# Forced Resonance of a Slightly-Curved Hydraulic Pipe Fixed at Two Ends

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**Abstract:** The ideally straight hydraulic pipe is inexistent in reality. The initial curve caused by the manufacturing or the creep deformation during the service life will change the dynamic character of the system. The current work discusses the effect of the initial curve on the hydraulic pipe fixed at two ends for the first time. Based on the governing equation obtained via the generalized Hamilton's principle, the potential energy changing with the height of the initial curve is discussed. The initial curve makes the potential energy curve asymmetric, but the system is always monostable. The initial curve also has very important influence on natural frequencies. It hardens the stiffness of the first natural mode at first and then has no effect on this mode after a critical value. On the contrast, the second natural frequency is constant before the critical value but increases while the height of the initial curve exceeds the critical value. On account of the initial value, the quadratic nonlinearity appears in the system. Forced resonance is very different from that of the ideally straight pipe under the same condition. Although the 2∶1 internal resonance is established by adjusting the height of the initial curve and the fluid speed, the typical double-jumping phenomenon does not occur under the initial curve given in the current work. This is very different from the straight pipe in the supercritical region. The work here claims that the initial curve of the hydraulic pipe should be taken into consideration. Besides, more arduous work is needed to reveal the dynamic characters of it.

**Key words:** forced resonance; hydraulic pipe; slightly-curved pipe; harmonic balance method

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## 0 Introduction

The hydraulic system is an important part for machine, engine, airplane and many engineering machineries. It usually contains lots of pipes, which convey the hydraulic oil with high pressure and high speed. Under excitations and pulsating pressure or speed, the piping system may yield drastic vibration, which will damage the system. Hence, many scholars paid their attention to this field, including pipes in other fields[1-2].

For those curved pipes designed on purpose, the dynamic character is discussed based on the initial curve intrinsically. The early model of the semi-circular arc pipe conveying fluid was proposed by Chen[3]. He found that the curved pipe showed instability similarly to a straight pipe when the fluid velocity exceeded a certain critical value based on an assumption that the centerline of the curved pipe was inextensible. However, it is controversial. The stability was not to be lost while one treated the centerline extensible. Consequently, Chung and Jung proposed a more accurate model for the semi-circular fluid-conveying pipe[4]. They discussed the natural character of this kind of pipes. Ni and his coworkers developed this model to the microscale region[5]. They also discussed the dynamic response of the semi-circular arc pipes conveying fluid via the simulation[6]. In 2017, Zhao et al. discussed the dynamic response of this kind of pipe by the analytical method[7]. These investigations indicate that the dynamic character of the curved pipe is complex. For straight

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pipes, the model is based on that the static equilibrium configuration is always straight. However, the manufacturing error and the creep deformation during the service life will make this assumption out of operation. The straight static equilibrium configuration becomes slightly curved. Consequently, the slightly curved hydraulic pipe must be discussed.

Sinir investigated a slightly curved pipe with the SIN function shape in 2010[8]. He claimed that the initial curve would increase the critical flow velocities and the natural frequencies of the system, and decrease the amplitude of the resultant motions. Latterly, Wang et al. also found that the critical flow velocity at which buckling instability occurred was higher than that for a pipe without imperfections[9]. In 2017, Li et al. discussed the forced resonance of a slightly curved pipe[10]. With the amplitude of initial imperfection increasing, the hardening behavior of the system was transformed into the soft behavior. Recently, the model of slightly curved pipe has been developed to the non-planar type[11] and the supercritical region[12]. All these investigations assumed that the pipe was simply supported at two ends. But for a real hydraulic pipe, the supports at two ends are more close to the fixed restriction.

The softening phenomenon also can be found for slightly curved beams. Oz et al. produced contributory work on this field[13]. They discussed the influence of different initial curve on the nonlinear response[14] and found that the 2∶1 internal resonance was possible for the case of parabolic curvature but not for that of sinusoidal curvature for a simply supported beam[15]. Huang and Chen discussed the bifurcation of the simply-supported slightly-curved beam via the incremental harmonic balance method[16-17]. The response was more complex than the beam without the initial curve as the system contained the quadratic nonlinearity and the cubic nonlinearity at the same time. Consequently, the vibration of the slightly curved beam must be controlled[17]. The boundary also has very important influence to the slightly curved beam[18]. These investigations also focused on the simply supported structure.

