Performance Analysis of Uplink Distributed Massive MIMO System with Cross-Layer Design over Rayleigh Fading Channel

WANG Hui[1], *YU Xiangbin*[1,2*], *LIU Fuyuan*[1], *LIU Tao*[1], *BAI Jiawei*[1]

1. College of Electronic and Information Engineering, Nanjing University of Aeronautics and Astronautics,
Nanjing 211106, P. R. China;
2. Key Laboratory of Wireless Sensor Network and Communication Shanghai Institute of Microsystem and
Information Technology, Chinese Academy of Sciences, Shanghai 200030, P. R. China

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Abstract: The performance of uplink distributed massive multiple-input multiple-output (MIMO) systems with cross-layer design (CLD) is investigated over Rayleigh fading channel, which combines the discrete rate adaptive modulation with truncated automatic repeat request. By means of the performance analysis, the closed-form expressions of average packet error rate (APER) and overall average spectral efficiency (ASE) of distributed massive MIMO systems with CLD are derived based on the conditional probability density function of each user's approximate effective signal-to-noise ratio (SNR) and the switching thresholds under the target packet loss rate (PLR) constraint. With these results, using the approximation of complementary error functions, the approximate APER and overall ASE are also deduced. Simulation results illustrate that the obtained theoretical ASE and APER can match the corresponding simulations well. Besides, the target PLR requirement is satisfied, and the distributed massive MIMO systems offer an obvious performance gain over the co-located massive MIMO systems.

Key words: distributed massive multiple-input multiple-output (MIMO) system; zero-forcing detection; cross-layer design; spectral efficiency; quality of service; packet error rate

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0 Introduction

As the age of the fifth generation (5G) communication is coming, distributed massive multiple-input multiple-output (DM-MIMO) technology is being paid more and more attention[1]. Different from co-located massive multiple-input multiple-output (CM-MIMO) systems, DM-MIMO systems can improve the quality of service (QoS) by reducing the distance between the user and the access points (APs), and distributed APs can increase the coverage and bring sufficient diversity[2].

In DM-MIMO, the distributed antenna system (DAS) is a reliable technology that maximizes cellular coverage in cellular networks[3-5]. DAS has gained lots of interest as a promising technology to satisfy rapidly growing demands for next generation wireless systems, because it is energy-saving and achieves more capacity over conventional co-located antenna systems (CASs)[6-8]. In DAS, a large number of single-antenna distributed APs are distributed in the cell, and all APs are connected to the central processing unit (CPU) through the backhaul link, where the received signal is processed. In addition to the above architecture, DM-MIMO benefits from channel hardening, which plays a vital role in the design of CM-MIMO. However, different from CM-MIMO, the distributed structure allows DM-MIMO to obtain larger gain relying on macro diversity. In addition, since the APs are relatively simple and cheap, the deployment of DM-MIMO is compara-

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*Corresponding author, E-mail address: yxbxwy@gmail.com.

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tively flexible, and applicable in various scenarios[9-11].

In the existing research, most papers focus on performance analysis and optimization on the physical layer of DM-MIMO systems. The uplink spectral efficiency (SE) of DM-MIMO systems was analyzed considering pilot pollution and channel aging in Ref.[12]. In Ref.[13], the SE of DM-MIMO downlink systems with hardware and channel damage was studied, and a power allocation algorithm was proposed to maximize the minimum user rate. A random pilot multiplexing scheme was proposed in Ref.[14], and the system SE expression was given. In addition, a method of sparse matrix was proposed in Ref.[14] to simplify signal processing and analyze the bit error ratio (BER).

To improve the SE and throughput of the system and satisfy the given QoS requirements at the same time, cross-layer design (CLD) has received much attention[15-17]. Usually, CLD combines adaptive modulation (AM) at the physical layer to enhance the throughput and truncated automatic retransmission request (ARQ) protocol at the data link layer to ensure the reliability of the system. Ref.[18] analyzed the performance of the co-located MIMO systems using CLD scheme under the Rayleigh channel. Furthermore, the CLD scheme of MIMO system with feedback delay over Nakagami-m fading channel was presented in Ref.[19]. In Ref.[20], the performance analysis of CLD scheme of multiuser MIMO system with antenna selection was presented. The effectiveness of CLD scheme in CM-MIMO systems over composite Rayleigh fading channel was verified in Ref.[21]. In addition, Ref.[22] investigated the effect that fading correlation had on the performance of CLD scheme in the MIMO system on Nakgami-m fading channels. In Ref.[23], a CLD scheme which considered packet combining was presented, where MIMO Nakagami fading channels were assumed, and retransmitted packets were not necessarily modulated using the same modulation format as in the initial transmission.

