Static Pull-In Analysis of a Composite Laminated Nano-beam with Flexoelectric Effect

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Abstract: Based on the new modified couple stress theory and considering the flexoelectric effect of the piezoelectric layers, the Euler Bernoulli nano-beam model of composite laminated materials driven by electrostatically fixed supports at both ends is established. The nonlinear differential governing equations and boundary conditions are derived by the Hamilton principle. The generalized differential quadrature method (GDQM) and the Newton Raphson method are used to numerically solve the differential governing equations. The influence of flexoelectric effect on the static and the dynamic pull-in characteristics of laminated nano-beams is analyzed. The results of the numerical calculation are in a good agreement with those in the literature when the model degenerated into a nano-beam model without flexoelectric effect. The stacking sequence, length scale parameter l and piezoelectric layer applied voltage V_p of the composite will affect the pull-in voltage, frequency and time-domain response of the structure. Given that the flexoelectric effect will reduce the pull-in voltage and dimensionless natural frequency of the structure, the maximum dimensionless displacement at the midpoint of the beam and the period of time-domain response should be increased.

Key words: flexoelectric effect; piezoelectric effect; pull-in; generalized differential quadrature method (GDQM)

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0 Introduction

Due to the development of science and technology, micro and nano electro mechanical system (M/NEMS) technology has made a major breakthrough. Among various actuation mechanisms, such as electrostatic actuation, optical actuation and thermal actuation, electrostatic actuation is one of the most widely used driving mechanisms. Electrostatically actuated micro-beam is one of the most typical structures. It is usually composed of two electrodes, in which the deformable electrode is suspended above the clamping electrode (Fig.1). When a voltage is applied between the suspended electrode and the clamping electrode, the deformable electrode at the top will bend towards the clamping electrode at the bottom due to electrostatic excitation. When the voltage between the two electrodes increases to the critical value, the deformation of the upper electrode will suddenly increase and finally collapse to the clamping electrode. This phenomenon is called pull-in instability[1-3]. For AC/DC voltage, it is called dynamic/static pull-in phenomenon[4-6]. If the pull-in phenomenon occurs in M/NEMS, the stability will be affected and the system will no longer be able to work effectively. Therefore, it is of great significance to study the suction characteristics of M/NEMS..

In the nanoscale, the strain gradient also plays an important role in the characterization of the mechanical behavior of structures[7-8]. The phenomenon of electric polarization caused by strain gradient is called flexoelectric effect. The flexoelectric effect is also a kind of electromechanical coupling effect like piezoelectricity. However, flexoelectric effect exits not only exists in asymmetric medium, but also in

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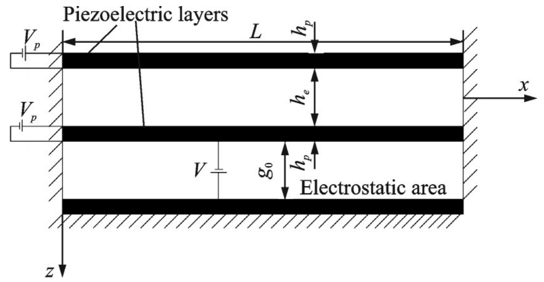


Fig.1 Schematic of clamped-Clamped NEMS actuator

asymmetric medium[9-11]. Compared with the piezoelectric effect, the flexoelectric effect is much weaker. But on nanscale, the flexoelectric effect can be designed to be of the same order as the piezoelectric. The reason is that the response of the flexoelectric effect is inversely proportional to the size of the structure[12-15]. Composite materials have been widely used in aerospace and other fields in recent years due to its advantages of low specific gravity, high specific strength and so on[16]. It is necessary to study the mechanical properties of composite nanostructures with flexoelectric effect in nanoscale.

1 Mathematical Formulation

The size dependence can be expressed using modified couple-stress theory in which the curvature tensor is utilized as deformation measures in addition to the conventional strain measures. According to the modified couple-stress theory, stress tensor and strain tensor can be written as

$$\begin{cases} \sigma_{ij} = \lambda \mathrm{tr}(\varepsilon)\delta_{ij}, \varepsilon_{ij} = \dfrac{1}{2}(u_{i,j} + u_{j,i}) \\ m_{ij} = 2l^2\mu, \chi_{ij} = \dfrac{1}{2}(\theta_{i,j} + \theta_{j,i}) \end{cases} \quad (1)$$

