Multi-agent Coordinated Control and Collision Avoidance with Unknown Disturbances

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Abstract: The formation problem of multi-agent systems via coordinated control is investigated, where the multiple agents can achieve the common velocity with leader and avoid collision during the evolution. In the real-world situation, the communication is often disturbed and inaccurate. Hence, the unknown disturbances are considered in the velocity measurements, which is assumed to be bounded and does not need to be modelled. Moreover, a complicated nonlinear interaction among agents is presented in the design of control. Based on the existing work of multi-agent systems, a flocking control protocol is proposed to address the formation problem in the dynamic topology. The stability analysis is given to prove that the velocities of all agents can converge to the velocity of leader and the stable motion with collision avoidance can be achieved eventually. Finally, some simulations are presented to verify the effectiveness of the proposed algorithm.

Key words: multi-agent systems; flocking; formation; unknown disturbances

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0 Introduction

The intelligent transportation[1-4] represents the main development direction of future transportation and has important influence in the fields of security, environment, military, etc. For the research of intelligent transportation, vehicle formation is a vital topic in recent years, where the automated vehicles can generate a collaborative group through perception and decision making.

With the development of computer technology and communication technology, a vehicle can be abstracted as an agent, and the vehicle formation problem can be addressed through the coordinated control of multi-agent systems[5-7]. Among the results of coordinated control, flocking control[8] is a distributed algorithm to achieve the consensus of velocity and avoid collision in the system. It is essentially the same as the vehicle formation problem, that is, to control the acceleration of the individual. There are three specific rules for flocking motion[9]: (1) Avoiding collision with other agents; (2) staying close with neighbor agents; (3) achieving common velocity with neighbor agents. Hence, it can be applied to the vehicle formation problem and solve practical issues.

Flocking control needs to consider the position restrictions between agents, which is not just a consensus problem. For the fixed and switching networks, Tanner et al.[10-11] constructed a new piecewise potential function, and its gradient can be acted as the repulsive force or attractive one to satisfy the above rules. A smooth potential function was proposed to guarantee the continuity of energy function by Olfati-Saber.[12], where the virtual leader was introduced to achieve the stable flocking. When the initial network was connected, the infinite potential functions were presented to ensure the connec-

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tivity during the evolution[13]. Moreover, the work was extended to the situation where only a fraction of agents have the information of virtual leader[14]. The authors utilized a dynamic strategy to study the connectivity-preserving flocking algorithm[15].

Most existing work with double-integrator dynamics needs to rely on the common assumption that the velocity is linear. However, in real applications, the feedback information may be uncertainty due to inaccurate measurements or unknown disturbances. Thus, a more reasonable assumption should be considered. For instance, the nonlinear dynamics were investigated in the work of Refs.[16-18], where the adaptive algorithms were proposed to achieve the convergence in the stability analysis. The flocking control protocol with nonlinear velocity measurements was studied[19-20]. Moreover, neural networks also can be a method to solve such nonlinear problems. The radial basis function neural network (RBFNN) was used to approximate the unknown continuous function[21-22].

For the study of nonlinear systems[16-18,23], the Lipschitz condition and Lipschitz-like condition are the general assumptions to address the nonlinear problems. However, it is not suitable to be used for unknown disturbances, where the uncertainty only can be known to belong to some given sets. Bauso et al.[24] investigated the consensus problems of single-integrator with unknown but bounded (UBB) disturbances. The ε-consensus of the position difference and the consensus of velocity difference can be achieved under the proposed control protocol. The consensus of double-integrator dynamics with UBB disturbances was considered[25]. Compared with the velocity consensus, the position of flocking is easier to achieve. Moreover, the flocking control was proposed to only rely on the velocity[26]. Hence, this paper prioritizes the disturbances in velocity.

Inspired by the work of Refs.[24-25], this paper generalizes the flocking formation of double-integrator multi-agent systems with UBB disturbances. Furthermore, most previous work only showed the disturbances in the situation where the velocity interaction between agents was simple. It is worth discussing that how to design the flocking control protocol when there are disturbances in the complicated velocity functions. Cao and Ren[27] constructed a new adjacent velocity function with the use of symbolic function, which could enhance the communication between agents.

Motivated by the methods in Refs.[27-28], a novel flocking protocol with UBB disturbances is proposed in this paper. Compared with the existing results, the main contributions of this work are as follows: (1) The disturbances in the velocity measurements between agents have a more general assumption, which is unknown but bounded; (2) the disturbances are considered in the complicated function between the velocity interaction; (3) the Barbalat lemma[29] is used to construct the stability analysis, which does not need to get the state of the minimum total energy.

The remainder of this paper is organized as follows. Some preliminaries are given in Section 1. In Section 2, the idea of the control algorithm is presented. The main results and theoretical analysis are discussed in Section 3. Numerical experiments shown in Section 4 demonstrate the effectiveness of the proposed control. Finally, Section 5 concludes this work.

1 Preliminaries

In this paper, the multi-agent system is considered with n agents moving in a m-dimensional Euclidean space. The communication topology among agents can be represented by an undirected graph $G(t)=\{v, e(t)\}$, where the set of vertices $v=\{1, 2, \cdots, n\}$ represents each individual agent, and a time-varying set of edges $e(t)=\{(i,j)\in v\times v\}$ represents neighboring relations among agents at time t. The agent i and the agent j have the communication link when $(i,j)\in e(t)$. The system is said to have dynamic topological structure if the neighboring set of any agents changes dynamically with time, otherwise the system has a fixed topology. This paper mainly considers the dynamic topological structure. A path is generated by an alternating sequence of distinct vertices and edges in the graph. The undirected graph is called connected if and only if there is a path between any pair of distinct nodes.

