Integrated Route Recovery of Aircraft and Aircrew Based on Column Generation Algorithm

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Abstract: The problem of abnormal flight recovery has always been the focus and difficulty in the field of civil aviation, and has important research significance. According to the recovery strategy, characteristics, and constraints of aircraft, aircrews and flights, this paper is based on the column generation algorithms. A mathematical optimization model for the integrated recovery of aircraft and aircrew in the case of temporary aircraft failures was established, and the corresponding solution algorithm was designed. At the same time, the influence of aircraft and aircrew on route selection was taken into account. Finally, the method of calling Cplex by Java was used. Part of the flight plan data actually operated by the company verifies the feasibility, accuracy and timeliness of the model and algorithm.

Key words: integrated recovery; route recovery; column generation; multi-label

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0 Introduction

An irregular flight refers to a flight that cannot be executed according to the original plan. Although the flight can make a comprehensive plan, the plan will never catch up with changes. There are too many random interference factors in the flight operation. Extreme weather, unexpected conditions, mechanical failures and other factors often disrupt the overall planning and re-arrangement, causing serious problems. Range of flight delays or adjustments brings many changes. Therefore, it is sufficient and necessary to schedule flights in real time under abnormal circumstances, re-optimize the allocation of aircraft and aircrew members, and improve the quality of flight recovery.

There are many variables involved in the recovery of abnormal flights, and it is difficult to model and solve them. Therefore, for many years, the problem of recovery of various resources of abnormal flights has been solved by the phased approach. The staged recovery is clearly defined and easy to solve. However, the later stage depends on the recovery results of the previous stage, and it is difficult to ensure global optimization. At the same time, aircraft and aircrews are the inherent resources of airlines, and they cooperate and restrict each other in time and space. Whether the flight can be executed normally depends on whether all resources are in place at the same time. Teodorović et al. was the first to study the problem of integrated flight recovery, and used a sequence method based on the dynamic programming algorithm to solve the integrated recovery of aircraft routes and aircrews[1]. In recent years, Jozefowiez et al. designed a heuristic algorithm to recover passengers and aircraft simultaneously[2]. Zhu et al. used the hybrid ensemble programming method to design a search algorithm to solve the integrated restoration constraint programming model of aircraft and aircrew[3]. Sinclai et al. proposed a heuristic algorithm of large neighborhood search to solve the integrated aircraft and passenger recovery problem[4]. Zhang et al. proposed a two-stage heuristic algorithm to solve the integrated aircraft and aircrew recovery problem, and simulat-

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ed three disturbance scenarios for testing[5]. Le et al. designed a special recursive algorithm and a paired storage tree method to obtain the feasible path of the aircraft and the aircrew[6]. Zhou et al. used the Benders decomposition algorithm to achieve flight schedule and aircraft path recovery under random disturbances[7].

In this paper, a mathematical model of resource assignment is used to optimize the integrated recovery of abnormal flight aircraft and aircrews under a single model. Aiming at the shortage of aircraft resources caused by temporary aircraft failures, an integrated recovery model of aircraft and aircrew was established, and flight connections were established based on the Hashgraph. The column generation algorithm framework was used to optimize the selection of feasible aircraft routes and aircrew routes based on multi-label algorithms. And when searching for the flight aircrew's route, it also took into account the possible delay of the aircraft. Finally, a calculation example verifies the correctness and effectiveness of the integrated recovery model and the proposed algorithm.

1 Problem Description

1.1 Definition of flight recovery problem

When the flight plan is disturbed or disturbed, in order to resume normal operations, the airline needs to adjust the aircraft path, aircrew scheduling, maintenance plan, passenger itinerary and ground support resources based on existing aircraft, aircrew and other resources and flight plans. These are collectively referred to as the airline's irregular flight recovery problems.