where σ_{ij} and ε_{ij} are the components of the Cauchy stress tensor and strain tensor, respectively. m_{ij}, χ_{ij} are the components of the deviatoric part of couple stress tensor and curvature tensor, respectively. δ_{ij} is the Kronecker delta, λ and μ are Lame's constants given in Eq.(2), l is a material length scale parameter which is needed in addition to the Lame's constants. It is worth mentioning that the parameter l is the square ratio of the curvature modulus to the shear modulus in mathematics and is an essentially property to measure the effects of couple stress. Then $\boldsymbol{\theta}$ is the rotation vector which is related to the displacement vector $\boldsymbol{u}$ such that

$$\begin{cases} \lambda = \dfrac{E\nu}{(1+\nu)(1-2\nu)}, \mu = \dfrac{E}{2(1+\nu)} \\ \theta_i = \dfrac{1}{2}\mathrm{curl}(u_i) \end{cases} \quad (2)$$

where E is the Young's modulus and ν the Poisson's ratio.

Based on the Euler-Bernoulli hypothesis, the displacement fields of the composite laminated beam (Fig.2) can be written as

$$u = -z\frac{\partial w}{\partial x},\ v = 0,\ w = w(x,t) \quad (3)$$

where u, v, and w are the displacements of global x-, y- and z-axes, respectively.

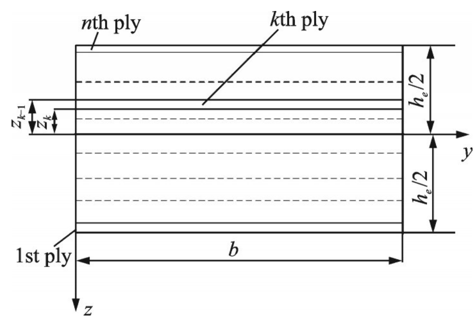


Fig.2 Schematic of composite laminated beam (Side view)

From Eqs.(1, 3), we can obtain the nonzero strain and rotation component as

$$\begin{cases} \varepsilon_x = -z\dfrac{\partial^2 w}{\partial x^2} \\ \theta_y = -\dfrac{\partial w}{\partial x} \end{cases} \quad (4)$$

For the kth layer of the composite, m_{ij} can be rewritten as

$$\begin{Bmatrix} m_{x'y'} \\ m_{y'x'} \end{Bmatrix} = \begin{bmatrix} C_{44}^k l_{kb}^2 & C_{55}^k l_{km}^2 \\ C_{44}^k l_{kb}^2 & C_{55}^k l_{km}^2 \end{bmatrix} \begin{Bmatrix} \dfrac{\partial \theta_{x'}}{\partial y'} \\ \dfrac{\partial \theta_{y'}}{\partial x'} \end{Bmatrix} \quad (5)$$

where we use the comma in the upper-right corner to denote the local coordinate. Therefore, (x', y', z') is the local coordinate of the kth layer, and the x' direction is the same as the fiber. For the parameters in the equation above, $C_{44}^k = G_{13}^k$, $C_{55}^k = G_{23}^k$, in which G_{13}^k, G_{23}^k are the shear modulus of the kth layer. l_{kb}, l_{km} are the material length scale parameters of the fiber and matrix of the kth layer.

For beams with large ratio of length-to-height, the influence of Poisson's ratio can be ignored, so Eq.(5) can be rewritten as Eq.(6), in which $\boldsymbol{\varepsilon}'^k$, C_{11}^k and ν_{21}^k can be expressed as Eq.(7).

$$\begin{bmatrix}\sigma_{x'}^k\\ m_{x'y'}^k\\ m_{y'x'}^k\end{bmatrix}=\begin{bmatrix}C_{11}^k & & \\ & l_{kb}^2C_{44}^k & l_{km}^2C_{55}^k\\ & l_{kb}^2C_{44}^k & l_{km}^2C_{55}^k\end{bmatrix}\begin{bmatrix}\varepsilon_{x'}\\ \chi_{x'y'}\\ \chi_{y'x'}\end{bmatrix} \tag{6}$$

$$\begin{cases}\boldsymbol{\varepsilon}'^k=[\ \varepsilon_{x'}\quad \chi_{x'y'}\quad \chi_{y'x'}\ ]^{\mathrm{T}}=\begin{bmatrix}\dfrac{\partial u'}{\partial x'} & \dfrac{\partial \theta_{x'}}{\partial y'} & \dfrac{\partial \theta_{y'}}{\partial x'}\end{bmatrix}^{\mathrm{T}}\\ C_{11}^k=\dfrac{E_1^k}{1-(\nu_{12}^k)^2}\\ \nu_{21}^k=\dfrac{\nu_{12}^kE_2^k}{E_1^k}\end{cases} \tag{7}$$

where E_1^k, E_2^k denote the Young's modulus of the kth layer and ν_{12}^k, ν_{21}^k the Poisson's ratio.

