Bending and Buckling of Circular Sinusoidal Shear Deformation Microplates with Modified Couple Stress Theory

*QING Hai**, *WEI Lu*

State Key Laboratory of Mechanics and Control of Mechanical Structures,
Nanjing University of Aeronautics and Astronautics, Nanjing 210016, P. R. China

(Received 15 January 2022; revised 13 February 2022; accepted 15 February 2022)

Abstract: The modified couple stress theory (MCST) is applied to analyze axisymmetric bending and buckling behaviors of circular microplates with sinusoidal shear deformation theory. The differential governing equations and boundary conditions are derived through the principle of minimum total potential energy, and expressed in nominal form with the introduced nominal variables. With the application of generalized differential quadrature method (GDQM), both the differential governing equations and boundary conditions are expressed in discrete form, and a set of linear equations are obtained. The bending deflection can be obtained through solving the linear equations, while buckling loads can be determined through solving general eigenvalue problems. The influence of material length scale parameter and plate geometrical dimensions on the bending deflection and buckling loads of circular microplates is investigated numerically for different boundary conditions.

Key words: circular microplates; size-effect; modified couple stress theory (MCST); general differential quadrature method (GDQM)

CLC number: TN925 **Document code:** A **Article ID:** 1005-1120(2022)01-0079-08

0 Introduction

Recent years, the micro- and nano-scale circular and annular plates are widely applied in micro-/nano-electro-mechanical systems such as accuators[1-2], sensors[3], resonators[4-5] and so on. Both experimental tests and molecule simulation have shown that the mechanical responses of micro- and nano-scale structures are size-dependent. Classical continuum mechanics fails to capture the size-dependent behaviors of micro- and nano-scale structures due to the absence of intrinsic length parameters. Several high-order continuum mechanical models, e.g. modified couple stress theory (MCST)[6-7] and strain gradient theory (SGT)[8-9], have been developed to address the size-dependent behaviors.

SGT: Gousias and Lazopoulos[10] derived the in-close form solution for static bending of clamped and simply-supported circular Kirchhoff plates. Ji et al.[11] compared the bending and vibration responses of circular Kirchhoff plate with different strain gradient theories. Li et al.[12] applied general differential quadrature method (GDQM) to study the nonlinear bending of circular Kirchhoff plate. Ansari et al.[13-14] applied GDQM to study thermal stability of annular Mindlin microplates and the nonlinear bending, buckling and free vibration of Mindlin plate. Mohammadimehr et al.[15] applied GDQM to study the dynamic stability of annular Mindlin sandwich plates. Zhang et al.[16] applied GDQM to study the bending, buckling and vibration of third-order shear deformable circular microplates.

MCST: Combining the orthogonal collocation point method and Newton-Raphson iteration method, Wang et al.[17] studied the nonlinear bending behavior of clamped and simply-supported circular

*Corresponding author, E-mail address: qinghai@nuaa.edu.cn.

How to cite this article: QING Hai, WEI Lu. Bending and buckling of circular sinusoidal shear deformation microplates with modified couple stress theory[J]. Transactions of Nanjing University of Aeronautics and Astronautics, 2022, 39(1): 79-86.

http://dx.doi.org/10.16356/j.1005-1120.2022.01.008

Kirchhoff plates. Ke et al.[18] employed GDQM to study the bending, buckling and vibration behaviors of annular Mindlin plate. Arshid et al.[19] studied the influence of Pasternak foundation on the bending and buckling response of annular/circular sandwich Mindlin sandwich microplate. Reddy and Berry[20] and Reddy et al.[21] derived the differential governing equations and developed finite element model for nonlinear axisymmetric bending of functionally graded circular Kirchhoff and Mindlin plates, respectively. Zhou and Gao[22] applied Fourier-Bessel series to study the linear bending of clamped circular Mindlin plate. Eshraghi et al.[23] applied GDQM to study the bending and free vibrations of thermally loaded FG annular and circular micro-plates based on Kirchhoff plate, Mindlin plate and third-order shear deformation theories. Sadoughifar et al.[24] employed GDQM to study the influence of Kerr elastic foundation on the nonlinear bending of thick annular and circular microplate based on two-variable shear deformation theory.

In this paper, MCST is applied to study the axisymmetric bending and buckling circular microplates based on sinusoidal shear deformation theory. The differential governing equations and boundary conditions are derived through the principle of minimum potential energy. Several nominal variable are introduced to simplify the mathematical expression, and the governing differential equations and boundary conditions are discretized with GDQM. The effect of material length scale parameter and plate dimensions as well as boundary conditions on bending deflections and buckling loads is investigated numerically.