The current work tries to discuss the natural character and the resonance of the hydraulic pipe fixed at two ends. The 2∶1 internal resonance condition is given to test that whether the 2∶1 internal resonance would happen under the given initial curve proposed in the current work. The harmonic balance method together with the pseudo-arc-length method is used to solve the complex steady state response.

## 1 Mechanical Modeling and Natural Frequencies

Fig.1 shows a section of hydraulic pipe of a fighter. The boundary of the pipe is simplified as the fixed constraint. The distance between the two supports is $L_p$ while the external and the inner diameters of the pipe are $D$ and $d$, respectively. The density of the pipe is $\rho_p$ and the Young's modulus is $E$. In the pipe, there is full of aviation hydraulic oil. To simplify the theoretical model, one can consider the hydraulic oil as incompressible Newtonian fluid without viscosity. With this assumption, the oil pressure is the same everywhere in the pipe. Therefore, the effect of the pressure of the hydraulic oil can be neglected. The hydraulic oil has the density of $\rho_f$ and moves with the velocity $\Gamma$. Values of these parameters are given in Table 1.

In ideal conditions, the pipe will be straight during the service life. But considering the manufacturing defect and the creep deformation, the pipe

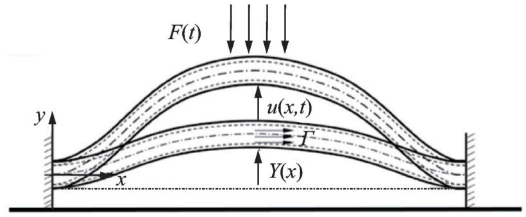



Fig.1 Schematic diagram of hydraulic pipe fixed at two ends

**Table 1 Physical parameters**

| Parameter | Notation | Value |
|---|---|---|
| Outer diameter of pipe / mm | $D$ | 6 |
| Inner diameter of pipe / mm | $d$ | 4.8 |
| Moment of inertia of the cross section / $m^4$ | $I_p$ | $3.756\times10^{-11}$ |
| Young's modulus of pipe / GPa | $E$ | 194 |
| Density of fluid / ($kg \cdot m^{-3}$) | $\rho_f$ | 872 |
| Density of pipe / ($kg \cdot m^{-3}$) | $\rho_p$ | 7 930 |
| Length of pipe / m | $L_p$ | 1 |

may have an initial curve. According to Ref.[10], the initial curve affects the dynamic characters of the system a lot. This will also be very important for the hydraulic pipe as it plays a very important role in the equipment. The initial curve is marked as $Y(x)$. Based on it, the governing equation of the hydraulic pipe will be established by the generalized Hamilton's principle.

The bending deformation of the pipe is $u(x,t)$. Thus, the kinetic energy of the pipe is

$$T_p = \frac{1}{2}\int_0^{L_p} \rho_p A_p u_{,t}^2 \mathrm{d}x \tag{1}$$

where $u$ is the lateral displacement along the pipe and $A_p$ the cross section of the pipe. The kinetic energy of the hydraulic oil along the pipe is

$$T_f = \frac{1}{2}\int_0^{L_p} \rho_f A_f (\Gamma^2 + u_{,t}^2)\mathrm{d}x \tag{2}$$

where $A_f$ denotes the cross section of the fluid. According to the Hamilton's principle, the potential energy needs to be taken into consideration. Thus, the potential energy of the pipe is

$$\delta U_p = \iiint_V E\varepsilon_x \delta\varepsilon_x \mathrm{d}V \tag{3}$$

where

$$\varepsilon_x = \sqrt{1 + (u+Y)_{,x}^2} - \sqrt{1 + Y_{,x}^2} \tag{4}$$

and it is the strain of a micro-body along the $x$ coordinate. Considering the virtual work of the uniformly distributed harmonic force along the pipe, the governing equation of the hydraulic pipe will be obtained as

$$\begin{aligned}&(\rho_p A_p + \rho_f A_f) u_{,tt} + 2\rho_f A_f \Gamma u_{,xt} + \rho_f A_f \Gamma^2 u_{,xx} + \\ &EI_p u_{,xxxx} - F_0 \cos(\Omega t) - \\ &\frac{EA_p}{2L_p}(Y_{,xx} + u_{,xx})\int_0^{L_p}(u_{,x}^2 + 2u_{,x}Y_{,x})\mathrm{d}x = 0\end{aligned} \tag{5}$$

with the boundary condition

$$\begin{cases} u(0,t) = 0, & u(L_p,t) = 0 \\ u_{,x}(0,t) = 0, & u_{,x}(L_p,t) = 0 \end{cases} \tag{6}$$

where $F_0$ is the amplitude of the excitation force and $\Omega$ the excitation frequency.