Let the neighbor set of the ith agent at time t is

$$N_i(t)=\{j:\|q_i-q_j\|<r, j=1,2,\cdots,n, j\neq i\} \quad (1)$$

where r is the sensing radius of the agent and q_i the position of agent i. Assume that all agents keep the same sensing radius $r>0$. Hence, the set of edges can be generated by $e(t)=\{(i,j): j\in N_i(t), i,j\in v\}$. The adjacent matrix $A(G)=(a_{ij})_{n\times n}$ associated with graph $G(t)$ is defined as $a_{ij}=1$ if $(i,j)\in e(t)$, and $a_{ij}=0$ otherwise. Define the Laplacian matrix as $\boldsymbol{L}(G)=\boldsymbol{D}(G)-\boldsymbol{A}(G)$, where $\boldsymbol{D}(G)$ represents the degree matrix of the graph and its ith diagonal element is the degree of the node i. It is clear that $L(G)$ is a symmetric and positive semi-definite matrix.

Lemma 1 (Barbalat lemma) Let f: R→R be a uniformly continuous function on the interval $[0,\infty)$. Suppose that $\lim_{t\to\infty}\int_0^t f(\tau)\mathrm{d}\tau$ exists and is finite, then $\lim_{t\to\infty} f(t)=0$.

2 Problem Formulation

The dynamic equation for each agent is described by

$$\begin{cases}\dot{\boldsymbol{q}}_i=\boldsymbol{p}_i \\ \dot{\boldsymbol{p}}_i=\boldsymbol{u}_i\end{cases} \quad i=1,2,\cdots,n \quad (2)$$

where $\boldsymbol{q}_i$, $\boldsymbol{p}_i$, $\boldsymbol{u}_i\in \mathrm{R}^m$ are the position, velocity and control input vectors of agent i, respectively. The purpose is to design control input to make the system approach to a stable motion while achieving the consensus of the velocities and avoiding collision. Generally, the control law $\boldsymbol{u}_i$ for agent i takes the form

$$\boldsymbol{u}_i=\boldsymbol{f}_i^{\alpha}+\boldsymbol{f}_i^{\beta}+\boldsymbol{f}_i^{\gamma} \quad i=1,2,\cdots,n \quad (3)$$

where $\boldsymbol{f}_i^{\alpha}$ is the gradient-based term which enforces each agent to converge to the desired position and avoid collision, $\boldsymbol{f}_i^{\beta}$ regulates the convergence of velocities among its neighbor and $\boldsymbol{f}_i^{\gamma}$ is a navigational feedback term to drive agents to track the virtual leader. The virtual leader for multi-agent system (2) is some certain purpose described by

$$\dot{\boldsymbol{q}}_\gamma=\boldsymbol{p}_\gamma \quad \dot{\boldsymbol{p}}_\gamma=f_\gamma(\boldsymbol{q}_\gamma,\boldsymbol{p}_\gamma) \quad (4)$$

where $\boldsymbol{q}_\gamma, f_\gamma, \boldsymbol{p}_\gamma\in \mathrm{R}^m$ are the position, acceleration, and velocity of the virtual leader, respectively. For simplicity, denote $f_\gamma(\boldsymbol{q}_\gamma,\boldsymbol{p}_\gamma)=\boldsymbol{0}$, which has no effect on the results of this paper.

Based on the ideal conditions, Olfati-Saber et al.[12] designed the control input (3) as follows

$$\boldsymbol{u}_i=-\sum_{j\in N_i(t)}\nabla_{q_i}\psi_\alpha(\|\boldsymbol{q}_i-\boldsymbol{q}_j\|_\sigma)-\sum_{j\in N_i(t)}a_{ij}(t)(\boldsymbol{p}_i-\boldsymbol{p}_j)-c_1(\boldsymbol{q}_i-\boldsymbol{q}_\gamma)-c_2(\boldsymbol{p}_i-\boldsymbol{p}_\gamma) \quad (5)$$

where c_1, c_2 are positive and the σ-norm of a vector is a map as

$$\|\boldsymbol{z}\|_\sigma=\frac{1}{\varepsilon}\left[\sqrt{1+\varepsilon\|\boldsymbol{z}\|^2}-1\right] \quad (6)$$

with a parameter $\varepsilon>0$. It is noted that the norm $\|\cdot\|_\sigma$ is differentiable everywhere.

