Improved Particle Swarm Optimization for Solving Transient Nonlinear Inverse Heat Conduction Problem in Complex Structure

ZHOU Ling, *ZHANG Chunyun*, *BAI Yushuai*, *LIU Kun*, *CUI Miao**

Key Laboratory of Advanced Technology for Aerospace Vehicles of Liaoning Province, State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian 116024, P.R.China

(Received 5 May 2021; revised 26 August 2021; accepted 12 September 2021)

Abstract: Accurately solving transient nonlinear inverse heat conduction problems in complex structures is of great importance to provide key parameters for modeling coupled heat transfer process and the structure's optimization design. The finite element method in ABAQUS is employed to solve the direct transient nonlinear heat conduction problem. Improved particle swarm optimization (PSO) method is developed and used to solve the transient nonlinear inverse problem. To investigate the inverse performances, some numerical tests are provided. Boundary conditions at inaccessible surfaces of a scramjet combustor with the regenerative cooling system are inversely identified. The results show that the new methodology can accurately and efficiently determine the boundary conditions in the scramjet combustor with the regenerative cooling system. By solving the transient nonlinear inverse problem, the improved particle swarm optimization for solving the transient nonlinear inverse heat conduction problem in a complex structure is verified.

Key words: improved particle swarm optimization; transient nonlinear heat conduction problem; inverse identification; finite element method; complex structure

CLC number: V19 **Document code:** A **Article ID:** 1005-1120(2021)05-0816-13

0 Introduction

For the traditional heat conduction problem, the temperature field of an object is calculated by giving the initial condition and boundary condition of the material property, which is called the direct heat conduction problem[1]. Many different innovation methods were proposed by scholars to solve the direct heat conduction problems. However, limited by various factors, solving the inverse problems of heat conduction under some circumstances is difficult. A large number of inverse methods were proposed and studied, which could help estimate unknown values or unmeasurable quantities by some known and easily measured information[2]. So, solving the inverse problems of heat conduction is of great importance.

In order to solve the inverse problems of heat conduction with excellent performances, numerous algorithms have been proposed and developed[3]. An enormous amount of research has been put into solving inverse problems of heat conduction[4]. Based on conjugate gradient method and discrepancy principle, Yang et al.[5] proposed a new anti-identification algorithm for the heat flow of disc brake system with time and space changes. Liu et al.[6] solved the problem with heat source and boundary condition recovery by multiple-scale polynomial Trefftz method. Mohasseb et al.[7] presented an innovative hybrid optimization algorithm to solve inverse heat conduction problems, which used genetic algorithm as the leading optimizer and used sequential quadratic programming to reduce the calculation time. In ad-

*Corresponding author, E-mail address: miaocui@dlut.edu.cn.

How to cite this article: ZHOU Ling, ZHANG Chunyun, BAI Yushuai, et al. Improved particle swarm optimization for solving transient nonlinear inverse heat conduction problem in complex structure[J]. Transactions of Nanjing University of Aeronautics and Astronautics, 2021, 38(5): 816-828.

http://dx.doi.org/10.16356/j.1005-1120.2021.05.010

dition, a number of other methods were innovatively proposed, such as the Levenberg-Marquardt method[8-10], the krill herd algorithm[11], the Fourier transform[12], the fundamental solutions[13], a domain decomposition method[14], the bee colony algorithm[15], the particle swarm optimization[16-17], and the genetic algorithm[18], which were introduced in Ref.[4].

Those algorithms could be divided into two groups, the gradient-based algorithm and stochastic algorithm[3]. For most inverse problems, the gradient-based methods could usually converge rapidly, and had high inverse identification accuracy. However, the properties of these methods were easily affected by some factors, e.g. cost function of the inverse problem, which made the optimization method fall into the local optimum and missed the global optimal solution. Moreover, sensitivity coefficients must be evaluated in this way. As for the stochastic methods, they have excellent global searching ability because of the randomness of the initial population and the calculation based on lots of possible solutions instead of an initial solution. However, stochastic methods are generally with a slow convergence rate, and a mass of iterations are required inevitably.

For linear inverse heat conduction problems, satisfactory results could be obtained by using gradient-based methods. But for nonlinear heat conduction problems (e.g. the present work), the sensitivity coefficient matrix is difficult to be precisely determined, and then sometimes the inversion results obtained by the gradient-based method are unreasonable. For example, with the same random number, the error of inverse identification would become smaller with the increase of measuring error, when the algorithm[4] was employed to solve the corresponding nonlinear inverse problem. However, compared with gradient-based methods, the particle swarm optimization (PSO) has some advantages in solving nonlinear heat conduction problems. The stronger ability of global search prevents the identification from falling into local optimum. Only a few parameters need to be input before calculation, so the operation is very convenient and estimation results are very stable. In addition, PSO algorithm combined interpolation method can quickly solve the nonlinear heat conduction problems without repeated calculation of direct problems. Therefore, to accurately and robustly identify the boundary conditions of a scramjet combustor with the regenerative cooling system in Ref.[4], this paper tries to propose a new stochastic algorithm, namely the improved PSO, to avoid the determination of the sensitivity coefficient matrix and ensure the accuracy, efficiency and robustness of inverse identification at the same time.

1 Principle of Traditional PSO

PSO was first proposed by Eberhart and Kennedy[19]. The idea of PSO was derived from the predation behavior of birds, which generated swarm intelligence and guided the inverse identification by consociation and competition of particles[20-21]. Recently, as a high efficiency, excellent stability and strong ability of global searching algorithm, the PSO algorithm has attracted wide extension of researchers. In the PSO algorithm, every particle in the searching space represented a possible solution, which adjusted the next site according to the flight experiences of individual and group[22-23]. There are M individual particles in D-dimensional searching space with velocity vector $\boldsymbol{V}$ and position coordinate X. The fitness values of individual particles are estimated by objective function according to their position coordinates. The biggest or smallest fitness values of individual particle and particle swarm encountered so far are named as local best position P_{i} and global best position P_{g}, respectively, which are regarded as individual flying experience and group particle's experience, and affect the positions of next-generation particle swarm. The basic PSO algorithm formula can be described as

$$V_i(t+1)=\omega V_i(t)+C_1R_1[P_{\mathrm{i}}(t)-X_i(t)]+C_2R_2[P_{\mathrm{g}}(t)-X_i(t)] \tag{1}$$

$$X_i(t+1)=X_i(t)+V_i(t) \tag{2}$$

where i denotes the ith individual particle, $i=1, 2, \cdots, N$; ω the inertia weight factor; $V_i(t)$ the velocity of the ith particle in tth generation; $V_i(t+1)$ the velocity of the ith particle in generation $(t+1)$;

$X_i(t)$ the location of the ith particle at generation t; $P_i(t)$ the individual particle best location at generation t; and $P_g(t)$ the global best location of particle swarm at generation t. C_1, C_2 denote the acceleration coefficients; and R_1, R_2 the random numbers which are uniformly distributed in [0, 1].

