 is the point on the clockwise side of the straight line where the directed line segment is located; the outer point about P_1P_2 is the point on the counterclockwise side of the line. The direction discriminant formula for the direction of point $P(x, y)$ is expressed as

$$D_{12}(x,y)=(x_2-x_1)(y-y_1)+(y_1-y_2)(x-x_1) \tag{5}$$

For any point $P(x,y)$, we can obtain as follows:

① When $D_{12}(x, y)<0$, $P(x, y)$ is the interior point of P_1P_2;

② When $D_{12}(x, y)>0$, $P(x, y)$ is the outer point of P_1P_2;

③ When $D_{12}(x, y)=0$, $P(x, y)$ is on P_1P_2.

Fig.5 reflects all cases where $P(x, y)$ is relatively inside and outside.

For the inflection point, it must be judged in a continuous time series point, and the concavity and convexity of the curve can be determined for three points on the curve that are not on the same straight line. Therefore, for the inflection point judgment, at least four consecutive points are required. Fig.6 shows the judgment of the inflection point of $P_3(x_3, y_3)$, where $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, $P_3(x_3, y_3)$, $P_4(x_4, y_4)$ are four consecutive points. Calculate $D_{12}(x_3, y_3)$ and $D_{23}(x_4, y_4)$, respectively. If $D_{12}(x_3, y_3)\cdot D_{23}(x_4, y_4)<0$, $P_3(x_3, y_3)$ is the inflection point of the curve.

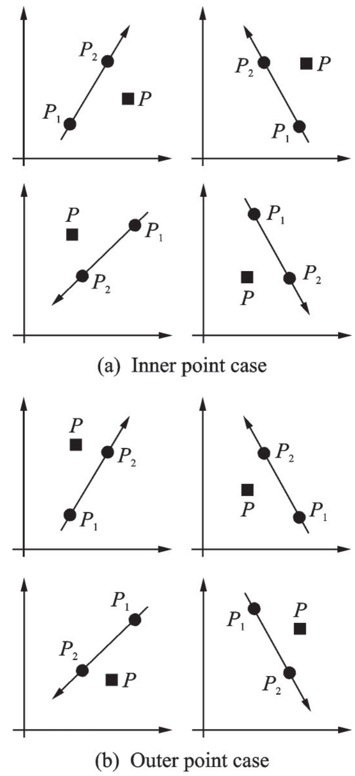

Fig.5 Judgement of point position

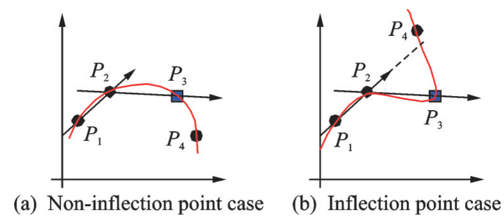

Fig.6 Judgment of inflection point

For the spatial line segment, you can first project the short line segment chain to the XY, YZ, and ZX planes, and then judge the inflection point. As long as the point is judged to be an inflection point on the projection curve in any plane, it can be considered as an inflection point on the fitting curve in the space where it is located.

(2) Feature data point set selection

Since the inflection points can only reflect the overall concave and convex features of the NURBS curve, for the concave and convex features between the inflection points, the corresponding feature points must also be extracted. The extreme points of curvature can reflect the concave and convex characteristics of the curve, and are widely used in path trajectory planning. Theoretical analysis and experiments have proved that if the NURBS curve is divided into intervals by each node of the node vector, there is at most one point of the maximum curvature in any one interval[14]. The inflection point is the node of the NURBS curve, so the maximum curvature point can be used to determine the concave and convex characteristics of the curve between the inflection points. Finally, the initial feature data point set is composed of the starting point, the end point, the curvature smoothing preset point, the inflection point, and the maximum curvature point between the inflection points.