The artificial potential function $\psi_\alpha(\cdot)$ used here is defined as

$$\begin{cases}\psi_\alpha(z)=\int_{d_\sigma}^{z}\phi_\alpha(s)\mathrm{d}s \\ \phi_\alpha(z)=\rho_h(z/r_\sigma)\phi(z-d_\sigma) \\ \phi(z)=\frac{1}{2}[(a+b)\sigma_1(z+c)+(a-b)]\end{cases} \quad (7)$$

where $\sigma_1(z)=z/\sqrt{1+z^2}$, $0<a\leqslant b$, $c=|a-b|/\sqrt{4ab}$, d is the desired distance of agents, and $\rho_h(\cdot)$ the bump function with $h\in(0,1)$.

$$\rho_h(z)=\begin{cases}1 & z\in[0,h) \\ 0.5\left[1+\cos\frac{\pi(z-h)}{1-h}\right] & z\in[h,1] \\ 0 & \text{Otherwise}\end{cases} \quad (8)$$

The above classic algorithm can achieve the stable flocking. However, in real applications, the actual control may be affected by other factors. For example, the available velocity measurements of other agents are not as accurate as the measurements of its own velocity. Due to the accuracy of the sensor or the complexity of information transmission, the measurements of velocity may have disturbances, and the transmission of speed information may become complicated. Hence, it is a challenging task to study the influence of the disturbances in a complicated velocity form.

3 Main Results

It is assumed that each agent can obtain its own velocity measurements accurately, and the neighbors' states may be disturbed. Denote the velocity measurements of agent k measured by agent i as

$$\boldsymbol{p}_{ik}=\boldsymbol{p}_k-\boldsymbol{d}_{ik} \tag{9}$$

where $\boldsymbol{p}_k$ is the true velocity of agent k and $\boldsymbol{d}_{ik}$ represents the unknown disturbances and satisfies the following assumption.

Assumption 1 The unknown disturbances $\boldsymbol{d}_{ik}$ are bounded, whose component satisfies $-\xi\leqslant \boldsymbol{d}_{ik}^{(m)}\leqslant \xi$, for $\xi>0$.

For the system (2), the control input with UBB disturbances is governed by

$$\begin{aligned}\boldsymbol{u}_i=&-\sum_{j\in N_i(t)}\nabla_{q_i}\psi_\alpha(\|\boldsymbol{q}_i-\boldsymbol{q}_j\|_\sigma)-c_1(\boldsymbol{q}_i-\boldsymbol{q}_\gamma)-c_2(\boldsymbol{p}_i-\boldsymbol{p}_\gamma)-\\&\sum_{j\in N_i(t)}a_{ij}\{D_{|N_i|\xi}[\sum_{k\in N_i(t)}a_{ik}(\boldsymbol{p}_i-\underbrace{\boldsymbol{p}_k+\boldsymbol{d}_{ik}}_{\boldsymbol{p}_{ik}})]-\\&D_{|N_j|\xi}[\sum_{k\in N_j(t)}a_{jk}(\boldsymbol{p}_j-\underbrace{\boldsymbol{p}_k+\boldsymbol{d}_{jk}}_{\boldsymbol{p}_{jk}})]\}\end{aligned} \tag{10}$$

where $|N_i|$ is the total number of the elements in the neighbor set N_i and $D_{|N_i|\xi}$: $\mathbf{R}^m\rightarrow\mathbf{R}^m$ a dead-zone function proposed by Bauso et al[24]., which is defined as

$$D_{|N_i|\xi}(\boldsymbol{x}+\boldsymbol{d})=\begin{cases}\boldsymbol{x}^{(m)}+\boldsymbol{d}^{(m)}-|N_i|\xi & \boldsymbol{x}^{(m)}+\boldsymbol{d}^{(m)}>|N_i|\xi\\0 & |\boldsymbol{x}^{(m)}+\boldsymbol{d}^{(m)}|\leqslant|N_i|\xi\\\boldsymbol{x}^{(m)}+\boldsymbol{d}^{(m)}+|N_i|\xi & \boldsymbol{x}^{(m)}+\boldsymbol{d}^{(m)}<-|N_i|\xi\end{cases} \tag{11}$$

where $\boldsymbol{x}^{(m)}$, $\boldsymbol{d}^{(m)}$ are the components of $\boldsymbol{x}$ and $\boldsymbol{d}$. Obviously, it can be concluded that

$$\boldsymbol{x}^{\mathrm{T}}D_{|N_i|\xi}(\boldsymbol{x}+\boldsymbol{d})\geqslant 0 \tag{12}$$

Remark 1 In control protocol (10), the disturbances and the irregular velocity interaction are both considered. Compared with the existing results, the proposed control is more general.

Remark 2 The construction of control protocol (10) uses the artificial potential function (7), which can guarantee the continuity of control.

Define the total energy of the system as follows

$$Q(\boldsymbol{q},\boldsymbol{p})=\frac{1}{2}\sum_{i=1}^{n}[U_i(\boldsymbol{q})+(\boldsymbol{p}_i-\boldsymbol{p}_\gamma)^{\mathrm{T}}(\boldsymbol{p}_i-\boldsymbol{p}_\gamma)] \tag{13}$$

where

$\boldsymbol{q}=[\boldsymbol{q}_1^{\mathrm{T}},\boldsymbol{q}_2^{\mathrm{T}},\cdots,\boldsymbol{q}_n^{\mathrm{T}}]^{\mathrm{T}}\equiv\mathrm{col}(\boldsymbol{q}_1,\boldsymbol{q}_2,\cdots,\boldsymbol{q}_n)\in\mathbf{R}^{nm}$,

$\boldsymbol{p}=\mathrm{col}(\boldsymbol{p}_1,\boldsymbol{p}_2,\cdots,\boldsymbol{p}_n)$ and

$$U_i(\boldsymbol{q})=\sum_{j=1,j\neq i}^{n}\psi_\alpha(\|\boldsymbol{q}_{ij}\|_\sigma)+c_1(\boldsymbol{q}_i-\boldsymbol{q}_\gamma)^{\mathrm{T}}(\boldsymbol{q}_i-\boldsymbol{q}_\gamma) \tag{14}$$

In the energy function, $U_i(\boldsymbol{q})$ represents the potential energy of the system, $(\boldsymbol{p}_i-\boldsymbol{p}_\gamma)^{\mathrm{T}}(\boldsymbol{p}_i-\boldsymbol{p}_\gamma)$ is the kinetic energy of the system. Clearly, Q is a positive semi-definite function.