1 Mathematical Modeling

The annular plate with thickness h, inner radius a and outer radius b is defined in a cylindrical coordinate system(r,z) where the r-axis is on the mid-plane and the z-axis is parallel to the thickness direction, as shown in Fig.1. Notice that the annular plate turns to be solid circular plate for $a=0$.

The displacement field of circular plate based on sinusoidal shear deformation theory is assumed as[25]

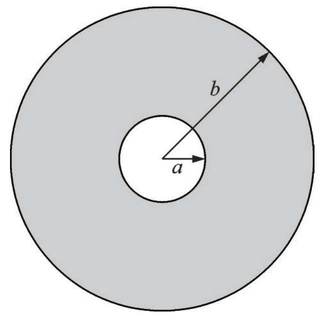


Fig.1 Schematic diagram of annular plate

$$[u_r,u_\theta,u_z]=[-zw'(r)+\sin(\beta z)\phi(r),0,w(r)] \tag{1}$$

where $\beta=\pi/h$ and "$'$" denotes the differentiation with respect to r.

According to the modified couple stress theory[7], the nonzero strain ε_{ij} and symmetric curvature components χ_{ij} can be expressed as

$$\begin{cases}\varepsilon_{rr}=-zw''+\sin(\beta z)\phi' \\ \varepsilon_{\theta\theta}=(-zw'+\sin(\beta z)\phi)/r \\ \gamma_{rz}=\beta\cos(\beta z)\phi \\ \chi_{r\theta}=[2w'-2rw''-\beta\cos(\beta z)(\phi-r\phi')]/4r\end{cases} \tag{2}$$

The virtual strain energy of the plate can be calculated as

$$\delta U=2\pi\Big\{[(rM_{rr})'-M_{\theta\theta}+M_{r\theta1}+(rM_{r\theta1})']\,\delta w\Big|_a^b+(rN_{r1}+rM_{r\theta2}/2)\,\delta\phi\Big|_a^b-r(M_{rr}+M_{r\theta1})\delta w'\Big|_a^b]+[(N_{\theta1}-(rN_{r1})'-((rM_{r\theta2})'+M_{r\theta2})/2+rQ_r)\delta\phi+(M'_{\theta\theta}-(rM_{rr})''-M'_{r\theta1}-(rM_{r\theta1})'')\delta w\,]\,\mathrm{d}r\Big\} \tag{3}$$

where

$$\begin{cases}[N_{r1},N_{\theta1}]=\int_{-h/2}^{h/2}[\sigma_{rr},\sigma_{\theta\theta}]\sin(\beta z)\,\mathrm{d}z \\ [Q_r,M_{r\theta2}]=\int_{-h/2}^{h/2}[\sigma_{rz},m_{r\theta}]\,\beta\cos(\beta z)\,\mathrm{d}z \\ [M_{rr},M_{\theta\theta},M_{r\theta1}]=\int_{-h/2}^{h/2}[z\sigma_{rr},z\sigma_{\theta\theta},m_{r\theta}]\,\mathrm{d}z\end{cases} \tag{4}$$

The virtual work done by the external axisymmetric loads is given by

$$\delta W=2\pi\Big\{rPw'\delta w\Big|_a^b+\int_a^b[rq-P(rw')']\,\delta w\mathrm{d}r\Big\} \tag{5}$$

where q and P are the distributed transverse load and inplane radial compressive force, respectively.

Based on the principle of minimum potential energy, the differential governing equations and boundary conditions can be expressed as

$$\begin{cases} N_{\theta 1}-(rN_{r1})'-[(rM_{r\theta 2})'+M_{r\theta 2}]/2+rQ_r=0 \\ M'_{\theta\theta}-(rM_{rr})''-M'_{r\theta 1}-(rM_{r\theta 1})''-rq+P(rw')'=0 \end{cases} \tag{6}$$

$$\begin{cases} (rN_{r1}+rM_{r\theta 2}/2)\,\delta\phi\big|_a^b=0 \\ [(rM_{rr})'-M_{\theta\theta}+M_{r\theta 1}+(rM_{r\theta 1})'-rPw']\,\delta w\big|_a^b=0 \\ [r(M_{rr}+M_{r\theta 1})]\,\delta w'\big|_a^b=0 \end{cases} \tag{7}$$

Based on the modified couple stress theory, the relation between general stress and strain components can be expressed as

$$\begin{cases} \sigma_{rr}=E(\varepsilon_{rr}+\nu\varepsilon_{\theta\theta})/(1-\nu^2) \\ \sigma_{\theta\theta}=E(\nu\varepsilon_{rr}+\varepsilon_{\theta\theta})/(1-\nu^2) \\ \sigma_{rz}=G\gamma_{rz} \\ m_{r\theta}=2Gl^2\chi_{r\theta} \end{cases} \tag{8}$$

where E, G and ν are Young's modulus, shear modulus, Poisson's ratio, respectively, and l is the material length scale parameter which describes the microstructural effect.