Let $\boldsymbol{P} = \{P_0, P_1, \cdots, P_l\}$ be a set of continuous discrete tool position points, where l is the number of the points, and the selection steps of the feature data point set are as follows:

① Estimate the radius of curvature R_i of the point P_i on the original short line segment chain based on the three-point circle, where $0 < i < l$.

② Use the curvature smoothing preset method to smooth the required curvature and obtain the curvature smooth preset point.

③ Apply the inflection point discrimination method to filter out the inflection point.

④ Apply the fast numerical algorithm of micro-neighborhood analysis[15] to extract the local maximum points of curvature between inflection points.

The points obtained according to the above steps, together with the start point and the end point of the original cutter position points, form the initial characteristic data point set of the curve to be fitted $\boldsymbol{Q}=\{Q_0, Q_1, \cdots, Q_m\}$, where $\boldsymbol{Q}\in\boldsymbol{P}$, $m\leqslant l$.

1.3.2 Node vector based on centripetal parameterization

In the parametric curve fitting problem, the distribution principle of the node vector value u_i is not unique. For the same set of value points, even if the same curve fitting algorithm is used, curve trajectories with different fitting effects will be obtained under different allocation principles. Reasonable parameterization can ensure that the coefficient matrix is reversible when solving the control points in the reverse direction, and the equation system will not produce ill-conditioned solutions, and at the same time make the generated curve smoother. Centripetal parameterization of type value points is a relatively reasonable parameterization method. Under the condition of centripetal parameterization, the difference between the parameter values of adjacent points is the ratio of the square root of the curve length at the two points to the square root of the entire curve length. Because the length of the NURBS curve cannot be accurately calculated, generally the sum of the lengths of all short line segments between the two value points is approximated as the curve length. Combined with the parameter normalization requirements, there are the following

$$\begin{cases} u_0 = 0 \\ u_t = u_{t-1} + \dfrac{L_{t-1,t}}{L_{\text{sum}}} \quad 1\leqslant t\leqslant m-1 \\ u_m = 1 \end{cases} \tag{6}$$

where $m+1$ is the number of initial feature data points, $L_{t-1,t}$ the length of the short line segment between adjacent feature points, and L_{sum} the sum of square roots of the length of the curve from the start point to the end point of this segment region. $L_{t-1,t}$ and L_{sum} are expressed as

$$L_{t-1,t} = \sqrt{\left\| Q_t - Q_{t-1} \right\|} \tag{7}$$

$$L_{\text{sum}} = \sum_{j-1}^{m} \sqrt{\left\| Q_j - Q_{j-1} \right\|} \tag{8}$$

where Q_i is the initial feature data point. In this paper, we select the B-spline basis function of degree three, and calculate the node vector $\boldsymbol{U}$ of the NURBS curve from Eqs.(6—8). $\boldsymbol{U}$ is expressed as

$$\boldsymbol{U} = [0,0,0,u_0,u_1,u_2,\cdots,u_{m-1},u_m,1,1,1]^{\mathrm{T}} \tag{9}$$

1.3.3 New NURBS fitting method based on curvature smoothing preset point constraint

The general least squares NURBS curve fitting does not pass through the intermediate data points except for the start and the end points, but this easily makes the radius of curvature of the fitted curve fluctuate larger, which causes the fluctuation of the feed speed to become larger. In this paper, we use a least squares NURBS curve fitting algorithm[16]. In the process of algorithm implementation, we select the weight factor $w_i = 1(0\leqslant i\leqslant n)$, the degree $p =$

3, and the node vector $U = [0, 0, 0, u_0, u_1, u_2, \cdots, u_{m-1}, u_m, 1, 1, 1]^{\mathrm{T}}$. On this basis, we use the established initial feature data point set $\boldsymbol{Q}$ to calculate a set of control vertices $\boldsymbol{V}$, and we propose a new NURBS fitting algorithm based on curvature smoothing preset point constraints to perform curve fitting on the selected initial data points. The curve is required to pass the curvature smoothing preset point accurately to ensure fitting accuracy during fitting.