Theorem 1 Consider a system of n agents with dynamics (2) steered by the control protocol (10). Under Assumption 1, if the initial energy $Q_0:=Q(t_0)$ is finite such that $Q(t_0)<\infty$, then it can be inferred that

(i) All agents will converge to the velocity of the virtual leader asymptotically.

(ii) The system asymptotically approaches to a configuration that is the local minimum of the artificial potentials energy.

(iii) If the initial energy is smaller than $\psi_\alpha(0)$, there is no collision happening between the mobile agents during the evolution.

The proof of part (i)

Denote the position and velocity errors between each agent and the virtual leader as $\tilde{\boldsymbol{q}}_i=\boldsymbol{q}_i-\boldsymbol{q}_\gamma$, $\tilde{\boldsymbol{p}}_i=\boldsymbol{p}_i-\boldsymbol{p}_\gamma$ and $\tilde{\boldsymbol{q}}_{ij}=\tilde{\boldsymbol{q}}_i-\tilde{\boldsymbol{q}}_j$, then one has

$$\dot{\tilde{\boldsymbol{q}}}_i=\tilde{\boldsymbol{p}}_i\quad \dot{\tilde{\boldsymbol{p}}}_i=\dot{\boldsymbol{p}}_i=\boldsymbol{u}_i \tag{15}$$

The control protocol and the energy can be rewritten as

$$\begin{aligned}\boldsymbol{u}_i=&-\sum_{j\in N_i(t)}\nabla_{\tilde{q}_i}\psi_\alpha(\|\tilde{\boldsymbol{q}}_{ij}\|_\sigma)-c_1\tilde{\boldsymbol{q}}_i-c_2\tilde{\boldsymbol{p}}_i-\\&a_{ij}\left\{D_{|N_i|\xi}\left[\sum_{k\in N_i(t)}a_{ik}(\tilde{\boldsymbol{p}}_i-\tilde{\boldsymbol{p}}_k+\boldsymbol{d}_{ik})\right]-\right.\\&\left.D_{|N_j|\xi}\left[\sum_{k\in N_j(t)}a_{jk}(\tilde{\boldsymbol{p}}_j-\tilde{\boldsymbol{p}}_k+\boldsymbol{d}_{jk})\right]\right\}\end{aligned} \tag{16}$$

and

$$Q(\tilde{\boldsymbol{q}},\tilde{\boldsymbol{p}})=\frac{1}{2}\sum_{i=1}^{n}[U_i(\tilde{\boldsymbol{q}})+\tilde{\boldsymbol{p}}_i^{\mathrm{T}}\tilde{\boldsymbol{p}}_i] \tag{17}$$

where

$$U_i(\tilde{\boldsymbol{q}})=\sum_{j=1,j\neq i}^{n}\psi_\alpha(\|\tilde{\boldsymbol{q}}_{ij}\|_\sigma)+c_1\tilde{\boldsymbol{q}}_i^{\mathrm{T}}\tilde{\boldsymbol{q}}_i \tag{18}$$

By the symmetry of potential function $\psi_\alpha(\|\tilde{\boldsymbol{q}}_{ij}\|_\sigma)$ and the adjacent matrix $\boldsymbol{A}$, the following property holds

$$\nabla_{\tilde{q}_i}\psi_\alpha(\|\tilde{\boldsymbol{q}}_{ij}\|_\sigma)=-\nabla_{\tilde{q}_j}\psi_\alpha(\|\tilde{\boldsymbol{q}}_{ij}\|_\sigma)=\nabla_{\tilde{q}_{ij}}\psi_\alpha(\|\tilde{\boldsymbol{q}}_{ij}\|_\sigma) \tag{19}$$

Take the time derivative of Q along the dynamics of system (15)

$$\dot{Q}(\tilde{\boldsymbol{q}},\tilde{\boldsymbol{p}})=\sum_{i=1}^n\tilde{\boldsymbol{p}}_i^{\mathrm{T}}\boldsymbol{u}_i+\frac{1}{2}\sum_{i=1}^n\sum_{j=1,j\neq i}^n\dot{\psi}_\alpha(\|\tilde{\boldsymbol{q}}_{ij}\|_\sigma)+\sum_{i=1}^n c_1\tilde{\boldsymbol{p}}_i^{\mathrm{T}}\tilde{\boldsymbol{q}}_i \tag{20}$$