Normally, abnormal flight recovery includes the stages of aircraft plan recovery, aircrew schedule recovery, passenger itinerary recovery, and an optimized new flight plan. Airlines generally adopt a phased recovery method: First resume the aircraft plan, second resume the aircrew schedule, and finally resume the passenger itinerary. The main reason for adopting this restoration sequence is that the aircraft is a scarce resource and the aircrew is relatively easy to obtain. The integrated restoration of irregular flights refers to the restoration of two or three of the three resources of aircraft, aircrew and passengers.

The recovery strategy of abnormal flights can be summarized from three aspects: For flights, normal execution, delay, and cancellation strategies are generally adopted; for aircraft, strategies such as executing the original flight, exchanging flights, calling empty flights, and stopping places are generally adopted; for the aircrew, strategies such as executing original flights, exchanging flights, adding aircrews, and calling backup aircrews are adopted.

1.2 Integrated problem optimization process

In some linear optimization problems, the number of constraints is limited, but the number of variables will explode as the problem size grows, so all variables cannot be explicitly expressed in the model. At this time, the simplex method seems powerless, so the column generation algorithm is proposed on the basis of the simplex method.

The column generation algorithm (CGA) is a method for solving large-scale linear programming problems. It can solve linear programming models with a huge number of variables. Its theoretical basis is proposed by Danzig in 1960. When solving a minimization problem, the main function of the column generation algorithm is to find a better lower bound for each search tree node.

There are many possible combinations of flight connections in the integrated restoration of aircraft and aircrews. The number of feasible routes for aircraft and aircrews is very large, that is, the scale of variables in the optimization model is huge. This paper will optimize the modeling and algorithm design based on CGA. The solution process is shown in Fig.1.

First, the problem of integrated recovery of abnormal flight aircraft and aircrews is divided into main problem (MP) and sub-problem (SP). Then, optimization models and solving algorithms are established. Due to the huge number of variables in MP, some of the variables (including at least one feasible solution) are selected to construct a restrain master problem (RMP). After initializing the route set, the simplex method is used to solve the RMP model to obtain the dual variable value. Combined the flight connection diagram, the available start time of the aircraft and the aircrew and the information of the airport, the flight arrival and departure time and the airport, the SP algorithm is used to solve the minimal cost and determine whether its value is negative or not. If not, the optimal solution is obtained, which is the optimal recovery plan. Otherwise, the two routes are added to the route set and solving the RMP is continued. The optimal solution is output through iteration.

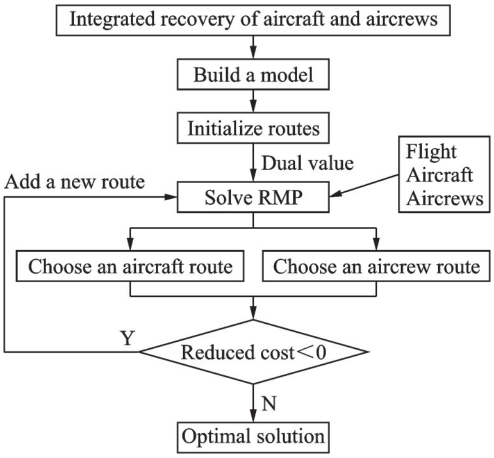

Fig.1 Flow chart for solving the integration problem

2 Model Building

2.1 Symbol definition

(1) Collection

K: Aircraft collection, $k \in K$;

P: Aircrew collection, $p \in P$;

F: Flight collection, $f \in F$;

M: Aircraft route collection, $m \in M$;

N: Aircrew route collection, $n \in N$.

(2) Parameter

C_f^1: Cost of canceling flight f;

C_f^2: Plus aircrew cost of flight f;

$C_{m,k}$: Cost of aircraft route m assigned to aircraft k (including delay cost and cancellation cost);

$C_{n,p}$: Cost of aircrew route n assigned to aircrew p (including delay cost and cancellation cost);

$$\alpha_{m,f} = \begin{cases} 1 & \text{Aircraft route } m \text{ including flight } f \\ 0 & \text{Others} \end{cases};$$

$$\beta_{n,f} = \begin{cases} 1 & \text{Aircrew route } n \text{ including flight } f \\ 0 & \text{Others} \end{cases}.$$