The Kelvin's material derivative is used in the current work to describe the viscoelasticity of the pipe, which means the following equation needs to be substituted into Eq.(5).

$$E = E + \mu \frac{\mathrm{d}}{\mathrm{d}t} \tag{7}$$

where $\mu$ is the visco-elastic coefficient of the pipe.

At last, the governing equation is

$$\begin{aligned}&(\rho_p A_p + \rho_f A_f) u_{,tt} + 2\rho_f A_f \Gamma u_{,xt} + \rho_f A_f \Gamma^2 u_{,xx} + \\ &EI_p u_{,xxxx} + \mu I_p u_{,xxxxt} - \\ &\frac{EA_p}{2L_p}(Y_{,xx} + u_{,xx})\int_0^{L_p}(u_{,x}^2 + 2u_{,x}Y_{,x})\mathrm{d}x - \\ &\frac{\mu A_p}{2L_p} u_{,xxt}\int_0^{L_p}(u_{,x}^2 + 2u_{,x}Y_{,x})\mathrm{d}x - \\ &\frac{\mu A_p}{L_p}(u_{,xx} + Y_{,xx})\int_0^{L_p}(u_{,x}u_{,xt} + u_{,xt}Y_{,x})\mathrm{d}x - \\ &F_0\cos(\Omega t) = 0\end{aligned} \tag{8}$$

Based on Eq.(8), the natural characters and dynamic characters can be discussed. Besides, one can find from it that the linear restoring force is affected by the initial curve. But the initial curve does not affect the linear damping of the system. Both of the nonlinear restoring force and the nonlinear damping are changed by the initial curve. Different with the bistable supercritical pipes conveying fluid[19], the slightly curved pipe here is a monostable system, although the governing equation here seems like that of the pipes conveying fluid. To validate this conclusion, the potential energy of the slightly curved pipe will be discussed. Before that, the initial curve should be defined. Considering the fixed boundary, one can use the following function to describe the initial curve

$$Y(x) = \frac{1}{2}A_0\left[1 - \cos\left(\frac{2\pi x}{L_p}\right)\right] \tag{9}$$

The shape of it is given in Fig.2. One can observe from it that the displacement and the rotation angle at two ends are zero, which satisfies the boundary condition Eq.(6). Consequently, the potential energy can be demonstrated changing with the initial curve in Fig.3. While $A_0$ is zero, the potential energy curve is symmetric about the 0 position. The symmetry disappears as $A_0$ is increasing. But the potential energy always has just one well.

By eliminating the excitation, the nonlinearity and the viscoelasticity in the governing equation, the natural frequencies can be obtained from the derived linear system. As a gyroscopic system, natural frequencies and resonance will be discussed

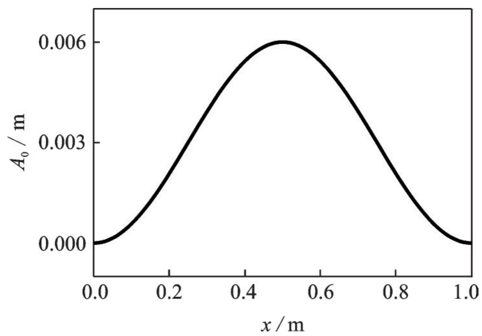



Fig.2 Initial curve of the slightly curved pipe

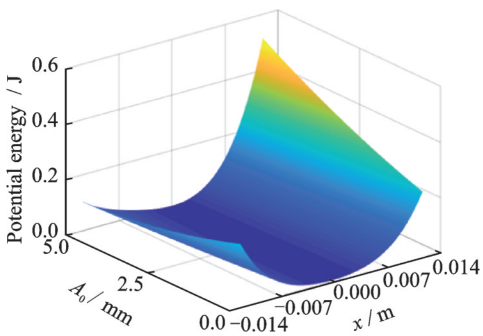



Fig.3 Potential energy changing with initial curve

based on natural modes of a static beam with fixed boundaries[20-21]. This means natural modes are discussed via