From the normative nature of NURBS and the weighting factors $w_i = 1 (0 \leqslant i \leqslant n)$, we can see that for each Q_k $(0 \leqslant k \leqslant m)$, there is a corresponding point $R(u_k)$ on the NURBS fitting curve, where $u_k (0 \leqslant k \leqslant m)$ is the corresponding parameter value. The NURBS curve fitting equation is expressed as

$$R(u_k) = \sum_{i=0}^{n} N_{i,p}(u_k) V_i \tag{10}$$

According to the parameter properties of NURBS, we can conclude that the basis function degree p, the number of control points $n+1$, and the number of nodes $m+1$ satisfy

$$m = n + p + 1 \tag{11}$$

The initial number of control points is $m-p$ obtained from Eq.(11).

In the fitting process, since the number of nodes is greater than the number of control points, there must be a fitting deviation between Q_k and $R(u_k)$, which is expressed as follows

$$e = |Q_k - R(u_k)| \quad 0 \leqslant k \leqslant m \tag{12}$$

For the sake of simplicity, Eq.(12) is expressed in matrix form as follows

$$\begin{aligned} \boldsymbol{e} &= |\boldsymbol{Q}_{(m+1)\times 1} - \boldsymbol{R}_{(m+1)\times 1}| = \\ &|\boldsymbol{Q}_{(m+1)\times 1} - \boldsymbol{N}_{(m+1)\times(n+1)} \boldsymbol{V}_{(n+1)\times 1}| \quad m \geqslant n \end{aligned} \tag{13}$$

where

$$\boldsymbol{N} = \begin{bmatrix} N_{0,p}(u_0) & \cdots & N_{n,p}(u_0) \\ \vdots & \ddots & \vdots \\ N_{0,p}(u_m) & \cdots & N_{n,p}(u_m) \end{bmatrix}$$

$$\boldsymbol{R} = [R_0, R_1, \cdots, R_m]^{\mathrm{T}}$$

$$\boldsymbol{Q} = [Q_0, Q_1, \cdots, Q_m]^{\mathrm{T}}$$

$$\boldsymbol{V} = [V_0, V_1, \cdots, V_n]^{\mathrm{T}}$$

In equations, $\boldsymbol{Q}$ and $\boldsymbol{N}$ are known, and $\boldsymbol{V}$ is to be solved. When $m > n$, the fitting problem requires the residual vector of the over-determined Eq.(13) to be the smallest under a certain norm $\|\boldsymbol{r}\|_t$, which is expressed as follows

$$\|\boldsymbol{r}\|_t = \|\boldsymbol{Q} - \boldsymbol{NV}\,]\|_t \tag{14}$$

For the curvature smoothing preset point, the fitting curve must pass them accurately, which is expressed as follows

$$\|\boldsymbol{Q} - \boldsymbol{NV}\,]\|_t = 0 \tag{15}$$

Usually $\|\boldsymbol{r}\|_t$ takes 2 norm and Eq.(15) can be converted to a linear least squares problem, to ensure that Eq.(14) is differentiable to $\boldsymbol{V}$. When rank $(\boldsymbol{N}) = n$ $(n < m)$, the only least squares solution of Eq.(15) is expressed as follows[17]

$$\boldsymbol{V} = (\boldsymbol{N}^{\mathrm{T}}\boldsymbol{N})^{-1}\boldsymbol{N}^{\mathrm{T}}\boldsymbol{Q} \tag{16}$$

According to the aforementioned node division method, that is, the selection of trajectory feature points, it can be ensured that each inner node interval in the defined domain contains at least one discrete point. Therefore, the matrix $\boldsymbol{N}^{\mathrm{T}}\boldsymbol{N}$ is positive definite, the Gaussian elimination method or LU decomposition method can be used to solve the linear equations to obtain $\boldsymbol{V}$, where V_i can be two-dimensional coordinates (x, y) or three-dimensional space coordinates (x, y, z).