(3) Decision variables

$$y_f = \begin{cases} 1 & \text{Flight } f \text{ cancelled} \\ 0 & \text{Others} \end{cases};$$

q_f: Number of plus aircrews for flight f;

$$x_{m,k} = \begin{cases} 1 & \text{Aircraft route } m \text{ including aircraft } k \\ 0 & \text{Others} \end{cases};$$

$$x_{n,p} = \begin{cases} 1 & \text{Aircrew route } n \text{ including aircrew } p \\ 0 & \text{Others} \end{cases}.$$

2.2 Mathematical model

This paper decomposes the problem of integrated recovery of abnormal flight aircraft and aircrew into a main problem based on resource assignment and a sub-problem based on path selection. Resource assignment is to use decision variables to assign various resources to various users in order to find an optimal solution. In the main problem, the aircraft and aircrew are assigned to the appropriate path (aircraft route and aircrew route, or flight string), and the sub-problem is a special weighted shortest path search problem.

2.2.1 Main problem model

(1) Objective function

When solving the problem of abnormal flight restoration, the optimization objective is to minimize restoration cost. The restoration cost includes flight cancellation cost, plus aircrew cost, delay cost, aircraft exchange cost and aircrew exchange cost. The mathematical expression is

$$\min z = \sum_{f \in F} C_f^1 y_f + \sum_{f \in F} C_f^2 q_f + \sum_{m \in M} \sum_{k \in K} C_{m,k} x_{m,k} + \sum_{n \in N} \sum_{p \in P} C_{n,p} x_{n,p} \tag{1}$$

(2) Constraints

Constraint 1 Aircraft coverage constraint for flights. This constraint guarantees that if flight f is cancelled, all feasible routes of the aircraft containing the flight will not be executed, that is, the flight cannot be executed by any aircraft; if it is not cancelled, there will be one and only one feasible route that contains the flight can be executed, that is, the flight can only be executed by one aircraft. The mathematical expression is

$$\sum_{m\in M}\sum_{k\in K}\alpha_{m,f}x_{m,k}+y_f=1 \quad \forall f\in F \tag{2}$$

Constraint 2 The flight aircrew coverage constraint. This constraint is similar to Constraint 1, but when the aircrew is covered, the possibility of adding an aircrew to the flight is increased, that is, if the flight f is not cancelled, it can be executed by one or more aircrews. The mathematical expression is

$$\sum_{n\in N}\sum_{p\in P}\beta_{n,f}x_{n,p}+y_f-q_f=1 \quad \forall f\in F \tag{3}$$

Eq.(3) adds a crew variable, which can take any value when flight f is executed. Generally, one crew can be added at most, but when flight f is cancelled, the value of the flight plus crew variable can only be 0. That is, if $y_f=1$, then $q_f=0$; if $y_f=0$, then $q_f=0, 1, 2, \cdots, N$. The mathematical expression in the planning model is

$$y_f+q_f/N\leqslant 1 \quad \forall f\in F \tag{4}$$

Constraint 3 Aircraft use constraints. This constraint guarantees that each aircraft can only be executed at most one feasible route, allowing the possibility that the aircraft will not perform flight tasks. The mathematical expression

$$\sum_{m\in M}x_{m,k}\leqslant 1 \quad \forall k\in K \tag{5}$$

Constraint 4 Aircrew assignment constraint. This constraint guarantees that each aircrew can only be executed at most one feasible route, and allows the possibility that the aircrew does not perform the task. The mathematical expression

$$\sum_{n\in N}x_{n,p}\leqslant 1 \quad \forall p\in P \tag{6}$$

In summary, the main problem mathematical model of the integrated recovery of aircraft and aircrew for abnormal flights in this paper is described as

$$\min z=\sum_{f\in F}C_f^1 y_f+\sum_{f\in F}C_f^2 q_f+\sum_{m\in M}\sum_{k\in K}C_{m,k}x_{m,k}+\sum_{n\in N}\sum_{p\in P}C_{n,p}x_{n,p} \tag{7}$$