However, there are few studies on the performance of DM-MIMO systems, especially those with CLD schemes. Performance analysis of DM-MIMO is a challenging problem compared with CM-MIMO, which can be considered as a special case of DM-MIMO. This is because channels between a user and the distributed APs are characterized by different path-loss and shadowing fading, which makes the regular analytical methods inapplicable.

Motivated by the analysis above, this paper will study the cross-layer design of uplink DM-MIMO systems over Rayleigh fading channel. First of all, based on the first item of the accurate BER formulas, approximate packet error rate (PER) formulas are given. Then the switching thresholds in physical layer are introduced based on the approximate formulas under the instantaneous packet loss rate (PLR) constraint. Furthermore, based on the conditional probability density function (PDF) of each user's approximate effective signal-to-noise ratio (SNR), the closed expressions of overall average SE and average PER of cross-layer design scheme in DM-MIMO system are inferred with the thresholds, which provides theoretical basis for the evaluation of its performance. Finally, with the approximation of complementary error functions, the approximate average PER and overall average SE are deduced to simplify the expressions.

1 Signal Model

As shown in Fig.1, we consider uplink DM-MIMO systems, in which L APs and K users are randomly distributed in the cell, and $L \gg K$. The users are equipped with a single antenna. All APs are connected to the CPU via the backhaul network, assuming that the backhaul link is perfect.

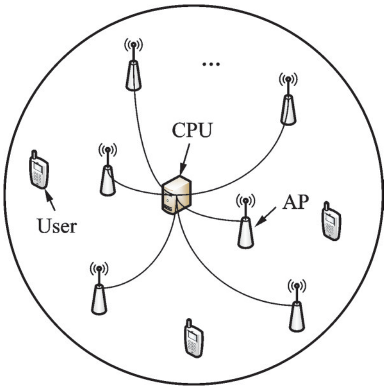

Fig.1 DM-MIMO system model

Suppose that the APs have perfect channel state information (CSI). The channel model contains small-scale fading that obeys Rayleigh fading and large-scale fading that includes path loss. The channel fading coefficient between the lth AP and the kth user is recorded as

$$g_{lk}=\beta_{lk}^{1/2}h_{lk} \tag{1}$$

where h_{lk} represents the small-scale fading between the lth AP and the kth user, and h_{lk} obeys the independent standard Gaussian distribution. $\beta_{lk}=(d_{lk}/d_0)^{-v}$ represents the large-scale fading, where d_{lk} represents the distance between the lth AP and the kth user, d_0 the minimum distance between all users and all APs, and v the path loss exponent.

The signal received by the lth AP is

$$y_l=\sqrt{p_{\mathrm{u}}^d}\sum_{k=1}^{K}g_{lk}x_k+n_l \tag{2}$$

where p_{u}^d represents the transmission power of the uplink signal; x_k the signal sent by the kth user and $\mathbb{E}\left(\left|x_k\right|^2\right)=1$, here $\mathbb{E}(\cdot)$ means the expectation operation and $|\cdot|$ the modulus operation; and n_l the additive Gaussian noise at the lth AP, which subjects to the standard complex Gaussian distribution.

All APs transmit their received signals to the CPU over the backhaul, and the signal matrix received by the CPU is

$$\boldsymbol{y}=\sqrt{p_{\mathrm{u}}^d}\,\boldsymbol{Gx}+\boldsymbol{n}=\sqrt{p_{\mathrm{u}}^d}\sum_{k=1}^{K}\boldsymbol{g}_k x_k+\boldsymbol{n} \tag{3}$$

where $\boldsymbol{y}=[y_1,\cdots,y_L]^{\mathrm{T}}$, and $\boldsymbol{x}=[x_1,\cdots,x_K]^{\mathrm{T}}$ represents the signal vector sent by all users. $\boldsymbol{G}=[\boldsymbol{g}_1,\cdots,\boldsymbol{g}_K]$ represents the channel between the APs and the users, where $\boldsymbol{g}_k=[g_{1k},\cdots,g_{Lk}]^{\mathrm{T}}$ represents the channel between the kth user and the APs, and $g_{lk}\sim CN(0,\beta_{lk})$. $\boldsymbol{n}=[n_1,\cdots,n_L]^{\mathrm{T}}$ is the additive Gaussian noise at APs.