$$\phi'''' = \beta^4 \phi \tag{10}$$

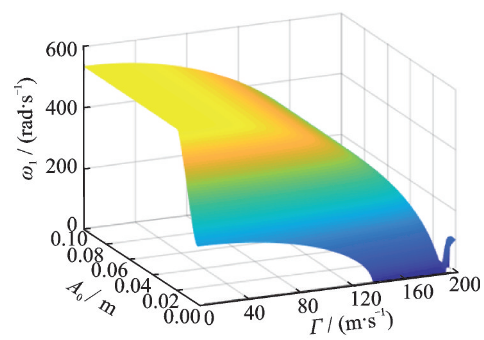
with the boundary value given below

$$\begin{cases} \phi(0)=0, & \phi(L_{\mathrm{p}})=0 \\ \phi_{,x}(0)=0, & \phi_{,x}(L_{\mathrm{p}})=0 \end{cases} \tag{11}$$

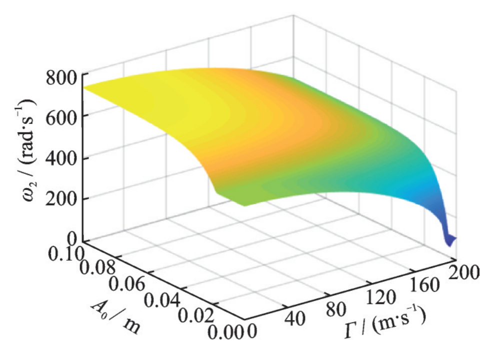
where $\phi$ is the natural mode and $\beta$ the modal eigenvalue. Things different from Refs.[20-21] are that the hydraulic pipe in the current work is slightly curved. But this is not a problem for solving the governing equation here as Refs.[20-21] have successfully used the series expansion to get the natural frequency and the resonance. Hence, the solution of the slightly-curved hydraulic pipe is still based on the modal expansion. The key for the accuracy of the solution is the convergence of the expansion. By comparing natural frequencies based on different expansions, an expansion with four modes can obtain convergent results for the first two natural frequencies.

As can be observed from Figs.4(a, b), natural frequencies $\omega_1$, $\omega_2$ are decreasing with the increase of the fluid speed. This is a recognized conclusion of the pipe conveying fluid. Meanwhile, the first natural frequency $\omega_1$ will increase with the height of initial curve at first. But this tendency disappears while the height exceeds a critical value. After that, the first natural frequency $\omega_2$ keeps the same. On the contrast, the second natural frequency is constant at first. After the critical height, it will increase with the height of the initial condition. Figs.4(c, d) explain this phenomenon more clearly in the 2D view. These two subpictures demonstrate the first two natural frequencies under the fluid speeds 0 m/s and 50 m/s, respectively. They all indicate the effect of the initial curve on the first two natural frequencies as described earlier. Besides, the first two natural frequencies have a point of intersection while the fluid speed is 0 m/s. But this intersection point disappears once the fluid flows.

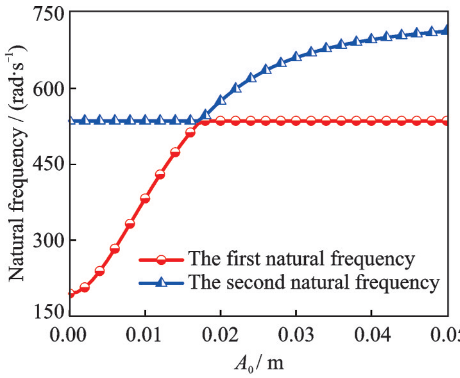
Figs.4(e, f) describe the first two natural frequencies changing with the initial curve and without the initial curve. With the given initial curve, the first natural frequency increases dramatically. But the second one changes little while the speed is not too high. The initial curve also changes the critical speed, which makes the first natural frequency be zero, to a high value. This indicates that the initial curve must be considered as the influence of it on the natural characters of the hydraulic system is considerable.