$$\begin{aligned}
\text{s.t.}\quad & \sum_{n\in N}\sum_{p\in P}\beta_{n,f}x_{n,p}+y_f-q_f=1 && \forall f\in F\\
& \sum_{m\in M}x_{m,k}\leqslant 1 && \forall k\in K\\
& \sum_{n\in N}x_{n,p}\leqslant 1 && \forall p\in P\\
& x_{m,k}\in\{0,1\} && \forall m\in M,\forall k\in K\\
& x_{n,P}\in\{0,1\} && \forall n\in N,\forall p\in P\\
& y_f\in\{0,1\} && \forall f\in F\\
& q_f\in\{0,1,2,\cdots\} && \forall f\in F
\end{aligned}$$

2.2.2 Restricted main problem model

The column generation algorithm is used to solve large-scale linear optimization problems, but the decision variables in the main problem model are integers, so the integer variables need to be relaxed to continuous variables to obtain the linear main problem. The variable relaxation is defined as

$$x_{m,k}\geqslant 0 \quad \forall m\in M,\forall k\in K \tag{8}$$

$$x_{n,p}\geqslant 0 \quad \forall n\in N,\forall p\in P \tag{9}$$

$$y_f\geqslant 0 \quad \forall f\in F \tag{10}$$

$$q_f\geqslant 0 \quad \forall f\in F \tag{11}$$

Restricting the main problem is to select a partial list to solve it, and restrict the variable scale in the linear main problem to a smaller size, which is equivalent to forcibly restricting other variables to non-basic variables. In the studied problem, modeling is based on the feasible routes of part of the aircraft and aircrew. Since it is not much different from the previous model, it will not be listed.

2.2.3 Subproblem model

The sub-problem is to choose the newly included base problem, that is, to search for new aircraft and aircrew feasible routes to join the restricted main problem. The solution of the sub-problem is used to judge whether the optimal solution of the main restriction problem has been obtained, and to provide a new column for the main restriction problem when the optimal solution is not obtained. The judgment basis is the test number of the non-basic variable, and the dual variable of the restriction main problem structure.

The reduced cost of executing the feasible route m for the aircraft k

$$C_{m,k}^{\mathrm{R}}=C_{m,k}-\sum_{f\in F}\alpha_{m,f}\varepsilon_f^1-\varepsilon_k \tag{12}$$

where ε_f^1 represents the dual variable of Constraint 1, and ε_k the dual variable of Constraint 3.

The reduced cost for the aircrew p to execute the feasible route n is

$$C_{n,p}^{\mathrm{R}}=C_{n,p}-\sum_{f\in F}\beta_{n,f}\varepsilon_f^2-\varepsilon_p \tag{13}$$

where ε_f^2 represents the dual variable of Constraint 2, and ε_p the dual variable of Constraint 4.

Since the objective function in the restricted main problem model is to minimize the total recovery cost, the sub-problem needs to select the route with the smallest value among the aircraft and aircrew routes with negative reduction costs to join the restricted main problem. The mathematical model of searching for aircraft route and aircrew route in the sub-problem is

$$\min \sum_{i\in F}\sum_{j\in F} d_{i,j}x_{i,j} \tag{14}$$

$$\text{s.t.} \quad \sum_{j\in \mathrm{adj}(i)} x_{i,j} - \sum_{j:i\in \mathrm{adj}(j)} x_{j,i} = \begin{cases} 1 & i=s \\ 0 & \text{Others} \\ -1 & i=t \end{cases}$$

$$x_{i,j}\in\{0,1\} \qquad \forall i,j\in F$$

where $d_{i,j}$ represents the reduced cost of each flight side in the flight connection diagram, and its calculation method will be specifically introduced in the sub-problem algorithm. $x_{i,j}$ is the flight side selection variable, which is a 0/1 variable: Value of 1 indicates that this flight edge is selected as the edge in the new aircraft route or aircrew route. $j\in \mathrm{adj}(i)$ indicates the previous flight node with i as j; s the source node; t the sink node; and F the collection of flight nodes.