The NURBS curve fitted based on the selected set of characteristic data points does not necessarily pass other characteristic points and other non-characteristic points except for the curvature smoothing preset point, so cutter position point deviation and chord height error may still exist. When these errors exceed the allowable range, this paper uses the median method to insert new feature points before and after the cutter position points that cause the deviation to obtain a new type value feature point set. Then update the node vector and the number of control points, and re-calculate the NURBS fitting calculation until all the cutter position point deviations meet the requirements. In addition, it is also necessary to consider that the discontinuities at the connection points of different curve fitting segments still cause sudden acceleration in the processing process. In order to ensure the continuity required in the design at the connection point of the fitting segment, cubic curve interpolation or Bezier curve interpolation is used to smooth the transition between curves[17].

2 Fitting Experiment and Analysis

In order to verify the effect of the curve fitting optimization algorithm proposed in this paper, a section of NURBS curve with known parameters is selected, and the CAD/CAM discretizes it into a tool path connected by short line segments. The proposed algorithm is used to re-fit the curve, and analyze the fitting error and curvature fluctuation by comparing with the original NURBS curve.

As shown in Fig.7, the original design curve to be processed comes from the dxf file generated in AutoCAD software. The NURBS parameters are as follows: $p=3$, $w_i=1$, $i=0$ $(0 \leqslant i \leqslant n)$, the number of control points $n+1=8$, the number of nodes $m+1=12$. The experiment uses MASTER CAM software to generate NC code, sets the dispersion tolerance to 10 μm, and generates 100 G01 short line segments, involving 101 cutter position points. For CAM software, theoretically, the path cutter position point after the NURBS curve is discretized on the curve. However, the NURBS curve must have a chord height error compared with the original design curve. Fig. 8 shows the original NURBS curve and discrete cutter position points directly displayed in a certain CNC system. After statistics, the chord height error of each discrete short line segment is all within 10 μm of the set error control value, as shown in Fig.9.

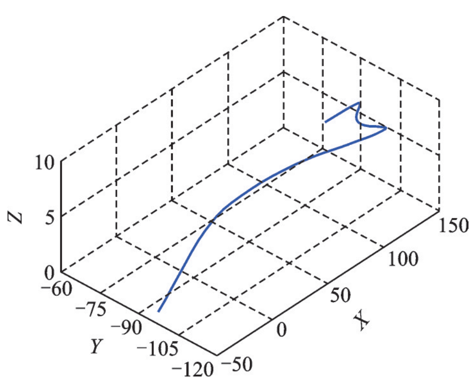

Fig.7 A NURBS curve with known parameters in space

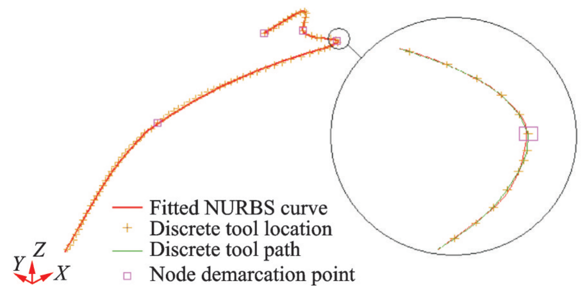

Fig.8 Original NURBS curve and discrete cutter position points displayed by the CNC system