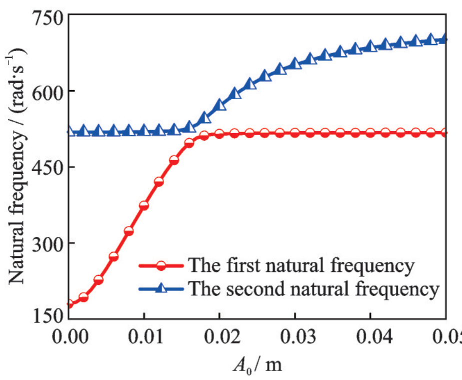

(a) The first natural frequency changing with initial curve and the fluid speed



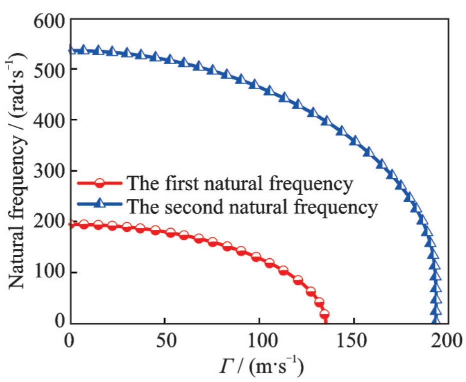

(b) The second natural frequency changing with initial curve and the fluid speed



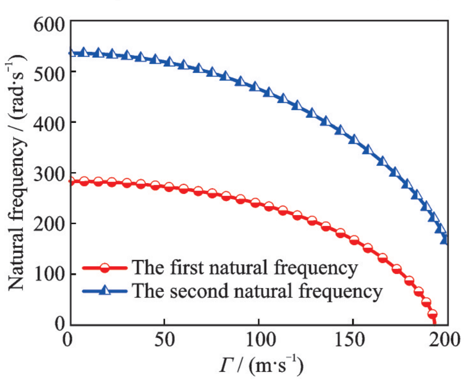

(c) The first two natural frequencies changing with the height of the initial curve while the fluid speed is 0 m/s




(d) The first two natural frequencies changing with the height of the initial curve while the fluid speed is 50 m/s




(e) The first two natural frequencies changing with the fluid speed without the initial curve while $A_0$ is 0 m




(f) The first two natural frequencies changing with the fluid speed with the initial curve while $A_0$ is 0.006 m

Fig.4 The first two natural frequencies changing with different parameters

As mentioned in Ref.[15], the 2∶1 internal resonance occurs under a special initial curve. In the current work, the first two natural frequencies are set to satisfy this internal resonance condition to validate that whether the 2∶1 internal resonance could happen with the shape of Eq.(9). The height of the initial curve in the current is set to 6 mm, which is just 0.6% of the length of the pipe. While the hydraulic oil flows at the speed 121.5 m/s, the 2∶1 internal resonance condition is established. Under this condition, the first natural frequency is 214.57 rad/s and the second one is 429.14 rad/s.

## 2 Harmonic Balance Method

The section above introduces the governing equation of the hydraulic pipe based on the initial curve, including the influence of it on the natural frequencies. In this section, the influence of the initial curve on the dynamic response will be discussed. Considering the nonlinearity may be strong, the harmonic balance method $\varphi$ is used in the current work. The solution of the system is expressed as

$$u(x,t)=\sum_{i=1}^{4}q_i(t)\phi_i(x) \tag{12}$$

where $\phi$ is the mode of the static beam with the fixed restraint. The partial differential governing equation Eq.(8) will be projected to the modal space via the Galerkin method. In this way, four coupled ordinary differential equations about variates $q_i$ ($i=1, 2, 3, 4$) will be produced. According to HBM, they are expanded as

$$q_i=a_{i,0}+\sum_{j=1}^{m}\left[a_{i,j}\cos\left(\mathrm{j}\Omega t\right)+b_{i,j}\sin\left(\mathrm{j}\Omega t\right)\right]$$
$$i=1,2,3,4 \tag{13}$$

Substituting Eq.(10) into the governing equation, making use of Eq.(11), there comes corresponding functions of harmonics. Collecting coefficients of different harmonics, one can obtain the steady state response functions. The response changing with the excitation frequency can be solved out from them.