Combination of Eq.(4) with Eqs.(2) and (8) gives

$$\begin{cases} N_{r1}=-D_{z1}(w''+\nu w'/r)+D_{z2}(\phi'+\nu\phi/r) \\ N_{\theta 1}=-D_{z1}(\nu w''+w'/r)+D_{z2}(\nu\phi'+\phi/r) \\ Q_r=D_{z3}\phi \\ M_{rr}=-D(w''+\nu w'/r)+D_{z1}(\phi'+\nu\phi/r) \\ M_{\theta\theta}=-D(\nu w''+w'/r)+D_{z1}(\nu\phi'+\phi/r) \\ M_{r\theta 1}=D_{z4}(\phi'-\phi/r)/2+Ghl^2(w'/r-w'') \\ M_{r\theta 2}=D_{z5}(\phi'-\phi/r)/2+D_{z4}(w'/r-w'') \end{cases} \tag{9}$$

where

$$\begin{cases} D=Eh^3/[12(1-\nu^2)] \\ [D_{z1},D_{z2}]=\dfrac{E}{1-\nu^2}\displaystyle\int_{-h/2}^{h/2}[zf(z),f^2(z)]\,\mathrm{d}z \\ D_{z3}=G\displaystyle\int_{-h/2}^{h/2}(f'(z))^2\,\mathrm{d}z \\ D_{z4}=Gl^2\displaystyle\int_{-h/2}^{h/2}f'(z)\,\mathrm{d}z \\ D_{z5}=Gl^2\displaystyle\int_{-h/2}^{h/2}(f'(z))^2\,\mathrm{d}z \end{cases} \tag{10}$$

Taking into account Eq.(9), the differential governing equations and boundary conditions can be expressed as

$$\begin{cases} [(D_{z1}+D_{z4})(w'/r-w''-rw''')- \\ \quad (D_{z2}+D_{z5})(\phi/r-\phi'-r\phi'')-rD_{z3}\phi]=0 \\ [(D+Ghl^2)(w'/r^2-w''/r+2w'''+rw^{(4)})- \\ \quad (D_{z1}+D_{z4})(\phi/r^2-\phi'/r+2\phi''+r\phi''')]=rq \end{cases} \tag{11}$$

$$\begin{cases} [D_{z1}(rw''+\nu w')-D_{z2}(r\phi'+\nu\phi)-D_{z4}(w'- \\ \quad rw'')/2+D_{z5}(\phi-r\phi')/4]\,\delta\phi\big|_a^b=0 \\ [(D+Ghl^2)(w'/r-w''-rw''')- \\ \quad (D_{z1}+D_{z4}/2)(\phi/r-\phi'-r\phi'')]\,\delta w\big|_a^b=0 \\ [D(rw''+\nu w')-Ghl^2(w'-rw'')- \\ \quad D_{z1}(r\phi'+\nu\phi)+D_{z4}(\phi-r\phi')/2]\,\delta w'\big|_a^b=0 \end{cases} \tag{12}$$

2 Numerical Solution Based GDQM

In order to simplify the mathematical expression, the following nominal variables are introduced

$$\begin{cases} R=b-a \\ \alpha=a/R \\ \eta=(r-a)/R \\ W(\eta)=w(r)/R \\ \Phi(\eta)=\phi(r) \end{cases} \tag{13}$$

Therefore, the differential governing equations and boundary conditions can be expressed in nominal form as

$$\begin{cases} (D_{z1}+D_{z4}/2)[W'''+W''/\eta_\alpha-W'/\eta_\alpha^2]- \\ \quad (D_{z2}-D_{z5}/4)[\Phi''+\Phi'/\eta_\alpha-\Phi/\eta_\alpha^2]+ \\ \quad D_{z3}R^2\Phi=0 \\ (D+Ghl^2)[W^{(4)}+2W'''/\eta_\alpha-W''/\eta_\alpha^2+ \\ \quad W'/\eta_\alpha^3]-(D_{z1}+D_{z4}/2)[\Phi'''+\Phi''/\eta_\alpha- \\ \quad \Phi'/\eta_\alpha^2+\Phi/\eta_\alpha^3]-R^3q=0 \end{cases} \tag{14}$$