In global coordinate, the constitutive equation of the kth layer can be expressed as

$$\sigma^k=\begin{Bmatrix}\sigma_x^k\\ m_{xy}^k\\ m_{yx}^k\end{Bmatrix}=\begin{bmatrix}Q_{11}^k & & \\ & \hat{Q}_{44}^k & \hat{Q}_{55}^k\\ & \hat{Q}_{44}^k & \hat{Q}_{55}^k\end{bmatrix}\begin{Bmatrix}\dfrac{\partial u}{\partial x}\\ \dfrac{\partial \theta_x}{\partial y}\\ \dfrac{\partial \theta_y}{\partial x}\end{Bmatrix} \tag{8}$$

in which Q^k can be expressed as follows

$$\begin{cases}Q_{11}^k=m^4C_{11}^k\\ \hat{Q}_{44}^k=m^4l_{kb}^2C_{44}^k+n^4l_{km}^2C_{55}^k-m^2n^2(l_{kb}^2C_{44}^k+l_{km}^2C_{55}^k)\\ \hat{Q}_{55}^k=n^4l_{kb}^2C_{44}^k+m^4l_{km}^2C_{55}^k-m^2n^2(l_{kb}^2C_{44}^k+l_{km}^2C_{55}^k)\end{cases} \tag{9}$$

where $m=\cos\psi^k$, $n=\sin\psi^k$, ψ^k is the angle of the ply. If $\psi^k=0$ or $\psi^k=\pi/2$, $m^2n^2=0$. In practice, $l_{kb}\gg l_{km}$, so we can assume that $l_{km}=0$. Besides, for isotropic materials, $C_{11}^k=E$, $C_{44}^k=C_{55}^k=G$, and $l_{kb}=l_{km}=l$, so we can simplify Eq.(9) as follows

$$\begin{cases}Q_{11}^k=C_{11}^k=E\\ \hat{Q}_{44}^k=\hat{Q}_{55}^k=l_{kb}^2C_{44}^k=l^2G\end{cases} \tag{10}$$

Based on the modified couple-stress theory, we can express the bending strain energy of the composite laminated beam U_2 as

$$U_2=\frac{1}{2}\int_{\Omega_e}\sigma_x\varepsilon_x\mathrm{d}\Omega+\frac{1}{2}\int_{\Omega_{e+p}}m_{yx}\chi_{yx}\mathrm{d}\Omega \tag{11}$$

where the subscripts e, p denote the composite laminated beam and piezoelectric layer. In Eq.(11), we also take the modified couple-stress of the piezoelectric layers into account.

The energy density of the piezoelectric layer can be expressed as[17]

$$U_1=\frac{1}{2}a_{kl}P_kP_l+\frac{1}{2}c_{ijkl}\varepsilon_{ij}\varepsilon_{kl}+d_{ijk}\varepsilon_{ij}P_k+f_{ijkl}P_i\eta_{jkl} \tag{12}$$

where a_{kl}, c_{ijkl} and d_{ijk} are the elements for the reciprocal dielectric susceptibility, elastic coefficient, and piezoelectric coefficient tensors, respectively. P_k, P_l, and P_i are the components for the polarization vector. f_{ijkl} is the flexoelectric coefficient tensor. The constitutive equation of the piezoelectric layer is

$$\begin{cases}\sigma_{ij}=\dfrac{\partial U_1}{\partial \varepsilon_{ij}}=c_{ijkl}\varepsilon_{kl}+d_{ijk}P_k\\ \tau_{ijk}=\dfrac{\partial U_1}{\partial \varepsilon_{ijk}}=f_{ijkl}P_i\\ E_i=\dfrac{\partial U_1}{\partial P_i}=a_{ij}P_i+d_{ijk}\varepsilon_{jk}+f_{ijkl}\eta_{jkl}\end{cases} \tag{13}$$