After linear detection with linear detector matrix $\boldsymbol{A}$, the received signal at the CPU becomes

$$\boldsymbol{r}=\boldsymbol{A}^{\mathrm{H}}\boldsymbol{y}=\sqrt{p_{\mathrm{u}}^d}\,\boldsymbol{A}^{\mathrm{H}}\boldsymbol{Gx}+\boldsymbol{A}^{\mathrm{H}}\boldsymbol{n} \tag{4}$$

In order to eliminate multi-user interference, zero-forcing (ZF) detection is used, i. e., $\boldsymbol{A}=\boldsymbol{G}(\boldsymbol{G}^{\mathrm{H}}\boldsymbol{G})^{-1}$. So the received signal can be expressed as

$$\boldsymbol{r}=\sqrt{p_{\mathrm{u}}^d}\,\boldsymbol{x}+\boldsymbol{A}^{\mathrm{H}}\boldsymbol{n} \tag{5}$$

Then the kth user's signal is

$$\mathrm{r}_k=\sqrt{p_{\mathrm{u}}^d}\,x_k+\boldsymbol{a}_k^{\mathrm{H}}\boldsymbol{n} \tag{6}$$

Thus the SNR for the kth user is

$$\gamma_k=\frac{p_{\mathrm{u}}^d}{\boldsymbol{a}_k^{\mathrm{H}}\boldsymbol{a}_k}=\frac{p_{\mathrm{u}}^d}{\left\|\boldsymbol{a}_k\right\|^2} \tag{7}$$

The SNR in Eq.(7) can be converted as follows

$$\gamma_k=\frac{p_{\mathrm{u}}^d}{\left\|\boldsymbol{a}_k\right\|^2}=\frac{p_{\mathrm{u}}^d}{\left[(\boldsymbol{G}^{\mathrm{H}}\boldsymbol{G})^{-1}\right]_{kk}}=p_{\mathrm{u}}^d\boldsymbol{g}_k^{\mathrm{H}}(\boldsymbol{I}_M-\boldsymbol{N})\boldsymbol{g}_k=$$
$$p_{\mathrm{u}}^d\boldsymbol{u}_k^{\mathrm{H}}\boldsymbol{\Lambda}_k^{1/2}(\boldsymbol{I}_M-\boldsymbol{N})\boldsymbol{\Lambda}_k^{1/2}\boldsymbol{u}_k \tag{8}$$

where $\boldsymbol{N}=\boldsymbol{G}_{[k]}(\boldsymbol{G}_{[k]}^{\mathrm{H}}\boldsymbol{G}_{[k]})^{-1}\boldsymbol{G}_{[k]}^{\mathrm{H}}$, $\boldsymbol{G}_{[k]}$ represents $\boldsymbol{G}$ with column vector $\boldsymbol{g}_k$ removed, and $\boldsymbol{u}_k\sim CN(0,\boldsymbol{I}_L)$, $\boldsymbol{\Lambda}_k=\mathrm{diag}\left\{\beta_{1k},\beta_{2k},\cdots,\beta_{Lk}\right\}$.

In massive MIMO systems, there are a large number of antennas, so $\boldsymbol{N}\rightarrow\bar{\boldsymbol{N}}=\left(\frac{K-1}{L}\right)\boldsymbol{I}_L$. Hence, we can get the approximate SNR for the kth user[24]

$$\tilde{\gamma}_k=\frac{p_{\mathrm{u}}^d(L-K+1)}{L}\boldsymbol{u}_k^{\mathrm{H}}\boldsymbol{\Lambda}_k\boldsymbol{u}=$$
$$\sum_{l=1}^{L}\frac{p_{\mathrm{u}}^d(L-K+1)\beta_{lk}}{L}\left|u_{lk}\right|^2 \tag{9}$$

where u_{lk} is the lth element of the vector $\boldsymbol{u}_k$. Because u_{lk} obeys the standard complex Gaussian distribution, $\left|u_{lk}\right|^2$ obeys the exponential distribution. From the characteristic of the exponential distribution, the approximate SNR can be seen as the sum of exponential variables with different parameters.

$$\widetilde{\gamma}_k=\sum_{l=1}^{L}Y_{lk} \tag{10}$$

where $Y_{lk}=\frac{p_{\mathrm{u}}^d(L-K+1)\beta_{lk}}{L}\left|u_{lk}\right|^2\sim\gamma(1/\mu_{lk})$, here $\gamma(\cdot)$ represents the exponential distribution, and $\mu_{lk}=\frac{p_{\mathrm{u}}^d(L-K+1)\beta_{lk}}{L}$. Therefore, the PDF of the approximate signal-to-noise ratio $\widetilde{\gamma}_k$ is

$$f_{\tilde{\gamma}_k}(\chi)=\sum_{i=1}^{L}\frac{\zeta_i\exp\left(-\frac{\chi}{\mu_{ik}}\right)}{\mu_{ik}} \tag{11}$$

where $\zeta_i=\prod_{j=1,j\neq i}^{L}\left(1-\frac{\mu_{jk}}{\mu_{ik}}\right)^{-1}$.