$$\begin{cases} [W''(D_{z1}+D_{z4}/2)+W'(\nu D_{z1}-D_{z4}/2)/\eta_\alpha- \\ \quad \Phi'(D_{z2}+D_{z5}/4)-\Phi(\nu D_{z2}- \\ \quad D_{z5}/4)/\eta_\alpha)]\,\delta\Phi\big|_0^1=0 \\ [(D+Ghl^2)(W'''+W''/\eta_\alpha-W'/\eta_\alpha^2)-(D_{z1}+ \\ \quad D_{z4}/2)(\Phi''+\Phi'/\eta_\alpha-\Phi/\eta_\alpha^2)]\,\delta W\big|_0^1=0 \\ [W''(D+Ghl^2)+W'(\nu D-Ghl^2)/\eta_\alpha- \\ \quad \Phi'(D_{z1}+D_{z4}/2)-\Phi(\nu D_{z1}- \\ \quad D_{z4}/2)/\eta_\alpha]\,\delta W'\big|_0^1=0 \end{cases} \tag{15}$$

where $\eta_\alpha=\alpha+\eta$.

Based on the basic procedure of the GDQM, the nominal radial coordinate is discretized by N nodes

$$\eta_i = [1 - \cos((i-1)\pi/(N-1))]/2 \tag{16}$$

where $i = 1, 2, \cdots, N$.

According to Bellman et al.[26], and Wu and Liu[27], the function Φ and W can be approximated as

$$\begin{cases} \Phi = \sum_{k=1}^{N} L_k(\eta)\delta_k^{\Phi} \\ W = \sum_{k=1}^{N+2} \psi_k(\eta)\delta_k^{W} \end{cases} \tag{17}$$

where L_k and ψ_k are Lagrange and Hermite interpolation basis functions which are defined explicitly in[26-27], and

$$\begin{cases} \boldsymbol{\delta}^{\Phi} = [\ \Phi(\eta_1) \quad \Phi(\eta_2) \quad \cdots \quad \Phi(\eta_N)\ ]^{\mathrm{T}} \\ \boldsymbol{\delta}^{W} = [\ W(\eta_1) \quad W(\eta_2) \quad \cdots \quad W(\eta_N) \\ \qquad W'(\eta_1) \quad W'(\eta_N)\ ]^{\mathrm{T}} \end{cases} \tag{18}$$

Performing derivative respect to η on Eq.(17), one obtains

$$\begin{cases} \Phi(\eta_k) = X_{km}^{(i)}\delta_m^{\Phi} \\ W^{(i)}(\eta_k) = Y_{kn}^{(i)}\delta_n^{W} \end{cases} \tag{19}$$

where $X^{(i)}$ and $Y^{(i)}$ are the weighting coefficients of the ith-order derivative and Einstein summation convention is adopted in this paper. m and n vary from 1 to N and $N+2$, respectively.

$$\begin{cases} (D_{z1} + D_{z4}/2)[W''' + W''/\eta_\alpha - W'/\eta_\alpha^2] - \\ \quad (D_{z2} - D_{z5}/4)[\Phi'' + \Phi'/\eta_\alpha - \\ \quad \Phi/\eta_\alpha^2] + D_{z3}R^2\Phi = 0 \\ (D + Ghl^2)[W^{(4)} + 2W'''/\eta_\alpha - W''/\eta_\alpha^2 + \\ \quad W'/\eta_\alpha^3] - (D_{z1} + D_{z4}/2)[\Phi''' + \Phi''/\eta_\alpha - \\ \quad \Phi'/\eta_\alpha^2 + \Phi/\eta_\alpha^3] - R^3 q = 0 \end{cases} \tag{20}$$

$$\begin{cases} [W''(D_{z1} + D_{z4}/2) + W'(\nu D_{z1} - D_{z4}/2)/\eta_\alpha - \\ \quad \Phi'(D_{z2} + D_{z5}/4) - \Phi(\nu D_{z2} - \\ \quad D_{z5}/4)/\eta_\alpha)]\ \delta\Phi\big|_0^1 = 0 \\ [(D + Ghl^2)(W''' + W''/\eta_\alpha - W'/\eta_\alpha^2) - \\ \quad (D_{z1} + D_{z4}/2)(\Phi'' + \Phi'/\eta_\alpha - \\ \quad \Phi/\eta_\alpha^2)]\ \delta W\big|_0^1 = 0 \\ [W''(D + Ghl^2) + W'(\nu D - Ghl^2)/\eta_\alpha - \\ \quad \Phi'(D_{z1} + D_{z4}/2) - \Phi(\nu D_{z1} - \\ \quad D_{z4}/2)/\eta_\alpha]\ \delta W'\big|_0^1 = 0 \end{cases} \tag{21}$$