3 Algorithm Design

3.1 Flight connection graph generation algorithm

The flight connection graph includes flight nodes and flight edges. Each flight node contains the flight arrival and departure time and the arrival and departure airport information. Each flight edge points from the previous flight node to the subsequent flight node. Two virtual flight nodes are set: The source node and the sink node. All flight nodes point to the sink node, and the source node points to all flight nodes. The generation of the new aircraft route and aircrew route in the sub-problem needs to be based on the flight connection diagram. In this paper, the hash diagram is used to establish the flight connection based on the arrival and departure time of each flight and the departure airport. The generation algorithm is as follows:

Step 1 Traverse every two flight nodes i and j;

Step 2 Determine whether the two flight nodes meet the space and time constraints. The space constraint is that the two flights take off and land at the same airport, and the time constraint is that the two flights guarantee the minimum transit constraint under the maximum delay time limit;

Step 3 If the edge (i, j) meets the above constraints, the flight connection is established, and the edge (i, j) is added to the flight edge set.

3.2 Solving algorithm of the restricted problem

The solution process of the restricted main problem mainly uses the simplex method and the dual simplex method to linearly solve the mathematical model. The solution algorithm is shown below.

Input: Aircraft collection, aircrew collection, flight plan, aircraft route collection, aircrew route collection.

Output: The new flight route executed by the aircraft and aircrew.

Step 1 According to the current aircraft and aircrew route and the objective function and constraint equations in the mathematical model, establish the value coefficient matrix and augmented constraint matrix.

Step 2 Use the simplex method to solve the current model.

Step 3 Calculate the values of the dual variables ε_f^1, ε_f^2, ε_k and ε_p of each constraint.

Step 4 Solve the sub-problems. Use the dual variable values to obtain the route with the least reduced cost when each aircraft and each aircrew execute its own routes. Then, obtain the smallest aircraft route among the routes with the smallest reduced cost executed by different aircrafts, and the selection of the smallest aircrew route also uses this method.

Step 5 If the reduced cost of the aircraft route <0 and the reduced cost of the aircrew route <0, add these two routes to the aircraft route and aircrew route sets to form a new column;

Return to Step 1;

Else

End the iteration, the current solution is the optimal solution;

End if

Step 6 End.

3.3 Solving algorithm of the sub-problem

The sub-problem is similar to the weighted shortest path problem, which can be solved by the three-label form and the reverse tracking method. In the search of the aircraft route, the influence of aircraft failure maintenance and aircraft exchange, the minimum transit time and the maximum delay time are considered. In the flight aircrew route search, the basic framework is similar to the aircraft route search, but the aircraft related parameters are added in the search process, and the impact of the possible delay of the aircraft is taken into account. The specific solution algorithm is as follows:

Input: Flight node set, flight connection edge set, flight arrival and departure time, available start time of aircraft and aircrew, maximum delay time MDT, minimum transit time MCT, dual variable value.

Output: New aircraft route and aircrew route.

Step 1 Set three labels for each flight node f, which are the reduced cost of flight node f, the previous flight node and the delay time.

Step 2 Initialize the label value of each flight node (0, 0, 0).

Step 3 Update the label value of each flight node.

Compared with the aircraft route search, the flight aircrew route search increases the influence of the possible delay of the aircraft, and the other parts are roughly the same. So we will not repeat the update algorithm of the flight node during the aircraft route search. The specific algorithm is as follows:

(1) Traverse the combination of each flight node i and each aircraft k.

① Determine whether the crew, flight and aircraft combination (p, i, k) meets the conditions: {departure time of flight node $i <$ available start time of plane k, and delay time of flight node $i <$ available start time of plane k-flight node i departure time}. If this condition is met, calculate the difference between the available start time of aircraft k and the departure time of flight node i. If the value is greater than MDT, jump out of this cycle; otherwise, record the difference;

② Determine whether the conditions are met: {departure time of flight node $i <$ available start time of flight crew p, and delay time of flight node $i <$ available start time of flight crew p-departure time of flight node i}. If this condition is met, calculate the difference between the available start time of crew p and the departure time of flight node i. If the value is greater than MDT, it will jump out of this cycle; otherwise, the difference will be recorded;

③ Update the delay time of flight node i to the original label value and the maximum of the two differences.