2 Performance Analysis of Cross-Layer Design in DM-MIMO System

In this section, the performance of CLD scheme for DM-MIMO systems is studied, which combines adaptive M_n-ary quadrature amplitude modulation (QAM) at the physical layer and truncated ARQ at the data link layer. At the physical layer, the receiver estimates the channel to obtain CSI and sends it to the transmitter through the feedback channel. The transmitter chooses the best modulation mode for the next transmission based on the received CSI. At the data link layer, when the receiver receives the data, if the packet is wrong, a negative acknowledgement is sent back to request retransmission. Otherwise, an acknowledgement is sent. The packet which fails after the maximum number of retransmissions $N_r^{\max}$, will be abandoned.

2.1 Switching thresholds

In the cross-layer design scheme of DM-MIMO systems, N types of QAM modulation modes with different constellation sizes are selectable in the physical layer, and the stop-and-wait ARQ protocol is used in the data link layer, and the maximum number of retransmissions is set as $N_r^{\max}$. Given the system target packet loss rate PLR_0, the target PER is

$$\mathrm{PER}_0 = \mathrm{PLR}_0^{1/\left(N_r^{\max}+1\right)} \tag{12}$$

Assuming that each bit in the packet is not related with others, the instantaneous PER for the nth mode of modulation is

$$\mathrm{PER}_n(\chi) = 1-(1-\mathrm{BER}_n(\chi))^{N_b} \tag{13}$$

where N_b is the number of bits in each packet and $\mathrm{BER}_n(\chi)$ the BER of the nth mode of modulation.

The accurate BER formula consists of a number of complementary error functions. Furthermore, the BER formulas of 2QAM and 4QAM have only one item, and the first item of the higher-order modulation accounts for the most. Therefore, selecting the first complementary error function of the exact BER, the approximate PER can be expressed as

$$\mathrm{PER}_n(\chi) \approx 1-(1-\varphi_{n,1}\mathrm{erfc}(\sqrt{\phi_{n,1}\chi}))^{N_b} \tag{14}$$

Considering instantaneous PER constraints, i.e., $\mathrm{PER}_n(\chi) = \mathrm{PER}_0$, the switching thresholds for the system are

$$\chi_n^{\mathrm{th}} = \left[\mathrm{erfc}^{-1}\left((1-(1-\mathrm{PER}_0)^{1/N_b})\Big/\varphi_{n,1}\right)\right]^2 \Big/ \phi_{n,1} \tag{15}$$

2.2 Performance analysis

With PDF of the kth user's SNR given in Eq.(11), the average SE at the physical layer of the kth user based on the switching thresholds in Eq.(15) is

$$\overline{\mathrm{SE}}_k^{\mathrm{phy}} = \sum_{n=1}^{N} R_n \int_{\chi_n^{\mathrm{th}}}^{\chi_{n+1}^{\mathrm{th}}} \sum_{i=1}^{L} \zeta_i \frac{1}{\mu_{ik}} \exp\left(-\frac{\chi}{\mu_{ik}}\right) \mathrm{d}\chi = \sum_{n=1}^{N}\sum_{i=1}^{L} R_n \zeta_i \left(\exp\left(-\frac{\chi_n^{\mathrm{th}}}{\mu_{ik}}\right) - \exp\left(-\frac{\chi_{n+1}^{\mathrm{th}}}{\mu_{ik}}\right)\right) \tag{16}$$

where R_n is the rate corresponding to the modulation mode whose size of constellation graph is M_n, and $R_n = \log_2 M_n$.

The average PER of the kth user in a DM-MIMO system with CLD scheme is defined as

$$\overline{\mathrm{PER}}_k = \frac{\sum_{n=1}^{N} R_n \overline{\mathrm{PER}}_n^k}{\overline{\mathrm{SE}}_k^{\mathrm{phy}}} \tag{17}$$

where $\overline{\mathrm{PER}}_n^k$ represents the average PER of the kth user in DM-MIMO system when choosing the nth modulation mode. $\overline{\mathrm{PER}}_n^k$ can be calculated as follows

$$\overline{\mathrm{PER}}_n^k = \sum_{i=1}^{L} \frac{\zeta_i}{\mu_{ik}} \int_{\chi_n^{\mathrm{th}}}^{\chi_{n+1}^{\mathrm{th}}} \left(1-\left(1-\varphi_{n,1}\mathrm{erfc}\left(\sqrt{\phi_{n,1}\chi}\right)\right)^{N_b}\right) \exp\left(-\frac{\chi}{\mu_{ik}}\right) \mathrm{d}\chi = \sum_{i=1}^{L} \zeta_i \left(\left(\exp\left(-\frac{\chi_n^{\mathrm{th}}}{\mu_{ik}}\right) - \exp\left(-\frac{\chi_{n+1}^{\mathrm{th}}}{\mu_{ik}}\right)\right) - \frac{1}{\mu_{ik}} \int_{\chi_n^{\mathrm{th}}}^{\chi_{n+1}^{\mathrm{th}}} \left(1-\varphi_{n,1}\mathrm{erfc}\left(\sqrt{\phi_{n,1}\chi}\right)\right)^{N_b} \exp\left(-\frac{\chi}{\mu_{ik}}\right) \mathrm{d}\chi\right) \tag{18}$$