For a slender beam, as it is always assumed that electric field changes only in the z direction, the axial displacement η_{xxx} is so small compared with η_{xxz} that can be ignored, so we only take η_{xxz} into account in this paper. The electric enthalpy density function of the piezoelectric layer can be defined as

$$H_1=U_1-\frac{1}{2}\varepsilon_0\varphi_{,i}\varphi_{,i}+\varphi_{,i}P_i \tag{14}$$

where $\varepsilon_0=8.854\times10^{-12}\ \mathrm{C^2/(N\cdot m^2)}$ is the dielectric constant in vacuum, φ is the electric potential and can be expressed by electric field $-\varphi_{,i}=E_i$. For convenience, the tensor subscripts of material properties are abbreviated as: $c_{11}=c_{1111}$, $f_{13}=f_{3113}$, the subscript 1, 2 and 3 are the x-, y- and z- axes, respectively. We can expressed the electric enthalpy as

$$H=\int_{\Omega_p}H_1\mathrm{d}V \tag{15}$$

Considering the fringing field, the energy resaved between the two electrodes U_v is

$$U_v=\frac{1}{2}\int_0^LF_v\mathrm{d}x=\frac{1}{2}\int_0^L\left[\frac{\varepsilon_0bV^2}{g_0-w}+\frac{2\varepsilon_0V^2}{\pi}\left[\ln\left(\frac{\pi b}{g_0-w}+1\right)+1\right]\right]\mathrm{d}x \tag{16}$$

where V is the voltage applied between the two plates and w the displacement of the clamped electrode.

It is assumed that the composite laminated beam and the piezoelectric layer on its surface are perfectly aligned, so when the same voltage is applied to the piezoelectric layer on the upper and lower surfaces of the composite laminated beam, the axial force of tension or compression generated in the piezoelectric layer will lead to the same axial force in the composite laminated beam. It can be expressed as

$$T_p = \int_{\Omega_p} \sigma_{11}^E \mathrm{d}y\mathrm{d}z = 2bV_p \frac{d_{31}}{a_{33}} - 2bh_p \frac{d_{31} f_{13} \eta_{113}}{a_{33}} \tag{17}$$

where V_p denotes the voltage applied to the piezoelectric layer.

The axial force generated by the residual stress is

$$T_r = \int_{\Omega_e} \left(-Q_{11}^k z \frac{\partial^2 w}{\partial x^2} + \sigma_r \right) \mathrm{d}A = \sigma_r b h_e \tag{18}$$

where $\sigma_r = \sigma_0 (1-\nu)$. σ_r, σ_0 are the biaxial residual stress and the effective residual stress, respectively. ν is the Poisson's ratio.

For clamped-clamped beams, the force reduced by neutral plane stretching can be written as

$$T_a = \frac{b \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} Q_{11}^k \mathrm{d}z}{2L} \int_0^L \left(\frac{\partial w}{\partial x} \right)^2 \mathrm{d}x \tag{19}$$

The energy reduced by kinds of axial forces is

$$U_s = -\frac{1}{4} \int_0^L (2T_r + 2T_p + T_a) \left(\frac{\partial w}{\partial x} \right)^2 \mathrm{d}x \tag{20}$$

Casimir force plays an important role in the mechanical properties of electrostatic driven nanobeams, the work done by Casimir force per unit length is

$$U_c = \int_0^L \mathrm{d}x \int_0^{w(x)} F_c \mathrm{d}w = -\int_0^L \frac{\pi^2 \hbar c b}{720 (g_0 - w)^3} \mathrm{d}x \tag{21}$$

where $\hbar = 1.055 \times 10^{-34}\,\mathrm{J \cdot s}$ is the Plank's constant and $c = 2.998 \times 10^8\,\mathrm{m/s}$ is the speed of light in vacuum.

Since the work done by the external force is 0, the nonlinear differential governing equation and its boundary conditions for the clamped-clamped composite laminated nanobeam which is electrostatic actuated are established in this paper and can be obtained by Hamilton principle

$$\delta \int_{t_1}^{t_2} (H + U_2 + U_s + U_c + U_v) \mathrm{d}t = 0 \tag{22}$$

Substituting Eqs.(11, 15—16, 20—21) into Eq.(22), the nonlinear differential governing equation along the nanobeam can be given as