Due to the property (19), the time derivative of potential function is

$$\begin{aligned}\sum_{i=1}^n\sum_{j=1}^n\dot{\psi}_\alpha(\|\tilde{\boldsymbol{q}}_{ij}\|_\sigma)=&\sum_{i=1}^n\sum_{j=1}^n\dot{\tilde{\boldsymbol{q}}}_i^{\mathrm{T}}\nabla_{\tilde{q}_{ij}}\psi_\alpha(\|\tilde{\boldsymbol{q}}_{ij}\|_\sigma)-\sum_{j=1}^n\sum_{i=1}^n\dot{\tilde{\boldsymbol{q}}}_j^{\mathrm{T}}\nabla_{\tilde{q}_{ij}}\psi_\alpha(\|\tilde{\boldsymbol{q}}_{ij}\|_\sigma)=\\&\sum_{i=1}^n\sum_{j=1}^n\dot{\tilde{\boldsymbol{q}}}_i^{\mathrm{T}}\nabla_{\tilde{q}_i}\psi_\alpha(\|\tilde{\boldsymbol{q}}_{ij}\|_\sigma)+\sum_{j=1}^n\sum_{i=1}^n\dot{\tilde{\boldsymbol{q}}}_j^{\mathrm{T}}\nabla_{\tilde{q}_j}\psi_\alpha(\|\tilde{\boldsymbol{q}}_{ij}\|_\sigma)=\\&2\sum_{i=1}^n\sum_{j=1}^n\tilde{\boldsymbol{p}}_i^{\mathrm{T}}\nabla_{\tilde{q}_i}\psi_\alpha(\|\tilde{\boldsymbol{q}}_{ij}\|_\sigma)\end{aligned} \tag{21}$$

Bring Eqs. (16) and (21) into Eq. (20), we have

$$\begin{aligned}\dot{Q}=&\sum_{i=1}^n\left[\tilde{\boldsymbol{p}}_i^{\mathrm{T}}\sum_{j\in N_i(t)}^n\nabla_{\tilde{q}_i}\psi_\alpha(\|\tilde{\boldsymbol{q}}_{ij}\|_\sigma)+c_1\tilde{\boldsymbol{p}}_i^{\mathrm{T}}\tilde{\boldsymbol{q}}_i+\tilde{\boldsymbol{p}}_i^{\mathrm{T}}u_i\right]=\\&\sum_{i=1}^n\left[\tilde{\boldsymbol{p}}_i^{\mathrm{T}}\sum_{j\in N_i(t)}^n\nabla_{\tilde{q}_i}\psi_\alpha(\|\tilde{\boldsymbol{q}}_{ij}\|)+c_1\tilde{\boldsymbol{p}}_i^{\mathrm{T}}\tilde{\boldsymbol{q}}_i\right]+\\&\sum_{i=1}^n\left[-\tilde{\boldsymbol{p}}_i^{\mathrm{T}}\sum_{j\in N_i(t)}\nabla_{\tilde{q}_i}\psi_\alpha(\|\tilde{\boldsymbol{q}}_{ij}\|_\sigma)-c_1\tilde{\boldsymbol{p}}_i^{\mathrm{T}}\tilde{q}_i-c_2\tilde{\boldsymbol{p}}_i^{\mathrm{T}}\tilde{\boldsymbol{p}}_i\right]-\\&\sum_{i=1}^n\tilde{\boldsymbol{p}}_i^{\mathrm{T}}\sum_{j\in N_i(t)}a_{ij}\left\{D_{|N_i|\xi}\left[\sum_{k\in N_i(t)}a_{ik}(\tilde{\boldsymbol{p}}_i-\tilde{\boldsymbol{p}}_k+\boldsymbol{d}_{ik})\right]-\right.\\&\left.D_{|N_j|\xi}\left[\sum_{k\in N_j(t)}a_{jk}(\tilde{\boldsymbol{p}}_j-\tilde{\boldsymbol{p}}_k+\boldsymbol{d}_{jk})\right]\right\}=\\&-\sum_{i=1}^n\tilde{\boldsymbol{p}}_i^{\mathrm{T}}\sum_{j\in N_i(t)}a_{ij}\left\{D_{|N_i|\xi}\left[\sum_{k\in N_i(t)}a_{ik}(\tilde{\boldsymbol{p}}_i-\tilde{\boldsymbol{p}}_k+\boldsymbol{d}_{ik})\right]-\right.\\&\left.D_{|N_j|\xi}\left[\sum_{k\in N_j(t)}a_{jk}(\tilde{\boldsymbol{p}}_j-\tilde{\boldsymbol{p}}_k+\boldsymbol{d}_{jk})\right]\right\}-\sum_{i=1}^n c_2\tilde{\boldsymbol{p}}_i^{\mathrm{T}}\tilde{\boldsymbol{p}}_i\\&=-\tilde{\boldsymbol{p}}^{\mathrm{T}}L(t)D\left[L(t)\tilde{\boldsymbol{p}}+\boldsymbol{d}\right]-\sum_{i=1}^n c_2\tilde{\boldsymbol{p}}_i^{\mathrm{T}}\tilde{\boldsymbol{p}}_i\end{aligned} \tag{22}$$

where $D(z)=\mathrm{col}(D_{|N_1|\xi}(z_1), D_{|N_2|\xi}(z_2),\cdots, D_{|N_n|\xi}(z_n))$, $d=\mathrm{col}(d_1,d_2,\cdots,d_n)$ and $d_i=\sum_{j\in N_i(t)}d_{ij}$.