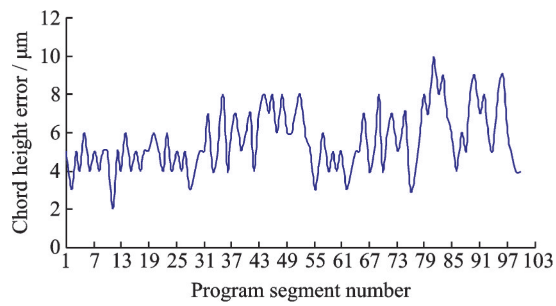

Fig.9 Chord height error of discrete tool path of original NURBS curve

2.1 Fitting error analysis

According to the curve fitting optimization method proposed in this paper, NURBS fitting is performed on the above-mentioned trajectory of discrete short line segments. Considering that the machining accuracy of the machine tool used in the experiment is generally 0.001—0.01 mm, the fitting error is set to 5 μm. Table 1 shows the results of NURBS fitting by dividing the short line segments into different segment regions. Due to the reconstruction of the node vector, the feature point set generated by the node division divides the original single-segment NURBS curve into four segments. The number of original feature points generated by each node vector is not the same. The number of feature points for generating the node vector is about 2/3 of that of discrete cutter position points, and the number of control vertices is about 1/3 of that of discrete cutter position points.

Comparison of the trajectory error between the optimized fitting NURBS curve and the original NURBS curve is the best basis for evaluating the fitting effect. According to the equidistant parameters, the original NURBS curve and the optimized fitting

Table 1 NURBS fitting results of tool path

Parameter	Sequence number of the fitted path segment			
	1	2	3	4
Number of cutter position points	30	30	26	21
Number of node vectors	24	24	22	18
Number of control vertices	14	14	10	8
Maximum fitting error /μm	3.1	3.6	4.2	4.9
Time of fitting operation/ μs	215	232	183	165

curve are subdivided into 300 parts for curve drawing. Fig.10 shows the fitting error of the curve after optimized fitting, and Fig. 11 is a histogram of fitting error distribution. It can be seen from Fig.10 that most of the fitting errors are distributed below 3 μm, and the maximum error is about 5 μm. Compared with the maximum chord height error of 10 μm for discrete tool path of original NURBS curve in Fig.9, the fitting accuracy is doubled.

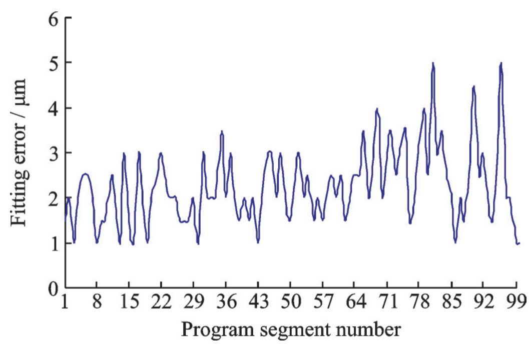

Fig.10 Fitting error of discrete tool path of the optimized NURBS curve

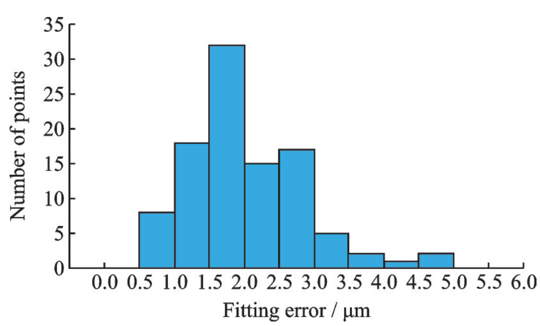

Fig.11 Fitting error distribution histogram

2. 2 Curvature volatility analysis

In the case of continuous short line segments, for cutter position points with a large radius of curvature, the interpolation speed is more limited by the maximum feed rate of the machine tool, and less limited by the normal acceleration. For cutter position points with small curvature radius, the slight change of the coordinates of the curvature smoothing preset point has a great influence on the curvature radius. Two results are shown in Fig.12, where the continuous line represents the radius of curvature calculated directly according to the original NURBS curve, and the dashed line represents the radius of curvature calculated after the curvature smoothing pre-adjustment process. From Fig.12, we can conclude that the curvature of the cutter position point with a small curvature radius can be effectively increased, and the curvature fluctuation can be reduced.