The differential governing equations and boundary conditions can be expressed in discrete form, and there are $2N+2$ linear equations. One can express the discrete linear equation in matrix form as

$$[\boldsymbol{K} - \hat{P}\boldsymbol{n} - \hat{\omega}^2\boldsymbol{M}]\boldsymbol{d} = \boldsymbol{q} \tag{22}$$

where $\boldsymbol{K}$, $\boldsymbol{n}$, and $\boldsymbol{M}$ are stiffness, geometrical stiffness and mass matrices based on GDQM, respectively. $\boldsymbol{d} = [\ \boldsymbol{\delta}^{W} \quad \boldsymbol{\delta}^{\Phi}\ ]^{\mathrm{T}}$, $\boldsymbol{q}$ is the load vector.

For a circular plate ($a = \alpha = 0$), according to the L' Hospital' s rule, the boundary conditions (Eq.(15)) at $\eta = 0$ can be approximated as

$$\begin{cases} W' = 0 \\ (D + Ghl^2)W''' - (D_{z1} + D_{z4}/2)\Phi'' = 0 \\ D_{z1}W'' - D_{z2}\Phi' = 0 \end{cases} \tag{23}$$

Therefore, the boundary condition at $\eta = 0$ for solid circular plates can be expressed in discrete form as

$$\begin{cases} X_{1n}^{(1)}\delta_n^{W} = 0 \\ (D + \mu hl^2)]X_{1n}^{(3)}\delta_n^{W} - (D_{z1} + D_{z4}/2)Y_{1m}^{(2)}\delta_m^{\Phi} = 0 \\ X_{1n}^{(2)}D_{z1}\delta_n^{W} - D_{z2}Y_{1m}^{(1)}\delta_m^{\Phi} = 0 \end{cases} \tag{24}$$

3 Numerical Results and Discussion

In this section, the influence of the material length scale parameter and geometrical dimensions on the bending and buckling responses of circular plates is investigated numerically. The material parameters are adopted as following[28]: $E = 1.44$ GPa, $\nu = 0.38$, $l = 17.6$ μm.

Fig.2 illustrates the influence of R/h on nominal bending deflection (NBD) and nominal buckling load (NBL) of circular solid plates without microstructural effect under clamped and hinged boundary conditions, where CM indicates current model and

$$\begin{cases} \mathrm{NBD} = w(0)/[qR^4/(64D)] \\ \mathrm{NBL} = PR^2/D \end{cases} \tag{25}$$

Meanwhile, results based on Kirchhoff and Mindlin plate theories[29] are plotted for comparison. Fig.2 shows that, with the increase of R, bending deflection and buckling loads based on CM approach to those based on Kirchhoff plate theory. In addition, compared with bending deflection based on

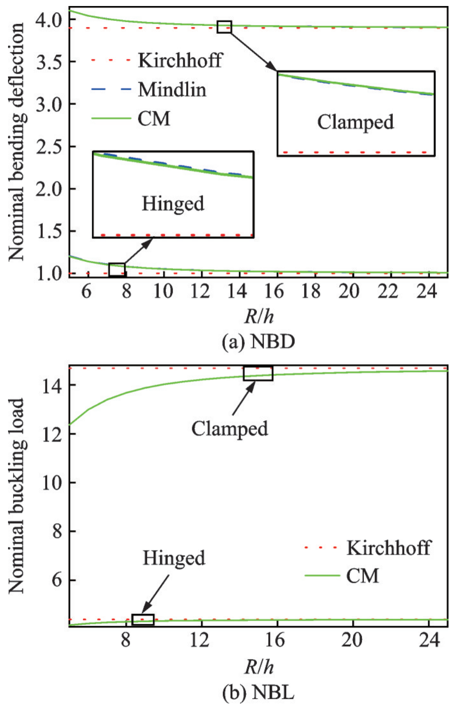


Fig.2 Validation of current model NBD and NBL of solid circular plates without microstructural effect

Mindlin plate theory, CM would provide higher and lower prediction for bending deflection of clamped and hinged plates, respectively.

Fig.3 illustrates the influence of h/l on normalized bending deflection and buckling load of circular solid microplates under clamped and hinged boundary conditions, in which, normalized bending deflection and buckling load are defined as the ratio between with and without microstructural effect. Meanwhile, solid and dash lines represent results for solid circular hinged and clamped microplates. It can be seen that bending deflections decrease and buckling loads increase with the decrease of h/l. In addition, the microstructural effect on clamped plates is larger than on hinged plates.