From Assumption 1, such inequality holds

$$-|N_i|\xi<d_i<|N_i|\xi \tag{23}$$

Hence, it can be concluded combined with Eq.(12) that

$$\dot{Q}(t)\leqslant 0\qquad\forall t>0 \tag{24}$$

which indicates

$$Q(t)\leqslant Q(0)\qquad\forall t>0 \tag{25}$$

Next, we will prove the convergence of the velocity errors between agents and virtual leader. Define $\phi(t)=\tilde{\boldsymbol{p}}^{\mathrm{T}}\tilde{\boldsymbol{p}}$, and it is obvious that $\dot{\phi}(t)=2\tilde{\boldsymbol{p}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{p}}}$. It follows from Eqs.(17), (18) and (25) that $\tilde{\boldsymbol{q}}(t)$ and $\tilde{\boldsymbol{p}}(t)$ are bounded. Combined with the definition of potential function, it can be seen that $\dot{\phi}(t)$ is bounded.

Denote $\{t_k\}_0^\infty$ as any time sequence with $t_{k+1}>t_k$ for all k satisfying $t_0=0$ and $\lim_{k\to\infty}t_k=\infty$, one has

$$\lim_{t\to\infty}(Q(t)-Q(0))=\int_0^\infty\dot{Q}(t)\mathrm{d}t=\sum_{k=0}^\infty\int_{t_k}^{t_{k+1}}\dot{Q}(\tau)\mathrm{d}\tau \tag{26}$$

Define $\Delta(k,r)=\int_k^{k+r}\dot{Q}(\tau)\mathrm{d}\tau$.

It is clearly $\Delta(k,r)\leqslant 0$ due to $\dot{Q}\leqslant 0$. Noticeably, the right of Eq.(26) is bounded, it must follow that

$$\lim_{k\to\infty}\Delta(k,r)=0\qquad r\geqslant 1 \tag{27}$$

According to the process of Eq.(22), for any $r\geqslant 1$, it implies

$$\Delta(k,r)=\int_k^{k+r}\dot{Q}(\tau)\mathrm{d}\tau\leqslant -c_2\int_k^{k+r}\sum_{i=1}^n\tilde{\boldsymbol{p}}_i^{\mathrm{T}}(\tau)\tilde{\boldsymbol{p}}_i(\tau)\mathrm{d}\tau \tag{28}$$

Therefore

$$0\leqslant\int_k^{k+r}\phi(\tau)\mathrm{d}\tau\leqslant-\frac{1}{c_2}\Delta(k,r) \tag{29}$$

which indicates that

$$\lim_{k\to\infty}\int_k^{k+r}\phi(\tau)\mathrm{d}\tau=0\quad r\geqslant 1 \tag{30}$$

Since

$$\int_0^\infty\phi(\tau)\mathrm{d}\tau=\sum_{k=0}^\infty\int_k^{k+1}\phi(\tau)\mathrm{d}\tau \tag{31}$$

$\lim_{t\to\infty}\int_0^t\phi(\tau)\mathrm{d}\tau$ exists and is bounded. By the Lemma

1, it has $\lim_{t\to\infty}\tilde{\boldsymbol{p}}^{\mathrm{T}}\tilde{\boldsymbol{p}}=0$, which implies

$$\tilde{\boldsymbol{p}}_1=\tilde{\boldsymbol{p}}_2=\cdots=\tilde{\boldsymbol{p}}_n=\boldsymbol{0} \tag{32}$$

and then

$$\boldsymbol{p}_1=\boldsymbol{p}_2=\cdots=\boldsymbol{p}_n=\boldsymbol{p}_\gamma \tag{33}$$

The proof of part(i) is completed.

The proof of part (ii)

The proof of part (i) shows that $\dot{\boldsymbol{p}}_1=\dot{\boldsymbol{p}}_2=\cdots=\dot{\boldsymbol{p}}_n=\dot{\boldsymbol{p}}_\gamma=\boldsymbol{0}$, which indicates that $\boldsymbol{u}_i=\boldsymbol{0}$. It follows from Eqs.(10,14) that

$$\nabla_{q_i}\left[\sum_{i=1}^{n}U_i\right]=0 \tag{34}$$

which indicates the configuration converges asymptotically to a fixed configuration that is an extreme of all agent global potentials. This completes the proof of part (ii).

The proof of part (iii)

Assume any agents b and s collide at time t_c, there is $\boldsymbol{q}_b(t_c)=\boldsymbol{q}_s(t_c)$. We can infer that

$$V(\boldsymbol{q})=\frac{1}{2}\psi_\alpha\left(\left\|\boldsymbol{q}_{ij}\right\|_\sigma\right)=\psi_\alpha\left(\left\|\boldsymbol{q}_b-\boldsymbol{q}_s\right\|_\sigma\right)+\frac{1}{2}\psi_\alpha\left(\left\|\boldsymbol{q}_{ij}\right\|_\sigma\right)>\psi_\alpha(0) \tag{35}$$

Based on the proof of part(i) it can be easily known that

$$Q(0)\geqslant Q(t)\geqslant V(t)>\psi_\alpha(0) \tag{36}$$

which contradicts the assumption in theorem. Thus, no collision happens during the evolution. The proof of theorem is completed.