$$D_2 \frac{\mathrm{d}^4 w}{\mathrm{d}x^4} - \left(\frac{2bV_p d_{31}}{a_{33}} + T_r + T_a \right) \frac{\partial^2 w}{\partial x^2} = \frac{\pi^2 \hbar c b}{240 (g_0 - w)^4} + \frac{\varepsilon_0 b V^2}{2 (g_0 - w)^2} \left(1 + \frac{0.65 (g_0 - w)}{b} \right) \tag{23}$$

$$\begin{cases} D_2 = b \int_{z_k}^{z_{k+1}} \sum_{k=1}^{n} (Q_{11}^k z^2) \mathrm{d}z + b \int_{z_k}^{z_{k+1}} \sum_{k=1}^{n} \hat{Q}_{55}^k \mathrm{d}z + \alpha_1 \\ \alpha_1 = \left(c_{11} - \frac{\varepsilon_0 d_{31}^2}{1 + a_{33} \varepsilon_0} \right) I_z - \frac{2A_1 f_{13}^2}{a_{33}} - \frac{d_{31}^2 A_1 (h_e + h_p)^2}{2a_{33} (1 + a_{33} \varepsilon_0)} \\ I_z = b \int_{-\left(\frac{h_e}{2} + h_p\right)}^{-\frac{h_e}{2}} z^2 \mathrm{d}z + b \int_{\frac{h_e}{2}}^{\frac{h_e}{2} + h_p} z^2 \mathrm{d}z \\ A_1 = b h_e \end{cases} \tag{24}$$

In piezoelectric layer, we also have

$$\begin{cases} (E_3 + \varphi_{,3}) = 0 \\ (-\varepsilon_0 \varphi_{,33} + P_{3,3}) = 0 \end{cases} \tag{25}$$

The boundary conditions of the nano-beam are

$$\begin{cases} \left[\alpha_1 \frac{\partial^3 w}{\partial x^3} - (I_2 Q_{11}^k + A \hat{Q}_{55}^k) w''' - \left(\frac{2bV_p d_{31}}{a_{33}} + T_r + T_a \right) \left(\frac{\partial w}{\partial x} \right) \right] \delta w \Big|_0^L = 0 \\ \left[D_2 \frac{\partial^2 w}{\partial x^2} - \frac{2A_1 f_{13} V_p}{a_{33} h_p} - \frac{A_1 V_p^2}{h_p^2} - b h_p \frac{d_{31} f_{13}}{4a_{33}} \left(\frac{\partial w}{\partial x} \right)^2 \right] \delta w' \Big|_0^L = 0 \\ (-\varepsilon_0 \varphi_{,33} + P_{3,3}) \delta \varphi \left(\Big|_{\frac{h_e}{2}}^{\frac{h_e}{2} + h_p} + \Big|_{-\frac{h_e}{2} - h_p}^{-\frac{h_e}{2}} \right) = 0 \end{cases} \tag{26}$$

Besides, the boundary conditions of a clamped-clamped nanobeam should also be satisfied

$$\begin{cases} w \Big|_{x=0,L} = 0, \quad \frac{\partial w}{\partial x} \Big|_{x=0,L} = 0 \\ \varphi \Big|_{z=\pm\frac{h_e}{2}} = 0, \quad \varphi \Big|_{z=\pm\left(\frac{h_e}{2} + h_p\right)} = V_p \end{cases} \tag{27}$$

By introducing the following nondimensional parameters

$$\begin{cases} X = \frac{x}{L},\ W = \frac{w}{g_0} \\ \beta_1 = \frac{L^2 (T_p + T_r)}{D_2},\ \beta_2 = \frac{g_0^2 b \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} Q_{11}^k \mathrm{d}z}{D_2},\ \beta_3 = \frac{\varepsilon_0 b L^4}{2D_2 g_0^3} \\ \beta_4 = \frac{0.65 g_0 \beta_3}{b},\ \beta_5 = \frac{\pi^2 \hbar c b L^4}{240 D_2 g_0^5} \end{cases} \tag{28}$$

the differential governing equation can be expressed as

$$\frac{\partial^4 W}{\partial X^4} - \left(\beta_1 + \beta_2 \int_0^l \left(\frac{\partial W}{\partial X}\right)^2 \mathrm{d}X\right)\frac{\partial^2 W}{\partial X^2} = \frac{\beta_5}{(1-W)^4} + \left(\frac{\beta_3 V^2}{(1-W)^2} + \frac{\beta_4 V^2}{(1-W)}\right) \tag{29}$$