The second item in the brace in Eq.(18) is recorded as $\Xi_{k,n}^{i}$, which is too complex to be derived as a closed expression. Hence, numerical integration is used.

$$
\begin{aligned}
\Xi_{k,n}^{i} = & \frac{1}{\mu_{ik}} \int_{\chi_n^{\text{th}}}^{\chi_{n+1}^{\text{th}}} \left(1 - \varphi_{n,1} \operatorname{erfc}\left(\sqrt{\phi_{n,1} \chi}\right)\right)^{N_{\mathrm{b}}} \cdot \\
& \exp\left(-\frac{\chi}{\mu_{ik}}\right) \mathrm{d}\chi = \frac{1}{\mu_{ik}} \frac{\pi}{Q} \Big(1 - \\
& \varphi_{n,1} \operatorname{erfc}\left(\sqrt{\phi_{n,1} x_q}\right)\Big)^{N_{\mathrm{b}}} \cdot \\
& xp\left(-\frac{x_q}{\mu_{ik}}\right) \sqrt{(x_q - \chi_n^{\text{th}})(\chi_{n+1}^{\text{th}} - x_q)}
\end{aligned} \tag{19}
$$

where Q is the order of the numerical integral, $x_q = \frac{\chi_n^{\text{th}} + \chi_{n+1}^{\text{th}}}{2} + \frac{\chi_{n+1}^{\text{th}} - \chi_n^{\text{th}}}{2} \cos\frac{(2q-1)\pi}{2Q}$. Substituting Eq.(19) into Eq.(18), we can obtain

$$
\overline{\text{PER}}_n^k = \sum_{i=1}^{L} \zeta_i \left(\exp\left(-\frac{\chi_n^{\text{th}}}{\mu_{ik}}\right) - \exp\left(-\frac{\chi_{n+1}^{\text{th}}}{\mu_{ik}}\right) - \Xi_{k,n}^{i} \right) \tag{20}
$$

Therefore, substituting Eqs. (16, 20) into Eq.(17), the average PER of the kth user is

$$
\overline{\text{PER}}_k = \frac{\sum_{n=1}^{N} R_n \sum_{i=1}^{L} \zeta_i \left(\exp\left(-\frac{\chi_n^{\text{th}}}{\mu_{ik}}\right) - \exp\left(-\frac{\chi_{n+1}^{\text{th}}}{\mu_{ik}}\right) - \Xi_{k,n}^{i} \right)}{\sum_{n=1}^{N} \sum_{i=1}^{L} R_n \zeta_i \left(\exp\left(-\frac{\chi_n^{\text{th}}}{\mu_{ik}}\right) - \exp\left(-\frac{\chi_{n+1}^{\text{th}}}{\mu_{ik}}\right) \right)} \tag{21}
$$

The average PER of the system is expressed as

$$
\overline{\text{PER}} = \frac{1}{K} \sum_{k=1}^{K} \overline{\text{PER}}_k \tag{22}
$$

When a packet still fails after $N_{\mathrm{r}}^{\max} + 1$ times transmissions, the packet is discarded, so the average number of transmission for the packet of the kth user is

$$
\overline{N}_k = \frac{1 - \overline{\text{PER}}_k^{N_{\mathrm{r}}^{\max}+1}}{1 - \overline{\text{PER}}_k} \tag{23}
$$

Combined with the average SE of the physical layer in Eq.(16), the overall average SE in DM-MIMO systems with CLD scheme is presented at the top of next page.

$$
\begin{aligned}
\overline{\text{SE}} = & \frac{1}{K} \sum_{k=1}^{K} \frac{\overline{\text{SE}}_k^{\text{phy}}}{\overline{N}_k} = \\
& \frac{1}{K} \sum_{k=1}^{K} \frac{1}{\overline{N}_k} \left(\sum_{n=1}^{N} \sum_{i=1}^{L} R_n \zeta_i \left(\exp\left(-\frac{\chi_n^{\text{th}}}{\mu_{ik}}\right) - \exp\left(-\frac{\chi_{n+1}^{\text{th}}}{\mu_{ik}}\right) \right) \right)
\end{aligned} \tag{24}
$$

Since the numerical integration in Eq.(19) is complex, we adopt the following approximation of complementary error functions to simplify performance analysis.