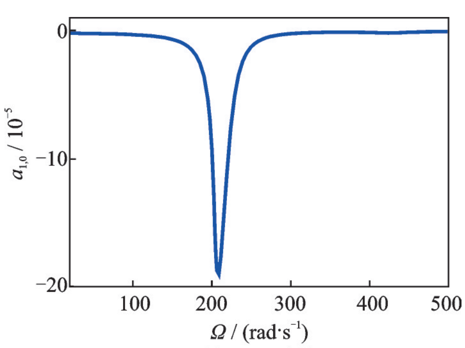
As the governing equation contains the cubic nonlinearity, $m$ in Eq.(11) will be set as 3 during the calculation. The strong nonlinearity may trigger complex resonance, which introduces jumping phenomenon to the response curve. The multiplicity of solution makes the coupled function difficult to be solved out. To overcome this trap, the Newton's method together with the pseudo-arc-length method is proposed. The detailed introduction of it can be found in Ref.[22].

All the natural modes of the static beam under the same restriction, the natural frequencies of the hydraulic pipe, the Galerkin truncation and the solution of the steady state response are carried out by the software MAPLE. The computational accuracy is set as $10^{-16}$. The judging criteria of the convergence of the solution is $\sum\left|\mathrm{Err}_i\left(y_c\right)\right| \leqslant 10^{-12}$, where $\mathrm{Err}_i$ is the scrap value of each coefficient function. At this step, the resonance of the hydraulic pipe with an initial curve can be obtained by substituting all parameters.

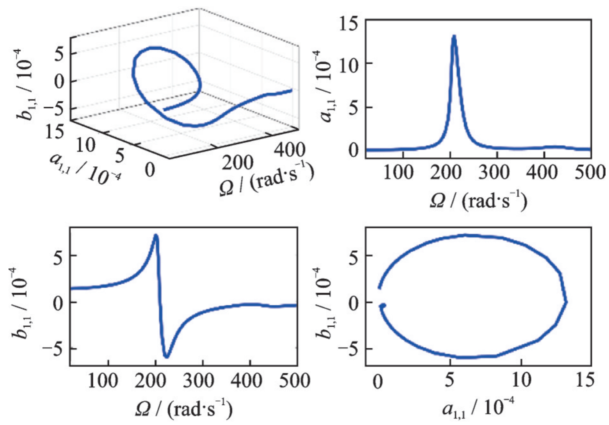
## 3 Numeric Examples and Discussion

The viscoelastic coefficient $\mu$ is set to $5\times 10^7$ N·s/m$^2$ while the excitation force is 0.5 N/m. Substituting these parameters into the steady state response functions, coefficients of harmonics are solved out. Reponses of each harmonic on the first two modes are demonstrated in Figs.5 and 6, respectively. On account of the quadratic nonlinearity, the zero-drift coefficient is nonzero. It is a component of the harmonic solution with zero-frequency. Besides, it arouses the response on the second harmonic.

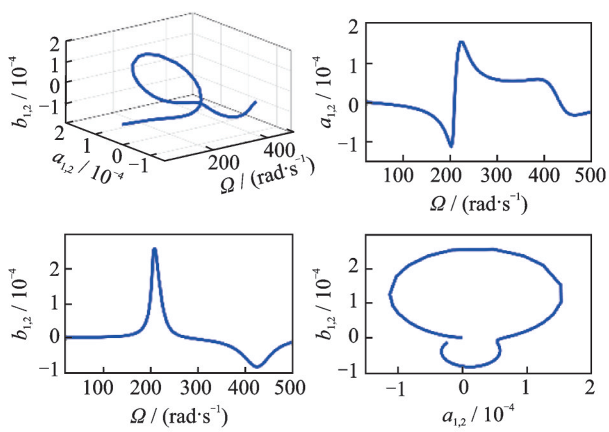
For the harmonics on the first mode, the 2nd order harmonic has the same magnitude with the 1st order one. The vibrating energy is transmitted to the quadratic nonlinearity. It is strange that, the response of the 3rd harmonic has a resonance peak at the second order of natural frequency. On the contrast, the responses of the zero-drift, the 1st harmonic and the 2nd harmonic are tiny in the second natural frequency region.