2 Improved PSO

2.1 Coefficients of improved PSO

For the inversion identification algorithm, its performance is not only affected by the mechanism of the algorithm itself, but also by the values of coefficients. As we know, an excellent inversion identification algorithm usually has powerful global exploration at the initial stage and outstanding local exploration at the last stage. However, the inertia weight factor ω and acceleration coefficients C_1 and C_2 in PSO method are constant, which limits the property of PSO method. Some studies show that the larger coefficients of inertia weight and acceleration C_1 and the smaller coefficient of acceleration C_2 can enhance the global searching ability. In contrast, the smaller coefficients of inertia weight and acceleration C_1, as well as the larger coefficient of acceleration C_2, maintain the local searching capability of the algorithm. Therefore, inertia weight factor and acceleration coefficients in improved PSO method have been redefined, which are shown as

$$\omega = \omega_{\text{ini}} - \frac{t}{M} \cdot (\omega_{\text{ini}} - \omega_{\text{fin}}) \tag{3}$$

$$C_1 = C_{1,\text{ini}} + \frac{C_{1,\text{fin}} - C_{1,\text{ini}}}{M} \cdot t \tag{4}$$

$$C_2 = C_{2,\text{ini}} + \frac{C_{2,\text{fin}} - C_{2,\text{ini}}}{M} \cdot t \tag{5}$$

where ω_{ini} denotes the initial value of ω and ω_{fin} the final value; M is an integer representing the biggest value of the generation. $C_{1,\text{ini}}$ and $C_{1,\text{fin}}$ denote the initial and final value of C_1, respectively; $C_{2,\text{ini}}$ and and $C_{2,\text{fin}}$ the initial and final value of C_2, respectively.

2.2 Iteration equation

The velocity of particles in the traditional PSO algorithm is only affected by the individual best position, group best position and the speed of the previous generation, in which iteration strategy cannot be adjusted in time according to the speed of convergence and population distribution. Therefore, the PSO method is easy to fall into the local optimum for solving the complicated inverse problem. To overcome this drawback of the traditional PSO algorithm, we propose the improved PSO method with a new evolution strategy in this paper. For the new improved PSO method, the randomness of the particles' evolution is increased while the fitness of global best position remains unchanged for a certain number of iterations and the value of particle diversity is very low, which enables the particles to escape from the region nearby the local optimum and to search for the global optimum. The iteration equation of the $(t+1)$th generation can be expressed as

$$\begin{aligned} V_i(t+1) = \omega V_t + R_1 C_1 [P_i(t) - X_i(t)] + \\ R_2 C_2 [P_g(t) - X_i(t)] \end{aligned} \tag{6}$$

$$X_i(t+1) = X_i(t) + V_i(t+1)$$

$$\text{diversity}(t) > \xi \text{ or } N_{\text{quit}} < n \text{ or } R_3 > N_{\text{quit}}/\alpha \tag{7}$$

$$V_{i,j}(t+1) = R_4(j)[L_{\max}(j) - L_{\min}(j)]\mu \tag{8}$$

$$X_{i,j}(t+1) = L_{\min}(j) + R_5(j)[L_{\max}(j) - L_{\min}(j)]$$

$$\text{diversity}(t) \leqslant \xi, N_{\text{quit}} \geqslant n, R_3 \leqslant N_{\text{quit}}/\alpha \tag{9}$$

where R_1, R_2, R_3, R_4 and R_5 denote random numbers uniformly distributed between 0 and 1, and ξ, n the restrictions of diversity(t) and N_{quit}, respectively. N_{quit} is the number of the algorithm convergence stagnation and α a parameter used to control the probability of particle initialization.

2.3 Particle diversity

The particle diversity is a parameter used to evaluate the dispersion degree of particle distribution in the whole searching space. In the process of identification, the particle diversity decreases slowly with the enhancement of particle tracking ability to the optimal position of group. Particle diversity is related to the ability of particles to jump out of the area near optimal local location. In some cases, the optimal position of the group is not the global optimal solution, and particles of traditional PSO algorithm cannot escape from the area near the local optimal solution at the last stage, which inevitably causes the algorithm to fail to identify. Particles diversity is defined as

$$\text{diversity}(t)=\frac{1}{N}\sum_{i=1}^{N}\sqrt{\sum_{j=1}^{D}\left[\frac{X_t(i,j)-\frac{1}{N}\sum_{i=1}^{N}X_t(i,j)}{L_{\max}(j)-L_{\min}(j)}\right]^2} \tag{10}$$

where $L_{\max}(j)$ and $L_{\min}(j)$ denote the maximum and minimum values of the jth dimension in the searching space, respectively; diversity(t) denotes particle diversity at generation t and D the total number of identified parameters.

3 Direct Problem: Three-Dimensional Transient Nonlinear Heat Conduction Problem

The direct problem is a three-dimensional transient nonlinear heat conduction problem, which is solved by the finite element method (FEM) in ABAQUS. Initial conditions, boundary conditions and physical property parameters of the model are certain, then the temperature field for inverse identification can be obtained. The energy equations of the direct problem can be described as

$$\rho(T)c(T)\frac{\partial T(x,y,z,t)}{\partial T}=\frac{\partial T(x,y,x,t)}{\partial x}\left[\lambda(T)\frac{\partial T(x,y,z,t)}{\partial x}\right]+\frac{\partial T(x,y,z,t)}{\partial y}\left[\lambda(T)\frac{\partial T(x,y,z,t)}{\partial y}\right]+\frac{\partial T(x,y,z,t)}{\partial z}\left[\lambda(T)\frac{\partial T(z,y,z,t)}{\partial z}\right] \tag{11}$$

with the initial condition

$$T(x,y,z,t)|_{t=0}=T(x,y,z) \tag{12}$$

and boundary conditions

$$-\lambda(T)\frac{\partial T(x,y,z,t)}{\partial n}|_{\Gamma_1}=q_{\Gamma_1}(x,y,z,t) \tag{13}$$

$$q(x,y,z,t)|_{\Gamma_2}=h(T_{\mathrm{w}}-T_{\mathrm{f}}) \tag{14}$$

4 Inverse Problem

For the inverse heat conduction problem, temperatures at some positions in the scramjet combustor are known, but some important unknown parameters of boundary conditions, e. g. heat flux in boundary, need to be inversely identified. In the process of inverse identification, the possible solutions are assessed by the minimization objective function, which are determined by the temperatures of estimated and measured. The objective function of the inverse problem is described as

$$S(q_1,q_2,\cdots,q_D)=\sqrt{\frac{1}{M}\sum_{i=1}^{M}\left[\frac{T_i^*-T_i(q_1,q_2,\cdots,q_D)}{T_i^*}\right]^2} \tag{15}$$

Iteration stops until the generation M or the objective function value of global best position less than a specified value f. The procedure for inverse identification is described as follows, and the flow chart is drawn in Fig.1.