$$
\operatorname{erfc}(x) \approx \frac{1}{\sqrt{\pi}\, x} \exp(-x^2) \tag{25}
$$

Substituting Eq.(25) into $\Xi_{k,n}^{i}$, we can infer that the approximate expression of $\Xi_{k,n}^{i}$ is

$$
\begin{aligned}
\tilde{\Xi}_{k,n}^{i} = & \frac{1}{\mu_{ik}} \sum_{i=0}^{N_{\mathrm{b}}} C_{N_{\mathrm{b}}}^{i} \left(-\frac{\varphi_{n,1}}{\sqrt{\pi \phi_{n,1}}} \right)^{N_{\mathrm{b}} - i} \cdot \\
& \Bigg(b_i^{-\frac{1+a_i}{2}} (\chi_n^{\text{th}})^{\frac{a_i - 1}{2}} \exp\left(-\frac{b_i \chi_n^{\text{th}}}{2}\right) W_{\frac{a_i-1}{2}, \frac{-a_i}{2}} (b_i \chi_n^{\text{th}}) - \\
& b_i^{-\frac{1+a_i}{2}} (\chi_{n+1}^{\text{th}})^{\frac{a_i - 1}{2}} \exp\left(-\frac{b_i \chi_{n+1}^{\text{th}}}{2}\right) W_{\frac{a_i-1}{2}, \frac{-a_i}{2}} (b_i \chi_{n+1}^{\text{th}}) \Bigg)
\end{aligned} \tag{26}
$$

where $a_i = -\frac{1}{2}(N_{\mathrm{b}} - i) + 1$, $b_i = \phi_{n,1}(N_{\mathrm{b}} - i) + \frac{1}{\mu_{ik}}$, and $W_{\lambda,\mu}(\bullet)$ is the Whitaker function. Therefore, the average PER of the kth user based on the approximate expression of the complementary error function is

$$
\overline{\text{PER}}_k^{\text{app}} = \frac{\sum_{n=1}^{N} R_n \sum_{i=1}^{L} \zeta_i \left(\exp\left(-\frac{\chi_n^{\text{th}}}{\mu_{ik}}\right) - \exp\left(-\frac{\chi_{n+1}^{\text{th}}}{\mu_{ik}}\right) - \tilde{\Xi}_i^{k,n} \right)}{\sum_{n=1}^{N} \sum_{i=1}^{L} R_n \zeta_i \left(\exp\left(-\frac{\chi_n^{\text{th}}}{\mu_{ik}}\right) - \exp\left(-\frac{\chi_{n+1}^{\text{th}}}{\mu_{ik}}\right) \right)} \tag{27}
$$

As a result, the approximate system average PER is

$$
\overline{\text{PER}}_{\text{app}} = \sum_{k=1}^{K} \frac{\overline{\text{PER}}_k^{\text{app}}}{K} \tag{28}
$$

Substituting Eq.(28) into Eq.(23), the corresponding average number of packet transmissions for the kth user is

$$
\overline{N}_k^{\text{app}} = \frac{1 - \left(\overline{\text{PER}}_k^{\text{app}}\right)^{N_{\mathrm{r}}^{\max}+1}}{1 - \overline{\text{PER}}_k^{\text{app}}} \tag{29}
$$

Based on Eqs.(16, 29), the approximate overall average SE in DM-MIMO systems with cross-layer design is given as follows

$$
\overline{\text{SE}}_{\text{app}} = \frac{1}{K} \sum_{k=1}^{K} \frac{\overline{\text{SE}}_k^{\text{phy}}}{\overline{N}_k^{\text{app}}} \tag{30}
$$

3 Simulation Results and Analysis

We will analyze and evaluate the performance of the CLD scheme in DM-MIMO systems, and compare the theory and simulation results of the average SE and average PER of the system to verify the validity of the theoretical formulas. We assume that the number of bits contained in each packet is $N_b = 1080$, and the sizes of QAM with gray-mapped constellation are $\{2, 4, 8, 16, 32, 64\}$. Unless otherwise stated, the target packet loss rate of the system is $\mathrm{PLR}_0 = 10^{-3}$, the cell radius is $R = 1000\,\mathrm{m}$, the reference distance is $d_0 = 100\,\mathrm{m}$, the number of randomly distributed APs is $L = 128$, the number of users is $K = 5$, and the path loss exponent is $v = 4.0$.