Remark 3 If f_γ is a known state, the control input can take effect directly by adding the known state in control input $\boldsymbol{u}_i+f_\gamma$. For the unknown state f_γ, the control problem is transformed into the systems with unknown nonlinear dynamics $\dot{\boldsymbol{q}}_i=\boldsymbol{p}_i$, $\dot{\boldsymbol{p}}_i=\boldsymbol{u}_i+f(\boldsymbol{p}_i)$, where $f(\cdot)$ satisfies the Lipschitz condition (e.g., Lorenz system, Chuas circuit and the Chen system). It follows from the results (22) that the last term of Eq.(22) will become $-(c_2-l)\tilde{\boldsymbol{p}}_i^{\mathrm{T}}\tilde{\boldsymbol{p}}_i$ with Lipschitz coefficient l. By selecting a large enough control coefficient c_2, the same convergence results still can be obtained. Moreover, for a more general condition of nonlinear dynamics such as bounded condition, the convergence results also can be obtained through adding a sign function term in the control protocol. Hence, we simplify $f_\gamma=\boldsymbol{0}$ for a clear presentation.

4 Simulation Verification

The simulation is based on MATLAB simulation environment. Under the control protocol (10), the example of 10 agents moving in the two-dimensional space is presented in this section. The initial configuration is shown in Fig.1(a), where the positions and the velocities of agents are randomly generated from $[0,12]\times[0,12]$ and $[0,1]\times[0,1]$, respectively. The solid lines denote the links if there is interaction between two agents, and the arrow of each agent represents its own velocity vector. Set the sensing radius of the agent to $r=4$ and the desired distance between adjacent agent to $d=3.3$. The corresponding parameters in control protocol (10) are $c_1=1, c_2=3, \varepsilon=0.01$, $a=5, b=8$ and $h=0.9$.

The disturbances of velocity measurement in the neighbor agents satisfy Assumption 1 with $\xi=1$. As a global information for the formation of agents, the initial position and velocity of the virtual leader are set as $\boldsymbol{q}_\gamma(0)=[8,8]^{\mathrm{T}}, \boldsymbol{p}_\gamma(0)=[0.5,0.5]^{\mathrm{T}}$.

With the increase of time, Fig.1(b) records the configuration of the multi-agent systems. It can be seen that all agents can move with the same velocity as the virtual leader and each agent can maintain the desired distance with its neighboring agents. The states of agents indicates that the stable motion can be achieved with the control protocol (10). Then, we will present the detailed data to further prove the effectiveness of the proposed control protocol.

The changes of agent velocities are shown in Fig.2. To observe the velocity relationships between the agents and the virtual leader, the differences of the relative velocity between agents and the virtual leader on x and y axes are depicted, respectively. Different line types represent different agents. The curves in Fig.2 can converge to the zero, meaning that the control protocol (10) can overcome the unknown disturbances and drive all agents

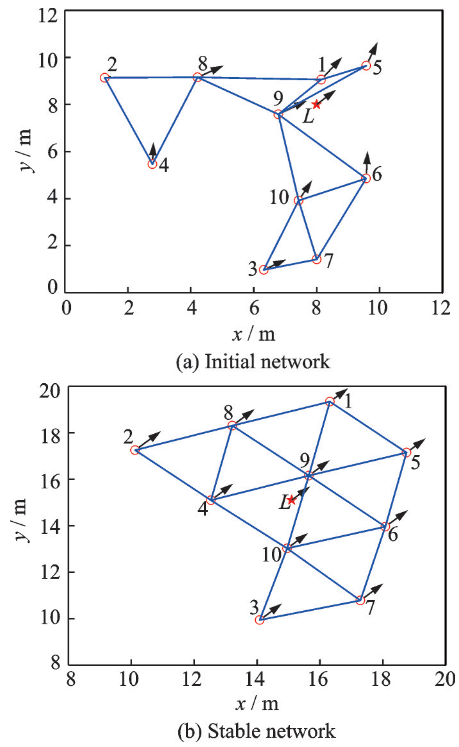

Fig.1 States of agents in the initial network and stable network

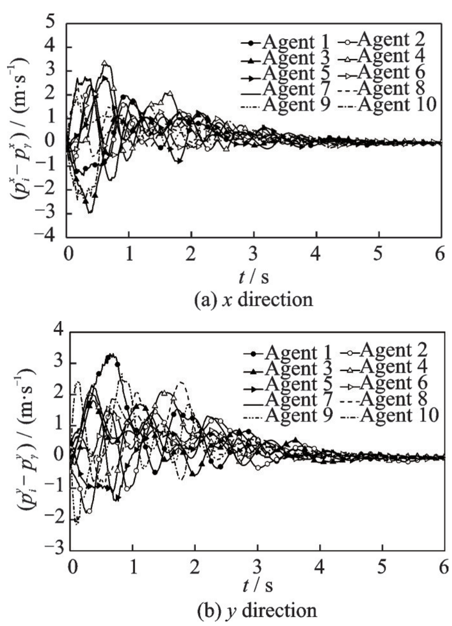

Fig.2 Velocity difference between agents and virtual leader

to keep the common velocity with the virtual leader. The velocity requirement in the flocking formation of multi-agent systems can be satisfied.

Figs.3(a, b) depict the position differences between agents and the virtual leader on the x and y axes, respectively. The results of the position information show that the initial irregular motion can gradually evolved into a stable formation, where the differences between the agents and the virtual leader can keep a certain distance and tend to be constant eventually, meaning the formation of the multi-agent systems will move with the guidance of the virtual leader. Moreover, the results in Fig.3 also indicate that there is no collision in the motion because the position of each agent is separated. It seems that the curves of agents 2 and 5 overlap on y axis. In Fig.3(a), the curves of agents 2 and 5 on x axis do not overlap and maintain a certain distance. Hence, the actual positions of agents 2 and 5 in the motion cannot collide. The cohesion, alignment and collision avoidance of flocking are proved.