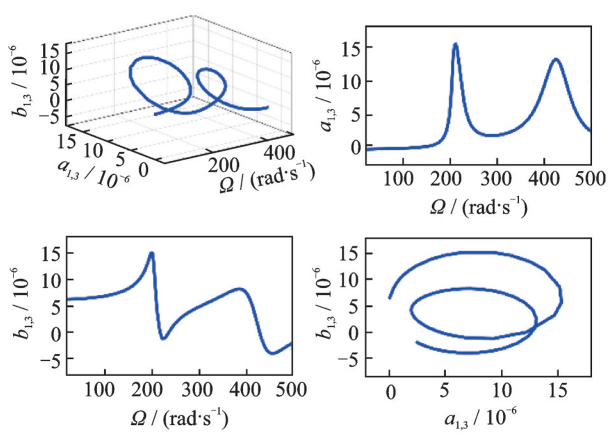
(a) Zero-drift



(b) The 1st harmonic



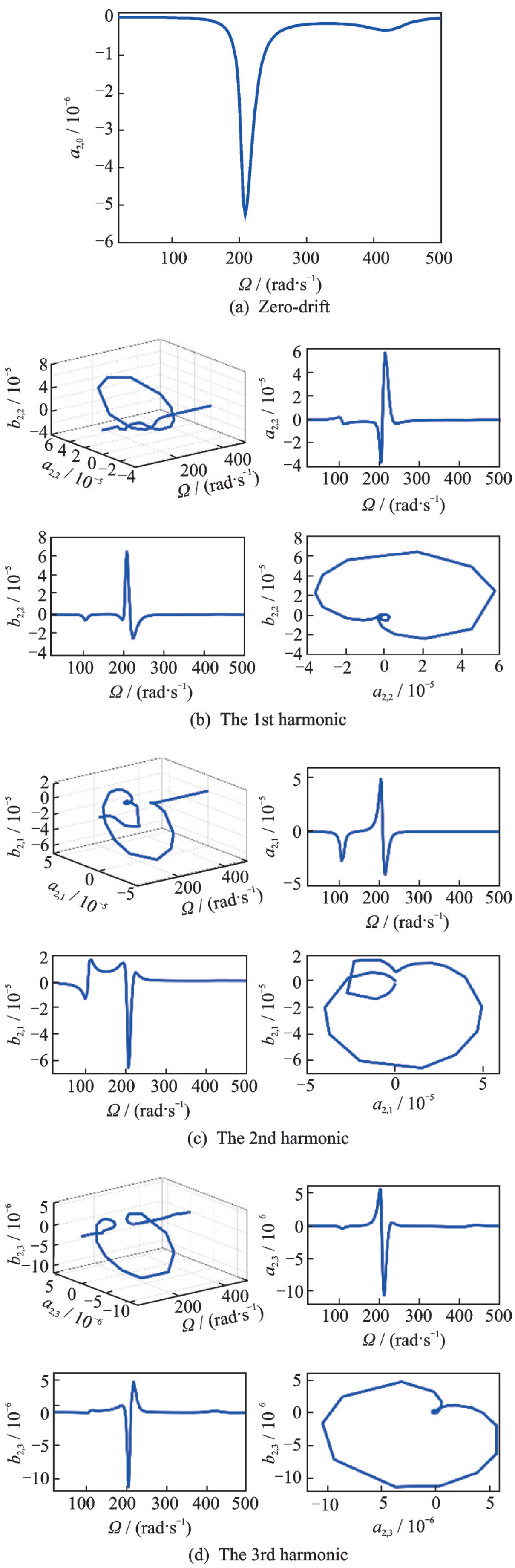
(c) The 2nd harmonic



(d) The 3rd harmonic

Fig.5 Harmonic responses of the first mode

Fig.6 shows the response of corresponding harmonics on the second mode. Although both of the structure and the excitation are symmetric, resonance also occurs on the asymmetric mode. This is a classical character of the pipe conveying fluid. However, this does not mean the secondary resonance will happen while the excitation frequency nears the second natural frequency.