In Fig.2, we plot the overall average spectral efficiency (ASE) of CLD scheme in DM-MIMO systems under different maximum numbers of retransmissions $N_r^{\max}$. As shown in Fig.2, the ASE of the system under the same SNR increases as $N_r^{\max}$ increases. The reason is that with the increase of $N_r^{\max}$, the probability of a packet discarded becomes lower, so more packets are successfully transmitted and the ASE of the system increases accordingly. Moreover, it should be noted that the ASE of $N_r^{\max} = 1, 2, 3$ is higher than that of $N_r^{\max} = 0$, and $N_r^{\max} = 0$ means that the CLD scheme is not used. It proves that the CLD scheme improves the system performance. In addition, the approximate results are close to the accurate results. This illustrates the validity of the approximate formula (30). Hence, the approximate formulas can be used to effectively evaluate the system.

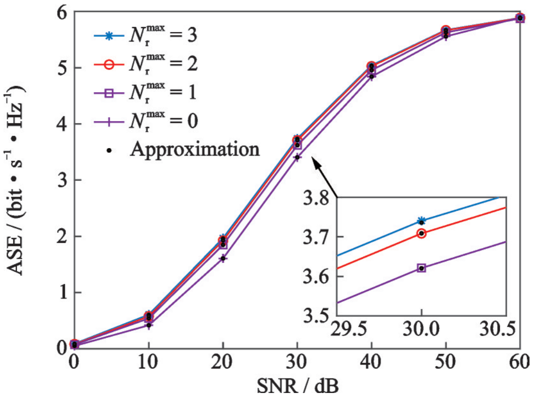

Fig.2 ASE of CLD scheme in DM-MIMO systems under different $N_r^{\max}$

Fig.3 shows the average packet error rate (APER) performance of the DM-MIMO systems when $N_r^{\max} = 0, 1, 2, 3$, respectively. We can find that the APER performance of the system deteriorates with the $N_r^{\max}$ increases. This is because that, under given target PLR, the target PER in Eq.(12) will increase with the increase of $N_r^{\max}$, namely, the limit on the system PER is gradually relaxed. In addition, combined with Fig.2, it is found that the ASE of $N_r^{\max} = 3$ increases little than that of $N_r^{\max} = 2$, but the APER performance of $N_r^{\max} = 3$ is worse. Consequently, it does not hold that larger $N_r^{\max}$ leads to better system performance, which also verifies that packets cannot be retransmitted indefinitely. At last, the approximate performance is close to the accurate performance. As the result, the approximate analysis can be used to effectively evaluate the system when the system has strict requirements for complexity.

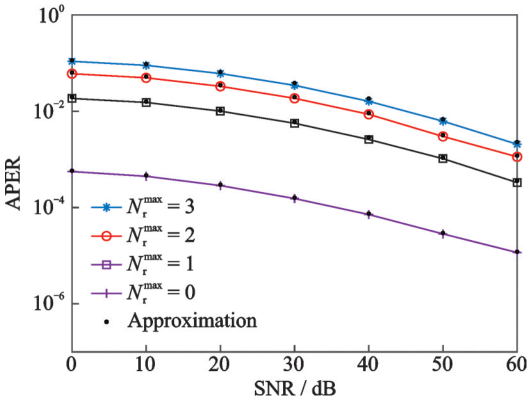

Fig.3 APER of CLD scheme in DM-MIMO systems under different $N_r^{\max}$

Figs.4, 5 compare the ASE and APER of CM-MIMO and DM-MIMO systems when $N_r^{\max} = 2$ and $N_r^{\max} = 0$, respectively. First of all, it can be found from the figures that the DM-MIMO system can obtain higher ASE and lower APER. This is because that the APs of the DM-MIMO system are dispersed in the cell to shorten the distance between the receiver and the user, thereby reducing the signal fading. Consequently, a higher-order modulation can be selected, so the ASE becomes higher. In CM-MIMO systems, the distance between the base station and users at the edge of cell is long, so the system performance is limited, namely, the ASE is lower. In summary, Figs.4, 5 illustrate the superiority of the DM-MIMO system. In addition, the system ASE of $N_r^{\max}=2$ is higher than that of $N_r^{\max}=0$, which is consistent with the previous analysis. Finally, we can observe that the theoretical curves and the simulation curves are basically consistent, which shows the effectiveness of the performance analysis.