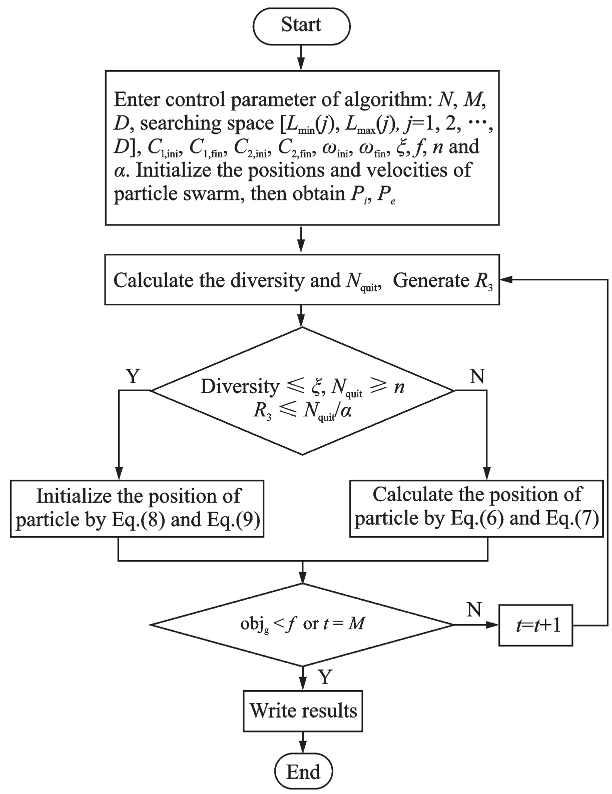


Fig.1 Flow chart of identification

Step 1 Prepare necessary data of the inverse problem and control parameters of the algorithm: Input the number of particles N, the maximum number of iteration M, number of identified parameters D and searching space $[L_{\min}(j), L_{\max}(j), j=1,2,\cdots,D]$. Input acceleration coefficients $C_{1,\mathrm{ini}}$, $C_{1,\mathrm{fin}}$, $C_{2,\mathrm{ini}}$, $C_{2,\mathrm{fin}}$, initial and final inertia weight factors ω_{ini} and ω_{fin}, restrictions ξ and n, control parameters α and tolerance value for objective function f.

Step 2 Initialize the positions of particle swarm in D-dimensional searching space and the velocities of particle swarm, and turn to Step 4.

Step 3 Calculate the velocities of particle swarm by Eq.(6) and the positions of particle swarm in D-dimensional search space by Eq.(7).

Step 4 Compute the values of temperatures at measuring positions by solving the direct problem, then the objective function of each particle can be obtained. Calculate the individual best position $P_i(t)$, global best position $P_g(t)$, the objective function value obj_i of $P_i(t)$ and the objective function value obj_g of $P_g(t)$.

Step 5 Calculate the diversity(t) of the particle swarm and N_{quit} at generation t. Generate a random number R_3, and compare with the criterion of running the iteration equation: If the conditions diversity$(t) \leqslant \xi$, $N_{quit} \geqslant n$ and $R_3 \leqslant N_{quit}/\alpha$ are achieved, turn to Step 6; Otherwise, turn to Step 7.

Step 6 Generate the random numbers R_4 and R_5, and initialize the positions and velocities of particles at generation $(t+1)$ by Eq.(8) and Eq.(9), then turn to Step 8.

Step 7 Generate the random numbers R_1 and R_2, and calculate the new position of particle according to Eq.(6) and Eq.(7). If the position is outside of the searching space at the jth dimension, make it to be equal to $L_{min}(j)$ or $L_{max}(j)$. Turn to Step 8.

Step 8 Calculate the objective function value of the new particles; Update the individual best position and the global best position. Check the stop criteria: If the value of obj_g is smaller than the specified value f or the number of current iteration is equal M, turn to Step 9; Otherwise, turn to Step 3.

Step 9 Output the computation, results and stop the program.

5 Examples of Improved PSO Algorithm for Solving Transient Nonlinear Inverse Problems

In this section, a model of outer wall structure[4] with steel mental material is used, and it is a part of the scramjet combustor in Ref.[4]. When the inversion methodology in Ref.[4] is applied to solve the corresponding nonlinear inverse problem, the results are unacceptable, due to the fact that the sensitivity coefficients cannot be evaluated accurately. Therefore, the improved PSO algorithm is used to solve the transient nonlinear inverse problem, in which sensitivity coefficients are unnecessary. The computer that was used to produce results of estimation has an Intel Core i7-4900 3.60 GHz CPU, 16.0 GB of memory.

In the physical model, the bottom surface is a wall of the combustor, and 14 regenerative cooling channels are designed inside. But only one of them is considered in this work because the heat transfer mechanism in each one is the same. The length, width and height of selected structure are 0.2, 0.005 and 0.019 m, respectively. The diameter of the channel is 0.001 5 m, and the density is 8 240 kg/m^3.

In order to investigate the property of improved PSO to solve the transient nonlinear heat conduction problem in complex structures, some physical property parameters, such as conductivity and specific heat of the selected structure, are not constant and change nonlinearly with temperature. The curves of temperature-dependent conductivity and specific heat are shown in Fig.2.

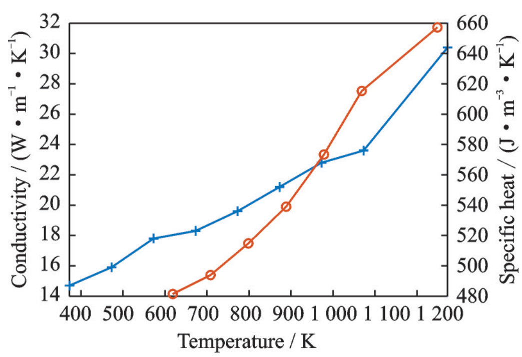


Fig.2 Temperature-dependent thermal properties

The initial temperature in the example is 1 200 K. The convective heat transfer boundary condition is imposed on the inner channel, and the temperature of the coolant is 300 K, which is a constant. Heat flux on the bottom surface is known. On the left, right, front, back and upper surfaces, the heat flux is zero. Boundary conditions of the selected structure are shown in Fig.3[4].