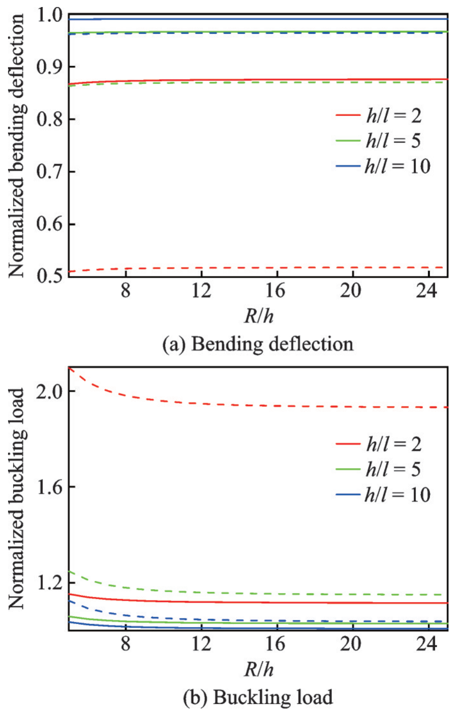


Fig.3 Microstructural effect on normalized bending deflection and buckling load of solid circular plates

Fig.4 illustrates the influence of buckling order on normalized buckling load of circular solid microplates under clamped and hinged boundary conditions, where solid, dash-dot and dash lines represent data for $h/l=2$, $h/l=5$ and $h/l=10$; 1—6 denote buckling order. It can be seen that the microstructural effect increases with the decrease of h/l and the increase of buckling order.

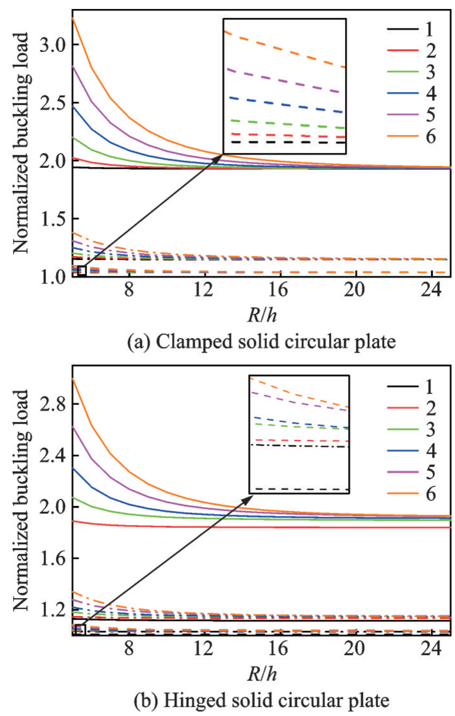
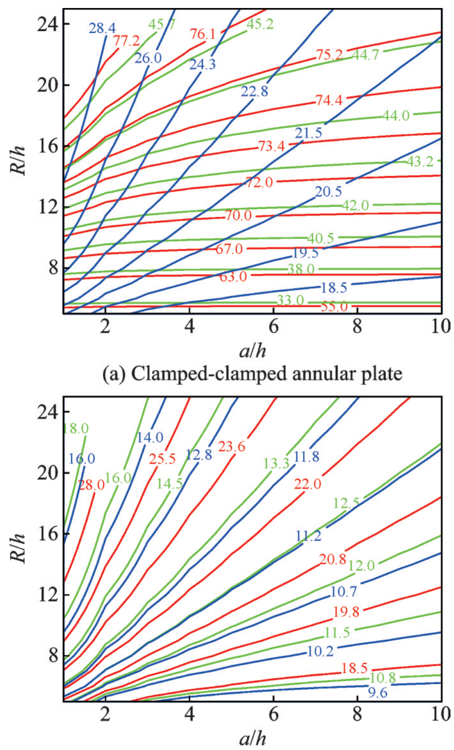


Fig.4 High-order normalized buckling loads of clamped and hinged solid circular plates

Fig.5 illustrates the influence of R/h and a/h on nominal buckling loads of annular microplates under clamped-clamped and hinged-hinged boundary conditions, in which, red, green and blue lines represent data for $h/l=2$, $h/l=5$ and $h/l=10$. It can be seen that NBLs increase consistently with the increase of R/h and decrease of a/h.