2 Results and Discussion

2.1 Verification of model and solution method

In this section, the static pull-in behaviors of a electrostatic actuated clamped-clamped composite laminated nanobeam are studied. Substituting Eq.(10) into Eqs.(18—19, 28), we can get the parameters when the laminations are [0°/90°/90°/0°] and [90°/0°/0°/90°] as follows

$$D_2 = \frac{7}{96} b h_e^3 C_{11}^k + \frac{1}{2} b(h_e + 2h_p) l_{kb}^2 C_{44}^k + \alpha_1 \tag{30}$$

$$D_2 = \frac{1}{96} b h_e^3 C_{11}^k + \frac{1}{2} b(h_e + 2h_p) l_{kb}^2 C_{44}^k + \alpha_1 \tag{31}$$

Then substituting Eqs.(30—31) into Eqs.(28—29), we can study the static pull-in behaviors of the clamped-clamped model by using GDQM[18].

In order to validate the model established in this paper and the solution method we used, the thickness of composite laminated beam is set to be 1 nm and the ply angle is assumed to be [0°/90°/90°/0°], the material scale parameter and the voltage applied on the piezoelectric layer are set to be zero, and the values of the remaining size parameters are shown in Table 1. The material constants of composite laminated beam are given as[19]: $E_2 = 6.98$ GPa, $E_1 = 25E_2$, $\mu_{12} = 0.25$, $G_{13} = 0.5E_2$, $G_{23} = 0.2E_2$. Table 2 shows the pull-in voltage and normalized deflection with the mid-plane stretching in different initial gaps. It is shown that the pull-in voltage and normalized deflection estimated by the current model and approach are in a good agreement with those in Ref.[18].

Table 1 Micron scale material constants

Parameter	MEM actuator	Piezoelectric layer
Width/μm	50	50
Height/μm	1	0.01
Young's modulus/GPa	169	78.6
d_{31}	—	1.87×10^8
a_{33}	—	0.79×10^8
Mass density/(kg•m^{-3})	2 331	7 500

Table 2 Pull-in voltage and normalized deflection with different initial gaps

g_0/μm	Ref.[18]		Present	
	V	w	V	w
1	3.84	0.337 4	3.84	0.337 4
2	10.64	0.302 5	10.64	0.302 3
3	19.39	0.297 0	19.38	0.296 1

2.2 Pull-in behaviors of the clamped-clamped composite laminated beam-like NEMS

In this section, we study the influence of flexoelectric effect on the pull-in behaviors under different conditions. The geometric parameters of the electrostatic actuated clamped-clamped composite laminated nanobeam are shown in Table 3. The material constants are still given as[10]: $E_2 = 6.98$ GPa, $E_1 = 25E_2$, $\mu_{12} = 0.25$, $G_{13} = 0.5E_2$, $G_{23} = 0.2E_2$.

Fig.3 shows the trend of nondimensional deflec-

Table 3 Nanoscale material constants

Parameter	MEM actuator	Piezoelectric layer
Width/nm	$b = 5(h_e + h_p)$	$b = 5(h_e + h_p)$
Height/nm	$h_e = 20$	$0.5h_e$
Length/nm	$L = 20(h_e + h_p)$	$L = 20(h_e + h_p)$
Initial gap	$g_0 = h_p$	$g_0 = h_p$
Young's modulus/ GPa	169	78.6
d_{31}	—	1.87×10^8
a_{33}	—	0.79×10^8
Mass density/(kg•m^{-3})	2 331	7 500

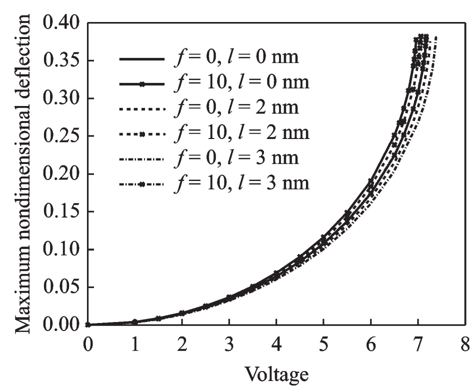
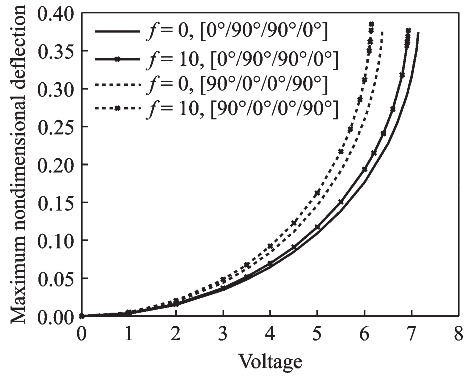