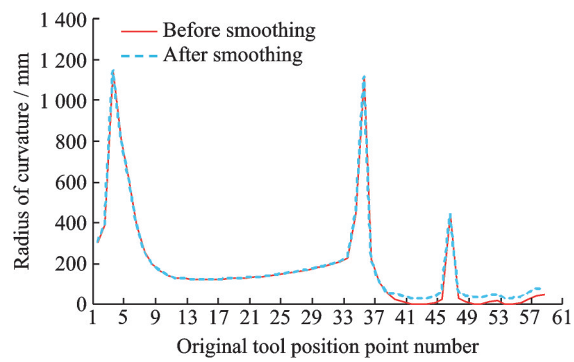

Fig.12 Curvature smoothing pre-adjustment result

In Table 2, P_0, P_1 and P_2 are three consecutive points in the plane that constitute two short line segment trajectories. After calculation, the radius of curvature before adjustment is 53.984 mm. Fine-tune the X coordinate of P_1 to 0.002 mm according to the curvature smoothing method, the adjusted radius of curvature is 121.025 mm, and the radius of curvature is expanded by about 2.24 times. Under the same constraint of the feed normal acceleration of the machine tool, according to the proportional relationship between the maximum feed rate and the square root of the radius of curvature, it can be concluded that the planned feed rate at this point is increased to 1.5 times the original feed rate. Therefore, curvature radius of a point on the high curvature part of the curve becomes larger after the curvature is pre-adjusted, which has a positive effect on reducing the fluctuation of the high curvature curve. In addition, because of the increase in the curvature radius, the speed planned at this point will also increase under the condition that the normal acceleration of the machine tool remains unchanged, which has a positive effect on improving the processing speed.

Table 2 Comparison of curvature radius before and after three-point curvature smoothing preset

Parameter	Before the preset	After the preset
Coordinates of P_0/mm	(211.083, 128.179)	(211.083, 128.179)
Coordinates of P_1/mm	(211.049, 127.566)	(211.047, 127.565)
Coordinates of P_2/mm	(211.008, 126.953)	(211.008, 126.953)
Radius of curvature/mm	53.984	121.025

3 Conclusions

For the current high-speed five-axis NC machining, the problem of interpolation impact still exists in the high curvature area of the NURBS fitting tool path. This paper proposes a new NURBS curve fitting optimization method based on the curvature smoothing preset point constraint. Through the fitting comparison experiment, the following conclusions are drawn.

(1) Adjusting the radius of the fitting circle of any three consecutive non-collinear points in the short line segment chain can reduce the curvature fluctuation of the fitted curve.

(2) The proposed NURBS curve fitting optimization method can improve the fitting accuracy of the curve and reduce the curvature fluctuation, thereby improving the speed stability of the curve interpolation.

(3) The results of the fitting example show that the accuracy of curve fitting is doubled, the curvature fluctuation of the high curvature part of the fitted tool path is reduced, and the feed speed is increased, which verifies the effectiveness of the proposed method.

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一种基于曲率平滑预调点约束的五轴高速数控加工路径NURBS拟合优化方法

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摘要：现有的曲线拟合算法主要集中在拟合误差的控制上，而忽略了在刀具路径的高曲率区域中原始离散刀具位置点取点不足的问题。这可能会导致进给轴驱动力的突然变化，从而导致进给速度的大幅波动。本文提出了一种基于曲率平滑预调点约束的NURBS曲线拟合优化方法。首先，通过对CAM软件生成的大量微线段优化分段，再对分段区间短线段进行曲率平滑调整；然后，构建反映拟合曲线曲率变化的特征点集作为拟合曲线的控制顶点，运用基于曲率平滑预调点约束的NURBS曲线拟合方法进行曲线拟合；最后，结合实例进行了拟合误差和曲线波动性分析，验证了该方法可显著改善高曲率刀具路径的曲率平滑性，减小拟合误差，提高进给速度。

关键词：曲率平滑；NC加工路径；NURBS曲线拟合；带权约束