(a) Clamped-clamped annular plate

(b) Hinged-hinged annular plate

Fig.5 Nominal buckling loads of clamped-clamped and hinged-hinged annular plates

4 Conclusions

Static bending and elastic buckling of circular microplates are investigated on the basis of modified stress couple theory and sinusoidal shear deformation theory. The differential governing equations and boundary conditions are derived through the principle of minimum total potential energy. Several nominal variables are introduced to simplify the mathematical expression. L'Hospital's rule is applied to deal with the boundary conditions of plate center for circular microplates. The general differential quadrature method is applied to discretize the differential governing equations and the boundary conditions, and a set of linear equations are obtained. Validation is performed through comparing the results of the proposed model without microstructural effect with those of the methods based on classic Kirchhoff and Mindlin plate theories.

Based on numerical results of this study, one can obtain the following conclusions:

(1) For circular solid microplates, bending deflections increase and buckling loads decrease with the increase of h/l, and nominal bending deflections and buckling loads approach to constants with the increase of R/h.

(2) For annular microplates, nominal buckling loads increase with the increase of R/h and decrease of a/h.

References

[1] FOX C H J, CHEN X, MCWILLIAM S. Analysis of the deflection of a circular plate with an annular piezoelectric actuator[J]. Sensors and Actuators a-Physical, 2007, 133(1): 180-194.

[2] TYLIKOWSKI A. Influence of bonding layer on piezoelectric actuators of an axisymmetrical annular plate[J]. Journal of Theoretical and Applied Mechanics, 2000, 38: 607-621.

[3] DONOSO A, CARLOS BELLIDO J. Distributed piezoelectric modal sensors for circular plates[J]. Journal of Sound and Vibration, 2009, 319: 50-57.

[4] YANG L, LI P, FANG Y, et al. A generalized methodology for thermoelastic damping in axisymmetric vibration of circular plate resonators covered by multiple partial coatings[J]. Thin-Walled Structures, 2021, 162: 107576.

[5] MA C, CHEN S, GUO F. Thermoelastic damping in micromechanical circular plate resonators with radial pre-tension[J]. Journal of Thermal Stresses, 2020, 43: 175-190.

[6] KOITER W T. Couple stresses in the theory of elasticity, Ⅰ and Ⅱ[J]. Proceedings Series B, Koninklijke Nederlandse Akademie van Wetenschappen, 1964, 67: 17-44.

[7] YANG F, CHONG A C M, LAM D C C, et al. Couple stress based strain gradient theory for elasticity[J]. International Journal of Solids and Structures, 2002, 39: 2731-2743.

[8] MINDLIN R D. Second gradient of strain and surface-tension in linear elasticity[J]. International Journal of Solids and Structures, 1965, 1: 417-438.

[9] LAM D C C, YANG F, CHONG A C M, et al. Experiments and theory in strain gradient elasticity[J]. Journal of the Mechanics and Physics of Solids, 2003, 51: 1477-1508.

[10] GOUSIAS N, LAZOPOULOS A K. Axisymmetric bending of strain gradient elastic circular thin plates[J]. Archive of Applied Mechanics, 2015, 85: 1719-1731.

[11] JI X, LI A Q, ZHOU S J. A comparison of strain gra-

dient theories with applications to the functionally graded circular micro-plate[J]. Applied Mathematical Modelling, 2017, 49: 124-143.

[12] LI A Q, JI X, ZHOU S S, et al. Nonlinear axisymmetric bending analysis of strain gradient thin circular plate[J]. Applied Mathematical Modelling, 2021, 89: 363-380.

[13] ANSARI R, SHOJAEI M F, MOHAMMADI V, et al. Size-dependent thermal buckling and postbuckling of functionally graded annular microplates based on the modified strain gradient theory[J]. Journal of Thermal Stresses, 2014, 37: 174-201.

[14] ANSARI R, GHOLAMI R, SHOJAEI M F, et al. Bending, buckling and free vibration analysis of size-dependent functionally graded circular/annular microplates based on the modified strain gradient elasticity theory[J]. European Journal of Mechanics A/Solids, 2015, 49: 251-267.

[15] MOHAMMADIMEHR M, EMDADI M, NAVI B R. Dynamic stability analysis of microcomposite annular sandwich plate with carbon nanotube reinforced composite facesheets based on modified strain gradient theory[J]. Journal of Sandwich Structures & Materials, 2020, 22: 1199-1234.

[16] ZHANG B, HE Y M, LIU D B, et al. A size-dependent third-order shear deformable plate model incorporating strain gradient effects for mechanical analysis of functionally graded circular/annular microplates[J]. Composites Part B-Engineering, 2015, 79: 553-580.

[17] WANG Y G, LIN W H, LIU N. Large amplitude free vibration of size-dependent circular microplates based on the modified couple stress theory[J]. International Journal of Mechanical Sciences, 2013, 71: 51-57.