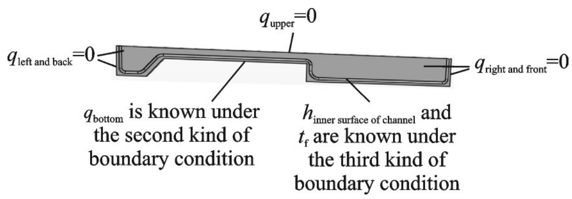


Fig.3 Boundary conditions

The unstructured grids are used in the FEM model for ABAQUS, which contains 23 188 nodes and 109 992 elements. The temperature field of the three-dimensional transient nonlinear heat conduction problem is shown in Fig. 4. It can be seen that temperature in the region near the middle of bottom surface increases with the increase of time, even the convective heat transfer has been imposed on the inner channel to cool the selected structure. This is because the heat fluxes on the bottom surface are so high that coolant in the channel cannot fully absorb the heat. In addition, the conductivity of the nonlinear problem increases with the increase of temperature, which accelerates the heat transfer rate in the selected structure and makes the temperature distribution in Fig. 4 more

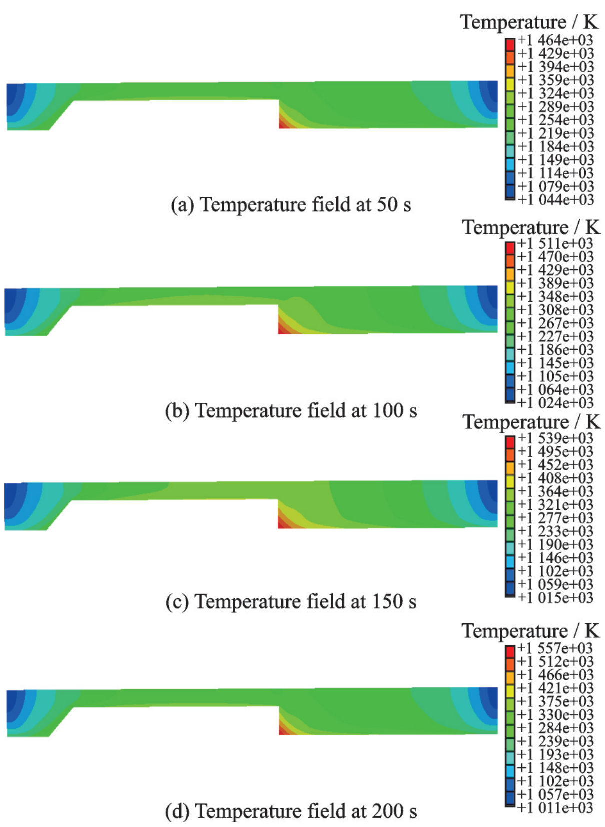


Fig.4 Nonlinear transient temperature fields of the selected structure

uniform, that is, the temperature difference between different positions in Fig.4 is smaller than that in Ref.[4].

Solutions to the direct problem will provide temperature information for the inverse identification of the boundary conditions.

5.1 Impact of measuring points' position

In the direct problem, each ABAQUS calculation needs about 3 min. Therefore, it is time-consuming to use the improved PSO algorithm to identify boundary conditions in the scramjet combustor with a regenerative cooling system, which needs multiple times to call ABAQUS. Long identification time is unfavorable to practice application. To overcome this deficiency, the database is used in this paper. We divide each dimension of searching space into Q equal parts, and the temperature measurements with each $P(i, j, k)$ $(i, j, k=1, 2, \cdots, Q+1)$ are obtained by solving the direct problem. All the temperature measurements are stored in the database. During the estimation using the improved PSO algorithm, there is no need to call ABAQUS to solve the direct problem. Instead, temperature measurements are obtained from the database by interpolation. Using the database in inverse analysis instead of calling ABAQUS can be reused in inversion processes. This methodology can significantly reduce the inversion time and improve the efficiency, which is favorable for practical applications, especially in real-time occasions.

By solving the inverse transient nonlinear heat conduction problem in a complex configuration, the capability of the improved PSO algorithm will be demonstrated in this part. Boundary conditions at three points, on the bottom surface of the model, are selected to be identified. The exact values of the identified heat fluxes are 460, 480, and 500 kW/m^2, respectively. To examine the accuracy and efficiency of the improved PSO algorithm, measuring temperatures at six points are numbered 1, 2, 3, 4, 5, and 6, and all the results summarized in the following part are averages of the best five estimation selected from several calculations for clear conclusions. Temperatures of selected points are measured every 10 s until 200 s. Some parameters of the improved PSO algorithm are defined: $D=3$, $C_{1,\mathrm{ini}}=C_{2,\mathrm{fin}}=2.5$, $C_{1,\mathrm{fin}}=C_{2,\mathrm{ini}}=0.5$, $\omega_{\max}=0.9$, $\omega_{\min}=0.4$, $\xi=20$, $n=10$, $\alpha=100$. It is necessary to point out that the values of the coefficients are based on results of multiple calculation, which can guarantee that the results are stable and accurate. Also, comparing with other kinds of interpolation functions, the cubic spline interpolation usually has high accuracy of results, so it is used in the inverse identification.

Generally speaking, the performances of the improved PSO algorithm are not only related to the algorithm itself, but also related to specific inverse problems, such as the number of measuring points, the position of measuring points, the number of particles, the measuring errors and so on.

To analyze the impact of positions of measuring points and numbers of measuring points on estimation, measuring points are divided into three groups: Group 1 contains points 1—3, group 2 contains points 4—6, and group 3 contains all the six points. Estimation results of the improved PSO algorithm with noiseless measurements are depicted in Tables 1—3, and the corresponding objective function value curves with different number of particles are shown in Fig.5. The program stops running if the iteration number reaches 50. The "Error" stands for the absolute error, which is an absolute value of the difference between the exact and identified value of heat fluxes, and the relative error is the ratio of absolute error to the exact value times 100%." It is observed that all the relative errors of identified heat fluxes are smaller than

Table 1 Estimation results of the improved PSO with different particles number in group 1

The number of particle	Error(E_{rel}/%)			Computation time/s	Best fitness
	1st	2nd	3rd		
100	0.000 7 (0.002)	2.700 9 (0.56)	0.000 0 (0.000 0)	1.251 0	$2.235\ 2\times10^{-8}$
1 000	0.000 0 (0.000 0)	0.222 2 (0.046 3)	0.000 0 (0.000 0)	1.361 8	$2.702\ 9\times10^{-9}$
10 000	0.000 0 (0.000 0)	0.034 2 (0.007 1)	0.000 0 (0.000 0)	2.810 4	$7.514\ 1\times10^{-10}$