(2) Traverse each subsequent flight node j of flight node i in the flight connection graph.

① Judge whether the flight and subsequent flight combinations (i, j) meet the conditions: {departure time of flight node $j <$ arrival time of flight node $i +$ delay time of flight node $i +$ MCT}. If this condition is met, calculate the delay time of flight node j, whose value is the sum of the arrival time of flight node i, the delay time of flight node i and MCT, and then minus the departure time of flight node j. If the delay time of flight node j is greater than MDT, jump out this cycle, and update the label of the next subsequent flight node; otherwise, continue the cycle to update the delay cost of flight node j;

② Determine whether the subsequent flight and crew combination (j, p) meets the conditions: {the flight node j originally planned to execute the crew is crew p}. If the conditions are met, the current cost of updating flight node j is the sum of the reduced cost flight node i, delay time of flight node j^* delay cost per unit time, the shadow price of flight node j and the shadow price of crew p; otherwise, update the current cost of flight node j to the sum of the reduced cost of flight node i, the delay time of flight node j^*, the delay cost per unit time, the shadow price of flight node j, the shadow price of crew p and the cost of crew exchange;

③ Judge whether the current cost of flight node j is less than the reduced cost of flight node j. If the conditions are met, update the reduced cost of flight node j to the current cost, and update the previous flight node of flight node j to flight node i.

Step 4 Using the reduced cost label of the sink node and the preorder label of each flight node to trace backward, the set of feasible routes for each aircraft and the set of feasible routes for each aircrew can be obtained, and the route with the smallest reduced cost is taken separately. The route with the smallest value among the routes with the least reduced cost for different aircraft and different aircrews is the two groups of flight routes listed.

3.4 The initial solution of the restricted main problem

There are two ideas for obtaining the initial solution. One is to use the aircraft route and aircrew route in the original flight plan as the initial solution, and the other is to use heuristic algorithms to randomly generate the aircraft route and aircrew route. Taking into account the efficiency of the solution and the characteristics of flight recovery, this paper uses the original aircraft route and aircrew route set as the initial solution.

4 Case Analysis

This paper selects the feasibility and accuracy of part of the flight plan data test model and algorithm of a certain domestic airline, involving three aircraft, four crews, 24 flights, and 11 airports. The specific data are shown in Table 1. The test parameters are the minimum flight transit time of 45 min, the maximum delay time of 120 min, the cost of flight cancellation ￥50 000 RMB/shift, the cost of additional crew ￥1 000 RMB/unit, the cost of aircraft exchange ￥1 000 RMB/time, the crew exchange cost ￥1 000 RMB/time, and the cost of flight delay ￥100 RMB/min. The test interference scenario is that the aircraft B2 is checked for failure at 06:00 and cannot be ready at 06:35. The estimated maintenance time is 3 h and can be resumed at 09: 35. The end of the recovery period is 24: 00. The computer used for the test is configured with 1.80 GHz Intel(R) Core (TM) i5-8265U CPU and 8.00 GB memory, and the operating system is Windows 10 64-bit. The test is based on Java and Cplex, and the test results are shown in Tables 2, 3.