[18] KE L L, YANG J, KITIPORNCHAI S, et al. Bending, buckling and vibration of size-dependent functionally graded annular microplates[J]. Composite Structures, 2012, 94: 3250-3257.

[19] ARSHID E, AMIR S, LOGHMAN A. Bending and buckling behaviors of heterogeneous temperature-dependent micro annular/circular porous sandwich plates integrated by FGPEM nano-composite layers[J]. Journal of Sandwich Structures & Materials, 2021, 23: 3836-3877.

[20] REDDY J N, BERRY J. Nonlinear theories of axisymmetric bending of functionally graded circular plates with modified couple stress[J]. Composite Structures, 2012, 94: 3664-3668.

[21] REDDY J N, ROMANOFF J, ANTONIO LOYA J. Nonlinear finite element analysis of functionally graded circular plates with modified couple stress theory[J]. European Journal of Mechanics a-Solids, 2016, 56: 92-104.

[22] ZHOU S S, GAO X L. A nonclassical model for circular mindlin plates based on a modified couple stress theory[J]. Journal of Applied Mechanics—Transactions of the ASME, 2014, 81(5): 051014.

[23] ESHRAGHI I, DAG S, SOLTANI N. Bending and free vibrations of functionally graded annular and circular micro-plates under thermal loading[J]. Composite Structures, 2016, 137: 196-207.

[24] SADOUGHIFAR A, FARHATNIA F, IZADINIA M, et al. Nonlinear bending analysis of porous FG thick annular/circular nanoplate based on modified couple stress and two-variable shear deformation theory using GDQM[J]. Steel and Composite Structures, 2019, 33: 307-318.

[25] LI Y S, REN J H, FENG W J. Bending of sinusoidal functionally graded piezoelectric plate under an in-plane magnetic field[J]. Applied Mathematical Modelling, 2017, 47: 63-75.

[26] BELLMAN R, KASHEF B G, CASTI J. Differential quadrature: A technique for the rapid solution of nonlinear partial differential equations[J]. Journal of Computational Physics, 1972, 10: 40-52.

[27] WU T Y, LIU G R. The generalized differential quadrature rule for fourth-order differential equations[J]. International Journal for Numerical Methods in Engineering, 2001, 50: 1907-1929.

[28] MA H M, GAO X L, REDDY J N. A microstructure-dependent Timoshenko beam model based on a modified couple stress theory[J]. Journal of the Mechanics and Physics of Solids, 2008, 56: 3379-3391.

[29] REDDY J N. Energy principles and variational methods in applied mechanics[M]. 3rd ed. New York: Wiley, 2017.

Acknowledgements This work was supported in part by the National Natural Science Foundation of China(No. 12172169) and the Priority Academic Program Development of Jiangsu Higher Education Institutions.

Author Prof. **QING Hai** received the B.S. degree in engineering mechanics from Xi'an Jiaotong University in 2002 and Ph.D. degree in solid mechanics from Tsinghua University in 2007, respectively. From 2007 to 2011, he was a postdoctoral research fellow in Risø National Laboratory, Technical University of Denmark, Denmark. From 2011 to

2014, he was senior engineer of Siemens Wind Power in Denmark. Since 2014, He has been with Nanjing University of Aeronautics and Astronautics (NUAA), where he is a currently full professor. His research has focused on composite materials and structures, computational solid mechanics and microscale continuum mechanics.

Author contributions Prof. **QING Hai** designed the study, complied the models, interpreted the results and wrote the manuscript. Ms. **WEI Lu** conducted the analysis and contributed to the discussion and background of the study. All authors commented on the manuscript draft and approved the submission.

Competing interests The authors declare no competing inrests.

(Production Editor: XU Chengting)

基于修正偶应力理论的正弦剪切变形圆板的弯曲屈曲研究

卿 海，魏 璐

(南京航空航天大学机械结构力学及控制国家重点实验室，南京 210016，中国)

摘要：利用修正偶应力理论(Modified couple stress theory, MCST)和正弦剪切变形理论，分析了圆形微孔板的轴对称弯曲和屈曲行为。通过最小总势能原理推导得到微分形式的控制方程和边界条件，并用引入的标称变量表示。应用广义微分求积法，将微分形式的控制方程和边界条件离散化，得到一组线性方程。通过求解线性方程组获得弯曲挠度，而屈曲荷载可以通过求解一般特征值问题得到。在不同的边界条件下，研究材料长度尺度参数和圆板的几何尺寸对弯曲挠度和屈曲载荷的影响。

关键词：圆形微孔板；尺寸效应；修正偶应力理论；广义微分求积法