Table 2 Estimation results of the improved PSO with different particles number in group 2

The number of particle	Error(E_{rel}/%)			Computation time /s	Best fitness
	1st	2nd	3rd		
100	0.000 0 (0.000 0)	0.000 0 (0.000 0)	0.000 0 (0.000 0)	1.178 6	$2.021\ 2\times10^{-8}$
1 000	0.000 0 (0.000 0)	0.000 0 (0.000 0)	0.000 0 (0.000 0)	1.368 6	$1.668\ 2\times10^{-9}$
10 000	0.000 0 (0.000 0)	0.000 0 (0.000 0)	0.000 0 (0.000 0)	2.899 6	$5.810\ 9\times10^{-11}$

Table 3 Estimation results of the improved PSO with different particles number in group 3

The number of particle	Error(E_{rel}/%)			Computation time /s	Best fitness
	1st	2nd	3rd		
100	0.000 0 (0.000 0)	0.000 0 (0.000 0)	0.000 0 (0.000 0)	2.336 4	$2.365\ 8\times10^{-8}$
1 000	0.000 0 (0.000 0)	0.000 0 (0.000 0)	0.000 0 (0.000 0)	2.648 2	$1.666\ 8\times10^{-9}$
10 000	0.000 0 (0.000 0)	0.000 0 (0.000 0)	0.000 0 (0.000 0)	5.449 4	$8.912\ 4\times10^{-11}$

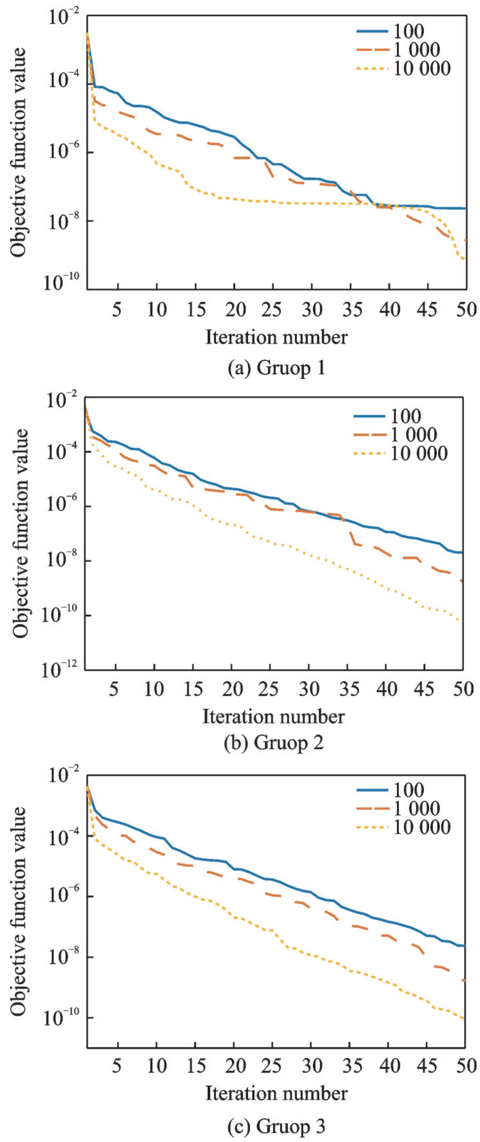


Fig.5 Objective function value curves with different number of particles

0.56%. Comparing the estimation of group 1 and group 2 can find that the results of group 1 obviously have bigger relative errors than group 2. This is because the position of point 2 in group 1 is on the right side of the upper surface, and temperature in that position only varies slightly with different boundary conditions on the bottom surface during the cooling process, so the inverse identification becomes more difficult. It is indicated that the accuracy of results can be affected by the positions of measuring points. Therefore, in order to improve the accuracy of the inversion, the measuring points whose temperatures change obviously with the boundary conditions should be selected.

5.2 Effects of measuring points' number

In addition to the position of measuring points, the influence of measuring points' number should be also analyzed. Results using measuring points in group 1 have been introduced before, and their accuracy is the worst among the results of the three groups of measuring points. Compared with group 1, group 3 has three more measuring points 4—6. Estimation results for group 3 are shown in Table 3. It is found that all the best fitness is less than 1.5×10^{-8} in group 3. However, the relative errors are zero when the particle numbers are 100, 1 000 and 10 000, which reveals the results in group 3 are very accurate. In addition, it indicates that more measuring points can improve the accuracy of estimation.

5.3 Estimation of PSO method

In order to examine the performances of the improved PSO method for solving the inverse nonlinear transient heat conduct problem in a complex structure, the PSO method is also used to retrieve the identified parameters with measuring points of group 1, and the particle numbers are 100, 1 000 and 10 000, respectively. The program stops running when the iteration number equals 50. Fig.6 shows the objective function value curves with three different number of particles. It is observed that the results of the first and the third parameters with different particle number are accurate in Table 4, whereas the relative errors of the second parameter are unsatisfied. Comparing the results of the PSO method with the improved PSO method, it is easy

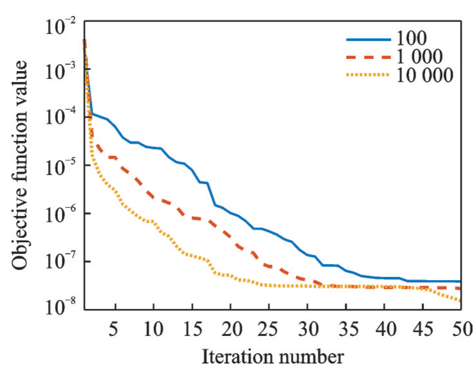


Fig.6 Objective function value curves with different number of particles

Table 4 Estimated results of PSO with different particles number

The number of particle	Error(E_{rel}/%)			Computation time/s	Best fitness
	1st	2nd	3rd		
100	0.001 6 (0.000 4)	6.248 9 (1.301 9)	0.000 0 (0.000 0)	1.190 2	$3.825\,5\times10^{-8}$
1 000	0.000 8 (0.000 2)	3.304 3 (0.688 4)	0.000 0 (0.000 0)	1.340 0	$2.740\,3\times10^{-8}$
10 000	0.000 4 (0.000 1)	1.483 5 (0.309 1)	0.000 0 (0.000 0)	2.850 4	$1.519\,6\times10^{-8}$

to see that relative errors of the second parameter of the PSO method are many times as much as that of the improved method. Hence, for inverse identification the boundary conditions in scramjet combustor, the improved PSO method shows higher accuracy than the PSO method. Also, the time shown in Table 1 and Table 4 is CPU time, which is used to compare the efficiency of the PSO method and the improved PSO method. The results show that both of the two methods with the same particle number take about the same time.