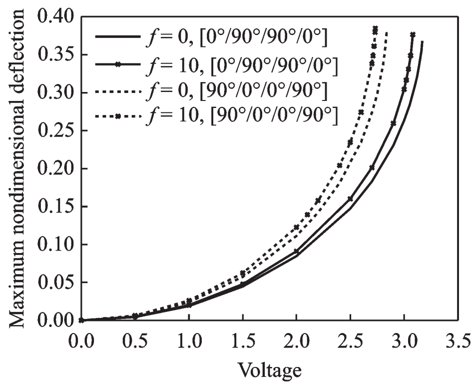
Fig.3 Nondimensional deflection at the midpoint of clamped-clamped composite laminated nanobeam with different scale parameters

tion at the midpoint of the electrostatic actuated clamped-clamped composite laminated nanobeam with the variation of voltage applied between two electrodes under different scale parameters, where f denotes flexoelectric coefficient. As can be seen from Fig.3, with the increase of voltage or scale parameter l, the nondimensional displacement at the midpoint of the nanobeam established in this paper all increases, too, and the structure may become unstable when voltage increases to a certain value. The pull-in voltage value of the structure become smaller when flexoelectric effect is taken into account. In the cases of $l=0, 2$, and 3 nm, the pull-in voltages are 6.96, 7.05, and 7.17 with flexoelectric effect while these voltages are 7.17, 7.26, and 7.38 without flexoelectric effect. we can see that the differences of pull-in voltage are all 0.21. It can be learnt that the influence of flexoelectric effect on the pull-in voltage is pretty stable with the change of scale parameters.

In Fig.4, the influences of different types of composite laminated cross-ply and length-height ratio on the flexoelectric effect are studied, and it is noted that in both cases $l=2$ nm. According to Fig.4, we can know that the pull-in voltage of [0°/90°/90°/0°] is bigger than that of [90°/0°/0°/90°]. It indicates that the composite laminated cross-ply can influence the pull-in characteristics of the structure and the pull-in voltage of the structure can be controlled by setting different cross-ply. In addition, by comparing Fig.4(a) with Fig.4(b), we can also get the influence of L/h ($h=h_e+h_p$) on the pull-in voltage. Obviously, the structure of which L/h is greater will be more likely to reach the pull-in state. When $L=20h$, the ratios of the increase of pull-in voltage due to the consideration of flexoelectric effect (ξ equals one minus the ratio of the pull-in voltage with flexoelectric effect to the pull-in voltage without flexoelectric effect) are 0.029 and 0.038, while these ratios are 0.028, 0.035 when $L=30h$, respectively. Therefore, the influence of flexoelectric effect on pull-in voltage will be changed when there are different composite laminated cross-ply



(a) $L=20h$



(b) $L=30h$

Fig.4 Nondimensional deflection at the midpoint of clamped-clamped composite laminated nanobeam with different cross-ply

and different L/h. In particular, under the condition that [90°/0°/0°/90°], the flexoelectric effect has a more significant influence on the pull-in voltage of the structure.

Next, we discuss the influence of flexoelectric effect when the composite laminated cross-ply is [0°/90°/90°/90°] and the scale parameter $l=2$ nm, with different residual stresses σ_0. The x-axis represents the voltage applied on the piezoelectric layer, while the y-axis represents the pull-in voltage. It can be seen from Fig.5 that the pull-in voltage of the nano-beam increases with the increase of residual stress but decreases with the increase of the voltage applied on the piezoelectric layer. When the flexoelectric effect is taken into account, it is clear that the pull-in voltage is smaller than that when the flexoelectric effect is ignored. Moreover, the pull-in voltage tends to decrease with the increase of the voltage applied on the piezoelectric layer, while the pull-in voltage tends to increase with the increase of