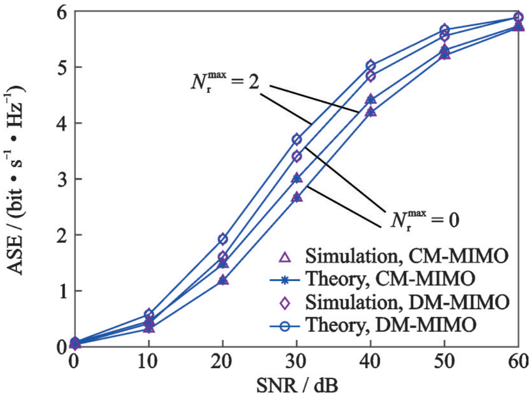

Fig.4 Comparison of ASE between CM-MIMO and DM-MIMO systems under different $N_r^{\max}$

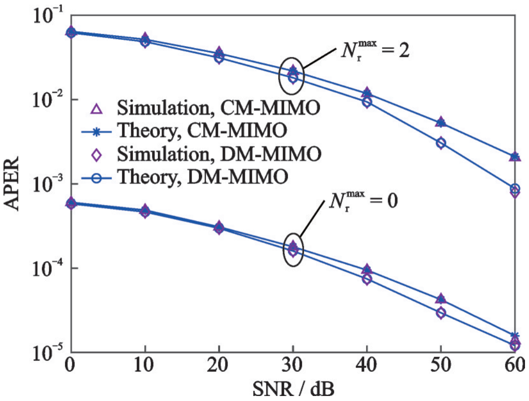

Fig.5 Comparison of APER between CM-MIMO and DM-MIMO systems under different $N_r^{\max}$

4 Conclusions

By combining discrete-rate AM with truncated ARQ protocol, the performance of uplink DM-MIMO systems with CLD scheme is investigated over Rayleigh fading channels. The APER and overall ASE of the system are analyzed. Firstly, using the ZF detection, we derive effective SNR and the corresponding conditional PDF. Secondly, under the target PLR constraint, the adaptive switching thresholds for CLD are obtained based on the approximate expression of PER. According to these results, the closed-form ASE at the physical layer is attained. Then with the help of the numerical integration, we deduce the theoretical APER and overall ASE, respectively, and the closed-form expressions are achieved. Finally, with the approximation of complementary error functions, the approximate APER and overall ASE are derived. It can be seen from the simulation results that the theoretical ASE and APER can match the corresponding simulation results well, proving the effectiveness of the theoretical analysis. Thus, the performance of DM-MIMO with CLD scheme can be assessed well. Besides, the impact of the number of the maximum retransmissions on system performance is also analyzed. The ASE performance can be improved through increasing the maximum retransmission number, but the improvement will become less for larger number of the maximum retransmission. Besides, the DM-MIMO can obtain superior performance over the CM-MIMO according to Figs.4, 5.

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Authors Ms. **WANG Hui** received her B.S. degree from Nanjing University of Aeronautics and Astronautics (NUAA) in 2019. Now she is a postgraduate of NUAA. Her research interests include massive MIMO and cross-layer design.

Prof. **YU Xiangbin** received his Ph.D. degree in communication and information systems from Southeast University in 2004. From 2010 to 2011, he was a research fellow in Department of Electronic Engineering, City University of Hong Kong. From 2014 to 2015, he was a visiting scholar in electrical and computer engineering, University of Delaware. Now he is a professor of NUAA. His research interests include distributed MIMO, adaptive modulation, precoding design, and green communication.

Author contributions Ms. **WANG Hui** designed the study, complied the models and wrote the manuscript. Prof. **YU Xiangbin** contributed to data and model components, and conducted the analysis. Mr. **LIU Fuyuan** provided the simulation results and interpreted the results. Dr. **LIU Tao** contributed to the discussion and background of the study. Mr. **BAI Jiawei** contributed to the simulation and prepared the drafts. All authors commented on the manuscript draft and approved the submission.

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瑞利衰落信道上行分布式大规模MIMO系统跨层设计性能分析

王　慧[1]，虞湘宾[1,2]，刘涪源[1]，柳　涛[1]，白嘉维[1]

(1.南京航空航天大学电子与信息工程学院，南京 211106，中国；
2.中国科学院上海微系统与信息技术研究所无线传感器网络与通信重点实验室，上海 200030，中国)

摘要：研究了在瑞利衰落信道下将离散速率自适应调制与自动重传技术相结合的上行分布式大规模多输入多输出(Multiple-input multiple-output, MIMO)系统的跨层设计(Cross-layer design, CLD)性能。通过性能分析，基于每个用户近似有效信噪比的条件概率密度函数和目标丢包率(Packet loss rate, PLR)约束下的切换阈值，推导出了采用CLD的分布式大规模MIMO系统的平均误包率(Average packet error rate, APER)和整体平均频谱效率(Average spectral efficiency, ASE)的闭式表达式。根据以上结果，使用互补误差函数的近似值，还可以推导出近似的APER和整体ASE。仿真结果表明理论分析是有效的，得到的理论ASE和APER能很好地匹配相应的仿真。此外，该系统可以满足目标PLR要求，并且分布式大规模MIMO系统比集中式大规模MIMO提供了明显的性能提升。

关键词：分布式大规模MIMO系统；迫零检测；跨层设计；频谱效率；服务质量；误包率