5.4 Efficiency and accuracy of the improved PSO method

In this section, the program stops running until the iteration number reaches 100 or the best fitness is less than 1×10^{-6}. Measuring points of group 1 are used in this analysis. In Table 5, it can be seen that the convergence speed for 20 000 particles is the most fast and only three iterations are needed, while 23 iterations are needed when the particle number is 5 000. It is observed that the iterations decrease with the increase of particle number, which is as expected. Also, results of three selected representative estimations with three different numbers of particles are chosen as observations, which are shown in Figs.7—8 and Table 5. From Fig.8, it is obvious that the objective function value of the improved PSO with 20 000 particles rapidly decreases. While the methods with 10 000 particles and 5 000 particles experience a period of stability in process of estimation, which cause the convergence speed slowly. So it inevitably needs more time for identification. Convergence curves of three heat fluxes with different particle numbers are shown in Fig.7. It can be concluded that an increasing in particle number can speed up the estimation. Computation time shown in Table 5 is only about 1 s. In addition, all the relative errors shown in Table 6 are less than 0.5%. It indicates that the improved PSO method

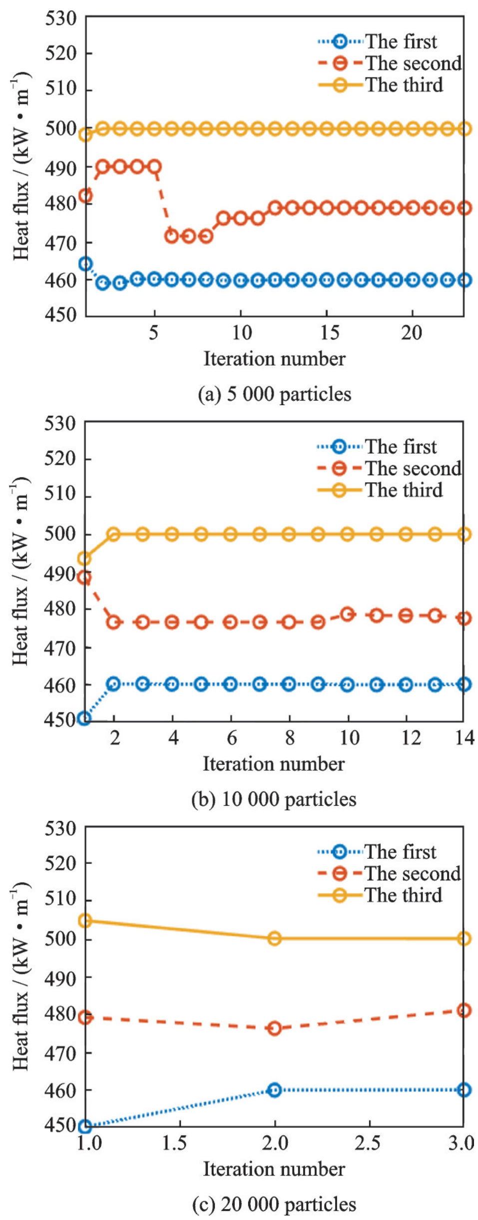


Fig.7 Convergence curves of three heat flux with different numbers of particles

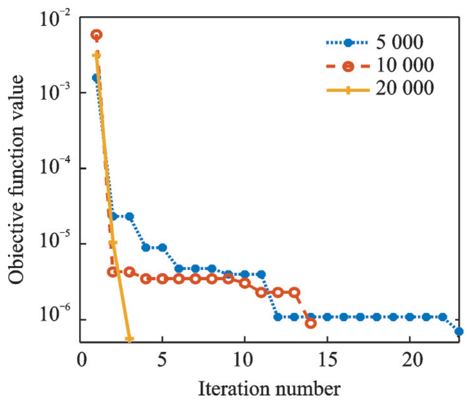


Fig.8 Objective function value curves with different numbers of particles

Table 5 Inversion time with different particle numbers

The number of particle	Iteration times	Computation time/s
5 000	23	1.059 0
10 000	14	0.953 0
20 000	3	0.816 0

Table 6 Estimated results of the improved PSO method with different particles numbers

The number of particle	Error(E_{rel}/%) 1st	2nd	3rd	Best fitness
5 000	0.023 6 (0.005 1)	0.948 6 (0.197 6)	0.001 0 (0.000 2)	$6.957\,6\times10^{-7}$
10 000	0.030 6 (0.006 7)	2.353 9 (0.490 4)	0.001 1 (0.000 2)	$8.935\,3\times10^{-7}$
20 000	0.018 0 (0.003 9)	1.017 3 (0.211 9)	0.000 5 (0.000 1)	$5.601\,6\times10^{-7}$

can predict the boundary conditions of nonlinear transient heat conduction problems in a complex structure efficiently and accurately.

5.5 Effects of measuring errors

In above analysis, the performances of the improved PSO method to estimate the inverse nonlinear heat conduction problem with noiseless temperature measurement are examined. However, noiseless data is unavailable in practice. Hence, it is necessary to investigate the property of the improved PSO method with certain measuring errors. A random error term is added to measuring temperature to simulate the measuring error, which is described as

$$T(x,y,z,t)=T(x,y,z,t)|_{\text{exact}}(1+\zeta\eta/2.576) \tag{16}$$

where ζ is a random number follow normally distributed with zero mean; η is a certain number used to control the range of measurement error, here η are set as 1%, 3% and 5%.

The ratio of the noise to the exact solution (NSR) of measuring temperature is depicted as Eq.(17). The NSR distributions with the measuring errors of 1%, 3% and 5% are shown in Fig. 9, which are turbulent.

$$\text{NSR}=\zeta\eta/2.576 \tag{17}$$

To assess the results of inverse identification, relative error is used in this work. The definition of relative error is described as

$$E_{\text{rel}}=\frac{\left|q(x,y,z,t)|_{\text{identified}}-q(x,y,z,t)|_{\text{exact}}\right|}{q(x,y,z,t)|_{\text{exact}}}\times 100\% \tag{18}$$