(a) Zero-drift

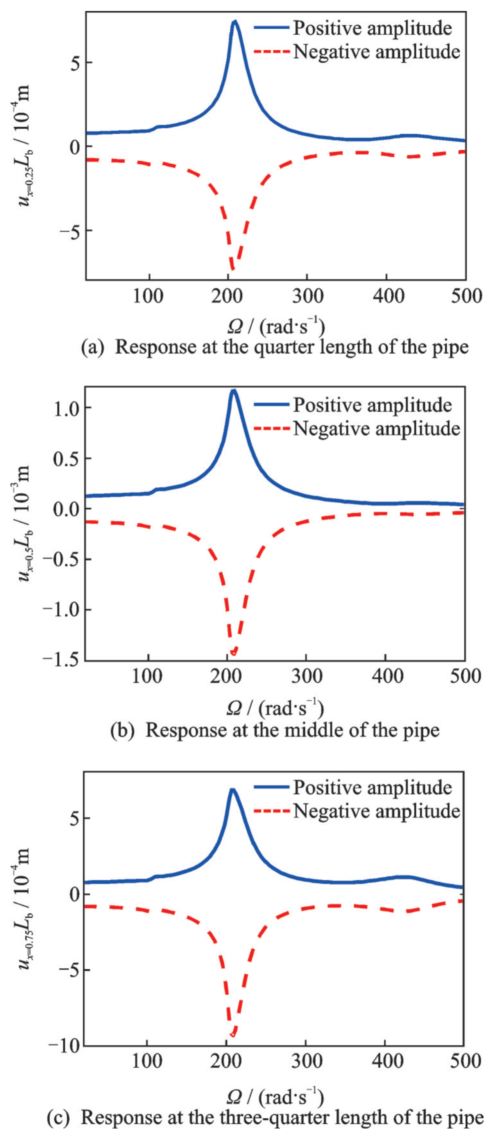
(b) The 1st harmonic

(c) The 2nd harmonic

(d) The 3rd harmonic

Fig.6 Harmonic responses of the second mode

The total response of the hydraulic pipe can be superposed by these harmonics together, including the spatial modes. Fig.7 shows the maximal and minimal responses of the middle point, the quarter point and the three-quarter point on the slightly curved hydraulic pipe. Obviously, the response curve is bended to the left, which means the initial curve softens the nonlinearity of the response. The response curve is asymmetric about the initial position. This is caused by the zero-drift part of the harmonic response. Although the forced resonance happens under the 2∶1 internal resonance condition, the typical double-jumping phenomenon does not occur. It means the 2∶1 internal resonance condition is not triggered under the given initial curve in the current work. This means the 2∶1 internal resonance is not just dominated by the internal resonance condition



(a) Response at the quarter length of the pipe

(b) Response at the middle of the pipe

(c) Response at the three-quarter length of the pipe

Fig.7 Total responses along the pipe

and the quadratic nonlinearity. For continuous, the shape of initial curve[15] and the boundary condition[23] also influences the 2∶1 internal resonance.

## 4 Conclusions

The work here investigated the force resonance under the 2∶1 internal resonance condition. The steady state response was solved out by the Harmonic balance method. Based on the discussions above, some conclusions were obtained.

(1) The initial curve hardens the stiffness of the first natural mode, which enlarges the first natural frequency and the critical fluid speed.

(2) The initial curve softens the nonlinear response. More harmonics are produced by the quadratic nonlinearity and the cubic nonlinearity.

(3) The 2∶1 internal resonance will not happen under the given shape in the current work, although the internal resonance condition and the quadratic nonlinearity occurs together.

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# 固定边界条件下微曲液压管道的受迫振动

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**摘要**：现实中理想的笔直管道是不可能存在的，由制造缺陷或者使役过程中的蠕变引起的初始位形会对液压管道系统的动力学特性产生较大影响。本文首次研究了两端固定约束下液压管道初始形状的影响，利用广义Hamilton原理建立系统动力学模型，并且基于该模型研究了管道势能随初始形状的变化。结果表明，初始形状使得系统势能曲线变得非对称，但系统依然是单稳态的，这与屈曲管道的性质不同。初始形状同样对系统的固有频率有较大影响，它硬化了一阶频率刚度。当初始形状高度超过一个临界值时，初始形状高度对该刚度不再产生影响。与此相反，二阶固有频率在初始形状高度较小时不受其影响，但是当初始形状高度超过临界值时，二阶频率开始增加。此外，由于初始形状的存在，系统动力学方程中产生了平方非线性，它使得系统响应远比理想笔直管道复杂。尽管通过调整初始形状高度以及液压流速建立了2:1内共振条件，系统并未出现2:1内共振典型的双跳跃现象，这与超临界输流管道特性有很大不同。研究表明液压管道建模过程中必须考虑管道的初始形状，并且对其动力学特性需要进行进一步的研究。

**关键词**：受迫振动；液压管道；微曲管道；谐波平衡法