Table 1 Part of one airline's flight plan

Aircraft	Flight	Departure airport	Arrival airport	Departure time	Arrival time	Aircrew
B1	1001	CKG	LFQ	0725	0855	P1
	1002	LFQ	TSN	0945	1105	
	1003	TSN	DLC	1345	1445	
	1004	DLC	JXA	1525	1710	P2
	1005	JXA	DLC	1750	1940	
	1006	DLC	TSN	2020	2130	
B2	1007	CKG	JNG	0635	0850	
	1008	JNG	WEH	1010	1130	
	1009	WEH	DLC	1250	1340	
	1010	DLC	WEH	1610	1700	P1
	1011	WEH	JNG	1805	1930	
	1012	JNG	CKG	2040	2325	
B3	1013	CKG	LFQ	0755	0915	P3
	1014	LFQ	TSN	1050	1155	
	1015	TSN	DLC	1245	1350	
	1016	DLC	TSN	1435	1605	
B4	1017	CKG	LZH	0640	0810	P4
	1018	LZH	HAK	0905	1030	
	1019	HAK	LZH	1120	1230	
	1020	LZH	CKG	1310	1445	
	1021	CKG	LLV	1535	1705	P5
	1022	LLV	TSN	1745	1855	
	1023	TSN	LLV	2010	2130	
	1024	LLV	CKG	2210	2405	

Table 2 Aircraft recovery results

Aircraft	Flight after resumption
B1	1001,1002,1003,1004,1005,1006
B2	1013,1014,1015,1016
B3	1007,1008,1009,1010,1011,1012
B4	1017,1018,1019,1020,1021,1022,1023,1024

Table 3 Aircrew recovery results

Aircrew	Flight after resumption
P1	1001,1002,1003,1010,1011,1012
P2	1017,1018,1019,1020
P3	1013,1014,1015,1016
P4	1004,1005,1006,1007,1008,1009
P5	1021,1022,1023,1024

The final recovery plan is to exchange the execution flights of aircrafts B2 and B3, and exchange the execution flights of aircrews P2 and P4. The total restoration cost is ￥24 500 RMB; the number of cancelled flights is 0; the number of additional aircrews is 0; and the solution time is 0.02 s. It can be seen from the solution results that the model and algorithm can realize the integrated recovery of the aircraft and the aircrew in the event of a temporary aircraft failure, and can solve the problem in a short time.

If the aircraft does not undergo an integrated recovery after a temporary failure, and postponement measures are taken for the disrupted flights, the original planned number of flight delays is 6, and the total delay time is 860 min.

According to the data, it can be seen that compared with the recovery results of the delayed and delayed measures, the number of delayed flights and the total length of flight delays in the integrated recovery calculation results are obvious. When a single aircraft fails, the integrated restoration result reduces the number of delayed flights by 2, and reduces the total flight delay time by 620 min. In horizontal comparison, it is reduced by 33.33% and 72.09%, respectively. Therefore, the flight execution plan after the integrated restoration can effectively absorb unnecessary delay time, avoid the delay of redundant flights, and reduce the loss of airline operating costs.

5 Conclusions

According to the recovery strategies, characteristics, constraints, etc. of aircraft, aircrews and flights, based on the column generation algorithm and multi-label algorithm, this paper decomposes the integrated recovery of aircrafts and aircrew in the case of aircraft temporary failures into the main problem of route assignment and route selection. In the sub-problem, the corresponding mathematical model and solution algorithm are established and designed. In the flight aircrew route search of the sub-problem, not only the time and other constraints are considered, but the possible delay effects of the aircraft are added, and both the aircraft and the aircrews are considered. Finally, the method of calling Cplex by Java is used to verify the feasibility, accuracy and timeliness of the model and the algorithm based on part of the flight plan data actually operated by an airline. It can be applied to the airline's small and medium-scale flight recovery. There are still many shortcomings in the research of this paper. For example, the scale of the calculation example is not large enough, and the constraints of the base airport are not taken into account, which will be the direction that needs to be considered in future.

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基于列生成的飞机和机组一体化恢复研究

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摘要:不正常航班恢复问题一直是民航领域研究的重点和难点,具有重要的研究意义和价值。本文根据飞机、机组和航班的恢复策略、特性、约束等,基于列生成算法和多标签算法,建立了飞机临时故障情况下的飞机和机组一体化恢复的数学优化模型,并设计了相应的求解算法,同时考虑到了飞机和机组对路线选择的影响。最后采用Java调用Cplex的方法,基于某航空公司实际运行的部分航班计划数据验证了模型和算法的可行性、准确性和时效性。

关键词:一体化恢复;路径恢复;列生成;多标签