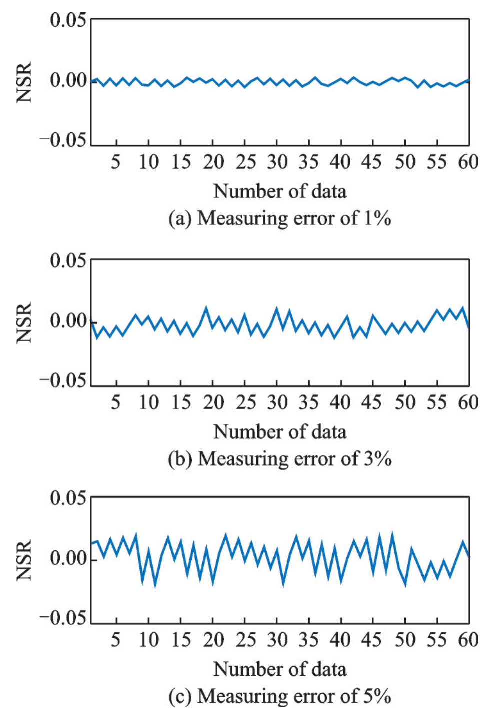
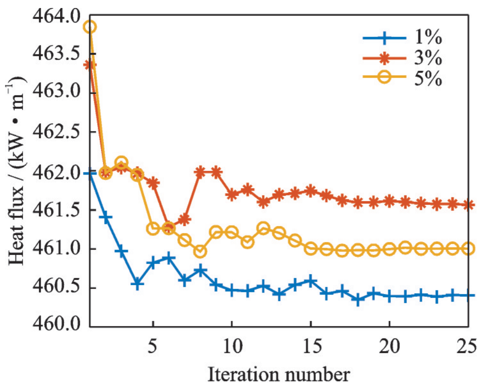
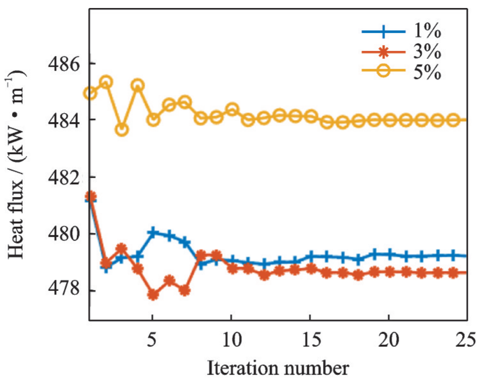


Fig.9 NSR distributions with different measuring errors

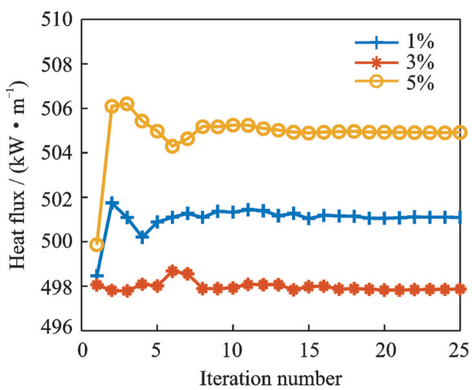
Measuring errors of 1%, 3% and 5% are used to examine the performance of the improved PSO method. The particle number is 25, and measuring points are No.1, No.2 and No.6. The program of the algorithm will stop running until iteration reaches 25. Fig.10 shows the convergence curves of three heat flux with different measuring errors. Fig.11 shows the convergence curves with different measur-



(a) Convergence curves of the first identified parameter (460) with different measuring errors



(b) Convergence curves of the second identified parameter (480) with different measuring errors



(c) Convergence curves of the third identified parameter (460) with different measuring errors

Fig.10 Convergence curves of three identified parameters with different measuring errors

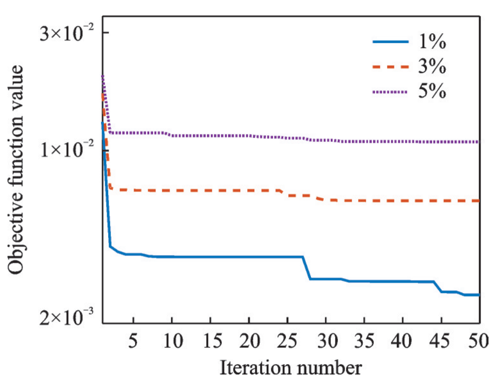


Fig.11 Objective function values with different random temperature measuring errors

ing errors of temperature, and results and corresponding relative errors of identification are listed in Table 7. It is observed that all the relative errors of identified parameters are smaller than 1%, even the biggest measuring error is 5%. In addition, relative errors of identification are smaller than measuring errors. It is validated that the improved PSO method has ability to overcome the inevitable measuring errors for identifying the boundary conditions in scramjet combustor with a regenerative cooling system accurately.

Table 7 Estimated results of improved PSO with different measuring errors

Measuring error ζ/%	Error(E_{rel}/%) 1st	2nd	3rd
1.0	0.405 4 (0.088 1)	0.778 4 (0.162 2)	1.089 7 (0.217 9)
3.0	1.564 8 (0.340 2)	1.359 4 (0.283 2)	2.133 3 (0.426 7)
5.0	1.004 1 (0.218 0)	4.016 1 (0.836 7)	4.909 7 (0.981 8)

6 Conclusions

An improved PSO is proposed and used to solve three-dimensional transient nonlinear inverse heat conduction problems with complex structures, then the boundary conditions in the scramjet combustor with a regenerative cooling system are identified. By analyzing the performances of the improved PSO method, several conclusions can be obtained as follows:

(1) Comparing with the traditional PSO method, the improved PSO method can solve the transient nonlinear inverse heat conduct problem more accurately with the same efficiency.

(2) The methodology of obtaining temperature measurements by using database can obviously reduce the estimation time, i.e., improve the efficiency.

(3) The improved PSO method can identify the boundary conditions in the scramjet combustor accurately and efficiently, for different measuring points and positions, which effectively solves the problem encountered by using the method in

Ref.[4].

(4) The improved PSO method can predict the boundary conditions in the scramjet combustor accurately, even in the case of containing certain measuring errors.

In general, it is verified the improved PSO method can solve the transient nonlinear inverse heat conduction problems in complex structures with good performances.

References

[1] GUO Zhouchao, LU Tao, LIU Bo. Inverse heat conduction estimation of inner wall temperature fluctuations under turbulent penetration[J]. Journal of Thermal Science, 2017, 26(2): 160-165.

[2] MOHEBBI F. Explicit sensitivity coefficients for estimation of temperature-dependent thermophysical properties in inverse transient heat conduction problems[J]. Computation, 2020, 8(4): 95.

[3] CUI Miao, ZHAO Yi, XU Bingbing, et al. Inverse analysis for simultaneously estimating multi-parameters of temperature-dependent thermal conductivities of an inconel in a reusable metallic thermal protection system[J]. Applied Thermal Engineering, 2017, 125: 480-488.

[4] CUI Miao, MEI Jie, ZHANG Bowen, et al. Inverse identification of boundary conditions in a scramjet combustor with a regenerative cooling system[J]. Applied Thermal Engineering, 2018, 134: 559-563.