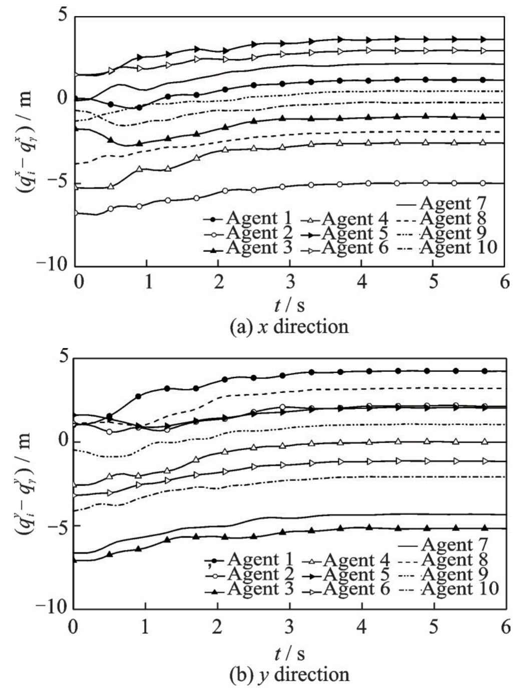

Fig.3 Position difference between agents and virtual leader

The trajectories of the multi-agent systems are depicted in Fig.4(a). A formation evolution process is illustrated clearly, where the velocities of the agents can get consensus with the velocity of the virtual leader, the position of the agents can keep a certain distance and avoid collision, and all agents will track to the virtual leader.

Furthermore, a comparison of convergence with existing method in Ref.[15] is presented in Fig.4(b). The solid curve represents the proposed method can take effect under disturbances, whereas the dash dot curve of Gao's method generates the oscillations due to the influence of disturbances. Therefore, the results in simulations further certify the correctness of the stability analysis and the effectiveness of the proposed control protocol.

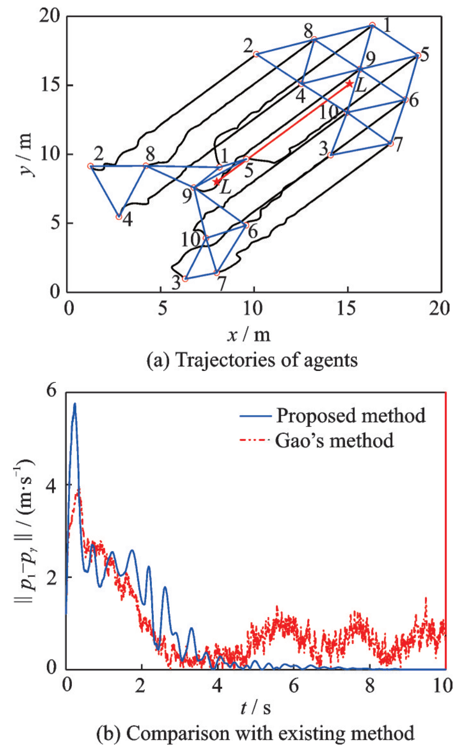

Fig.4 Trajectories of systems and comparison with existing method

5 Conclusions

This paper has investigated the formation of multi-agent with the flocking control of multi-agent system. The proposed control method for the stable motion concerns on the disturbances of velocity measurements and complicated interaction of velocities between agents. The stability analysis of proposed method utilizes the symmetry of undirected graph. When it is extended to the directed graph, it is necessary to eliminate the influence caused by the asymmetry, e.g., the balance directed graph with equal in-degree and out-degree can provide a feasible direction to achieve the convergence.

Our main contribution is as follows: (1) The assumptions of disturbance and the use of interaction function have been expanded and are more suitable for the real applications, resulting in the better robustness of the control process; (2) the disturbances in complicated interaction make the design of the control difficult; (3) the stability analysis verifies the convergence of the algorithm, where the velocities of all agents can converge to the consensus and the position of agents satisfy the minimum potential energy while avoiding collision. Some simulations are presented to illustrate the effectiveness of the theoretical results. The proposed method provides theoretical support in addressing the above issues through a basic model of agent dynamics. For better application in practice, the results will be extended to the more general systems corresponding to the practical systems in our future research work, such as rotation motion and position information with disturbances.

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未知干扰下具有避免碰撞机制的多智能体协同控制

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摘要:研究了基于协同控制的多智能体系统编队问题,其中,智能体在运动过程中可以实现与领导者速度的一致性以及避免碰撞。在实际中,通信经常受到干扰并且往往是不准确的,因此需要考虑在速度测量中的未知干扰。本文只考虑了未知干扰项的有界性,并不用将其模型化。此外,在设计控制时,本文考虑了一种智能体间复杂的非线性交互方式。基于多智能体系统现有的工作,提出了一种蜂拥控制方法来解决动态拓扑中的编队问题。在稳定性分析中证明了所有智能体的速度都可以收敛到领导者的速度,并且可以实现避免碰撞的稳定运动。最后,给出一些模拟结果来验证所提出算法的有效性。

关键词:多智能体系统;蜂拥;编队;未知干扰