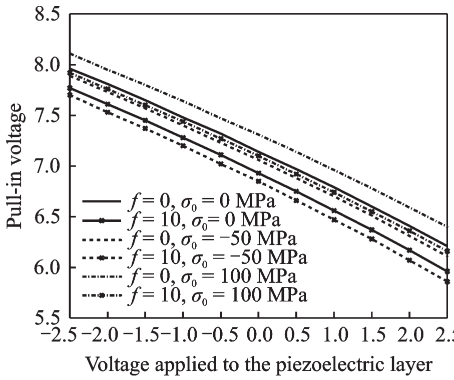


Fig.5 Influence of residual stresses and the voltage applied to the piezoelectric layer on the pull-in voltage

residual stress. In the cases of with and without flexoelectric effect, the trend of pull-in voltage variation is in a same way, and the difference value is not conspicuous, which indicates that ξ becomes larger when flexoelectric effect is taken into account with the increase of the voltage applied to piezoelectric layer. When $\sigma_0 = 0$ MPa, ξ increases from 0.024 to 0.04 when the voltage applied on the piezoelectric layer increases from -2.5 to 2.5. Besides, with the increase of σ_0, ξ becomes small ($\xi = 0.034$ and 0.032 when $\sigma_0 = 0$ MPa and 100 MPa). Therefore, when considering the residual stress, the influence caused by the flexural electric effect cannot be ignored.

In Fig. 6, the influence of flexoelectric effect on pull-in characteristics is studied with or without Casimir force. The composite laminated cross-ply is [0°/90°/90°/90°], the scale parameter $l = 2$ nm, and the residual stresses $\sigma_0 = 100$ MPa. The other

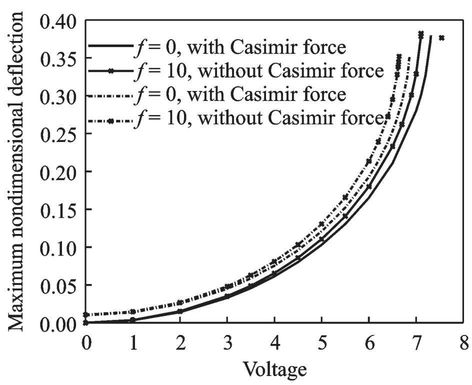


Fig.6 Nondimensional deflection at the midpoint of clamped-clamped composite laminated nanobeam with or without Casimir force

parameters which we have not mentioned now are still shown in Table 3. From Fig.6, we can see that with the considering of Casimir force, the pull-in voltage and displacement of the composite laminated nanobeam reduce whether considering flexoelectric effect or not. The result shows that in the nanoscale, Casimir force can produce a great influence on the structure's pull-in behaviors, Therefore, it is necessary to consider the Casimir force when analyzing and discussing nanostructures. When the flexoelectric effect is taken into account, the pull-in voltage and displacement of the structure all have a tendency to decrease, and $\xi =$ 0.034, 0.029 whether we take Casimir force into account or not. So we can know that when Casimir force is considered, the flexoelectric effect has a larger influence on the pull-in characteristics of the structure.

3 Conclusions

Based on the modified couple-stress theory, a clamped-clamped composite laminated Euler-Bernoulli nano-beam actuated by electrostatic model is established considering the flexoelectric effect. The differential governing equation and boundary conditions are established by using the Hamilton principle, and solved by using GDQM and the Newton-Raphson iterative method. The influence of flexoelectric effect on the pull-in characteristics of the proposed model is obtained from the analysis of the results. The result shows that the pull-in voltage of the structure will decrease considering the flexoelectric effect. In different conditions, the influences of flexoelectric effect on the pull-in characteristics are various, but for the proposed model, ξ is within 5%.

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考虑挠曲电效应的复合材料纳米梁静态吸合特性分析

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摘要：基于新修正偶应力理论，在考虑挠曲电效应的情况下，建立了两端固支静电驱动复合材料层合 Euler-Bernoulli 纳米梁模型，通过 Hamilton 原理导出其非线性微分控制方程以及边界条件，选用广义微分求积法（Generalized differential quadrature method, GDQM）和 Newton-Raphson 法对微分控制方程进行数值求解，分析了挠曲电效应对层合纳米梁静态和动态吸合特性产生的影响。结果表明：考虑挠曲电效应的复合材料层合纳米梁模型退化为不考虑挠曲效应的纳米梁模型后的数值计算结果与已有文献中数据相吻合。复合材料的铺层顺序、长度尺度参数 l 以及压电层施加电压 V_p 等参数对结构的吸合电压、频率以及时域响应都会产生影响。同时，考虑挠曲电效应会使得结构的吸合电压和量纲化为一的固有频率减小，梁中点最大量纲化为一的位移和时域响应的周期增大。

关键词：挠曲电效应；压电效应；吸合；广义微分求积法