[5] YANG Y C, CHEN W L. A nonlinear inverse problem in estimating the heat flux of disc in a disc brake system[J]. Applied Thermal Engineering, 2011, 31: 2439-2448.

[6] LIU C S, KUO C L, JHAO W S. The multiple-scale polynomial Trefftz method for solving inverse heat conduction problems[J]. International Journal of Heat and Mass Transfer, 2016, 95: 936-943.

[7] MOHASSEB S, MORADI M, SOKHANSEFAT T, et al. A novel approach to solve inverse heat conduction problems: Coupling scaled boundary finite element method to a hybrid optimization algorithm[J]. Engineering Analysis with Boundary Elements, 2017, 84: 206-212.

[8] LIU F B. A hybrid method for the inverse heat transfer of estimating fluid thermal conductivity and heat capacity[J]. International Journal of Thermal Sciences, 2011, 50: 718-724.

[9] GARCIA E, MERESSE D, POMBO I, et al. Identification of heat partition in grinding related to process parameters, using the inverse heat flux conduction model[J]. Applied Thermal Engineering, 2014, 66: 122-130.

[10] XIE Tao, HE Yaling, TONG Zixiang, et al. An inverse analysis to estimate the endothermic reaction parameters and physical properties of aerogel insulating material[J]. Applied Thermal Engineering, 2015, 87: 214-224.

[11] SUN Shuangcheng, QI Hong, ZHAO Fangzhou, et al. Inverse geometry design of two-dimensional complex radiative enclosures using krill herd optimization algorithm[J]. Applied Thermal Engineering, 2016, 98: 1104-1115.

[12] WIKSTROM P, BLASIAK W, BERNTSSON F. Estimation of the transient surface temperature and heat flux of a steel slab using an inverse method[J]. Applied Thermal Engineering, 2017, 27: 2463-2472.

[13] MIERZWICZAK M, KOLODZIEJ J A. The determination temperature-dependent thermal conductivity as inverse steady heat conduction problem[J]. International Journal of Heat and Mass Transfer, 2011, 54: 790-796.

[14] KHAJEHPOUR S, HEMATIYAN M R, MARIN L. A domain decomposition method for the stable analysis of inverse nonlinear transient heat conduction problems[J]. International Journal of Heat and Mass Transfer, 2013, 5: 125-134.

[15] HETMANIOK E. Inverse problem for the solidification of binary alloy in the casting mould solved by using the bee optimization algorithm[J]. Heat and Mass Transfer, 2016, 52: 1369-1379.

[16] CHOPADE R P, AGNIHOTRI E, SINGH A K, et al. Application of a particle swarm algorithm for parameter retrieval in a transient conduction-radiation problem[J]. Numerical Heat Transfer, Part A: Applications, 2011, 59: 672-692.

[17] ZHANG J, CHO H, MAGO P J, et al. Multi-objective particle swarm optimization (MOPSO) for a distributed energy system integrated with energy storage[J]. Journal of Thermal Science, 2019, 28(6): 1221-1235.

[18] MA Xiaoteng, CAO Qun, CUI Zheng. Optimization design of the grate cooler based on the power flow method and genetic algorithms[J]. Journal of Thermal Science, 2020, 29(6): 1617-1626.

[19] KENNEDY J, EBERHART R. Particle swarm optimization[J]. Swarm Intelligence, 1995, 1: 33-57.

[20] YOON K B, PARK W H. Inverse problem of flame surface properties of wood using a repulsive particle

swarm optimization algorithm[J]. Journal of Thermal Science, 2015, 24: 173-178.

[21] DING Yanming, ZHANG Wenlong, YU Lei, et al. The accuracy and efficiency of GA and PSO optimization schemes on estimating reaction kinetic parameters of biomass pyrolysis[J]. Energy, 2019, 176: 582-588.

[22] CAMPOS A D, POZO A T R, DUARTE E P. Parallel multi-swarm PSO strategies for solving many objective optimization problems[J]. Journal of Parallel and Distributed Computing, 2019, 126: 13-33.

[23] MATHIEU A C, LOUIS G. Comparison between five stochastic global search algorithms for optimizing thermoelectric generator designs[J]. Numerical Heat Transfer, Part A: Applications, 2019, 76: 323-347.

Acknowledgements This work was supported by the National Natural Science Foundation of China (Nos.12172078, 51576026) and the Fundamental Research Funds for the Central Universities in China (No.DUT21LK04).

Authors Mr. **ZHOU Ling** received the B.S. degree in thermal energy and power engineering in 2014, and received the M.S. degree in power engineering and engineering thermophysics in 2017. He is studying for a Ph.D. degree at Dalian University of Technology. His research is focused on inverse identification method and polygonal boundary element method.

Prof. **CUI Miao** received the B.S. and Ph.D. degrees in thermal engineering from Northeastern University, Shenyang, China, in 2003 and 2009, respectively. From 2009 to present, she has been with School of Aeronautics and Astronautics, Dalian University of Technology (DLUT), where she is currently a full professor. Her research has focused on numerical methods for solving multi-physics and the inverse problems.

Author contributions Prof. **CUI Miao** designed the study, provided financial assistance, and conducted the project administration and supervision. Mr. **ZHOU Ling** contributed to the data curation and investigation, researched the methodology and wrote the original draft. Miss **ZHANG Chunyun**, Mr. **BAI Yushuai** and Mr. **LIU Kun** revised the manuscript, and conducted the formal analysis together. All authors commented on the manuscript draft and approved the submission.

Competing interests The authors declare no competing interests.

(Production Editor: XU Chengting)

改进粒子群算法求解具有复杂结构的瞬态非线性热传导反问题

周 凌，张春云，白宇帅，刘 焜，崔 苗

(大连理工大学工业装备结构分析国家重点实验室，辽宁省航天飞行器先进技术重点实验室，大连116024，中国)

摘要：精确求解具有复杂结构的瞬态热传导反问题对于为耦合传热问题建模和结构优化设计提供关键参数具有重要意义。本文采用ABAQUS软件中的有限元法计算瞬态非线性热传导正问题，提出的改进粒子群算法(Particle swarm optimization，PSO)用于求解瞬态非线性热传导反问题；以具有再生冷却系统的超燃冲压发动机燃烧室不可接触表面的边界条件作为反向辨识参数，文中给出了一些数值测试用于检验该算法的性能。结果表明，新方法可以精确且有效地反向辨识具有可再生冷却系统的超燃冲压发动机燃烧室边界条件。通过求解瞬态非线性反问题，验证了改进例子群算法对求解具有复杂结构的瞬态非线性热传导反问题的有效性。

关键词：改进粒子群算法；瞬态非线性热传导问题；反向辨识；有限单元法；复杂结构