Dynamic Load Identification for Structures with Variable Stiffness Based on Extended Kalman Filter

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Abstract: We introduce the extended Kalman filter (EKF) method combined with the least square estimation to identify the unknown load acting on the time-varying structure and realize the tracking of the structural parameters of the time-varying system. Firstly, we propose the dynamic load identification method when the unknown parameters are stiffness coefficients. Then, a five-degree-of-freedom slowly-varying-stiffness structure is introduced to verify the effectiveness and the accuracy of the EKF method. The results show that the EKF method can accurately identify unknown loads and structural parameters simultaneously even considering noises in the input data.

Key words: extended Kalman filter; least square estimation; load identification; parameter identification

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0 Introduction

Various parameters (mass, stiffness and damping) in different aircraft structures may change over time, such as changes in the stiffness of the aircraft during flutter, and changes in the mass of the rocket during flight. In the field of engineering application technology, it is necessary to obtain the dynamic loads of these time-varying structures. It is important to obtain accurate dynamic load data of these time-varying structures. They will have a great impact on the reliability and safety of the structure.

Traditional dynamics problems are in the three of excitation, structural characteristics, and structural response. Both of them must be known and the remaining one must be solved. For time-varying structures, especially slowly-varying structures, the structure parameters will change slowly, which means that the parameters of the structure cannot be known in advance. To identify the load acting on the slowly changing structure, when only the structural response is known, the structural parameter changes and the load must be identified together, because they are interrelated. Therefore, a dynamic load identification algorithm that can simultaneously identify structural parameters and dynamic loads is meaningful.

The extended Kalman filter (EKF) is improved on the basis of the classic Kalman filter (KF)[1]. It substitutes the unknown physical parameter vector into the state vector to form the extended state vector together. In the KF system to identify the physical parameters of the structure. EKF was first used[2] for parameter identification of multi-degree-of-freedom linear structures. Domestically, EKF[3] is adopted in 1991 and reduced the state vector to a vector containing only stiffness and damping coefficients, successfully identifying the stiffness and damping of the structure and other physical parameters, reducing the amount of calculation and ensuring recognition accuracy. KF is a new form of identifying dynamic loads in the time domain. It does not need to rely on the impulse response function of the system, but only according to the mutual

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correction of the state update equation and the measurement update equation to obtain the load. In addition, KF is also suitable for online load identification, and has the characteristics of small memory occupation and fast calculation speed.

With the improvement of many scholars, the least square estimation (LSE) can also be used to identify the physical parameters of the structure. For example, the adaptive least square method is used to identify the changing process of structural parameters[4]; the sequential nonlinear least square method is proposed in 2006[5], which can also identify the changing process of structural parameters and can be used for nonlinear structures. However, the use of LSE means that the response of all measuring points on the structure must be known. Therefore, if there is a situation where some measuring points on the structure cannot be measured, LSE cannot be used.

1 Dynamic Load Identification Method Combined with EKF and LSE

For a n-degree-of-freedom system, when some parameters in the structure (mass, stiffness and damping) are unknown, combine these unknown parameters with the state vector $\boldsymbol{x}$ to form an extended state vector $\boldsymbol{z}$ as follows

$$\boldsymbol{z}(t)=\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{\alpha}\end{bmatrix}=\begin{bmatrix}\boldsymbol{p}(t)\\ \dot{\boldsymbol{p}}(t)\\ \boldsymbol{\alpha}\end{bmatrix} \tag{1}$$

where $\boldsymbol{\alpha}$ is the unknown parameter vector with a length of m, $\boldsymbol{x}(t)$ the state vector with a length of $2n$, $\boldsymbol{p}(t)$ the displacement vector, and $\dot{\boldsymbol{p}}(t)$ the acceleration vector. Assuming $\dot{\boldsymbol{\alpha}}=0$. The state update equation $\dot{\boldsymbol{z}}(t)$ and the measurement update equation $\boldsymbol{y}(t)$ of $\boldsymbol{z}(t)$ are nonlinear equations, can be rewritten as

$$\dot{\boldsymbol{z}}(t)=\begin{bmatrix}\dot{\boldsymbol{p}}(t)\\ -\boldsymbol{M}^{-1}\boldsymbol{K}\boldsymbol{p}(t)-\boldsymbol{M}^{-1}\boldsymbol{C}\dot{\boldsymbol{p}}(t)+\boldsymbol{M}^{-1}\boldsymbol{B}_u\boldsymbol{f}(t)\\ \boldsymbol{0}\end{bmatrix}= f^c(\boldsymbol{z}(t),\boldsymbol{f}(t)) \tag{2}$$

$$\boldsymbol{y}(t)=\boldsymbol{H}_0\boldsymbol{M}^{-1}(-\boldsymbol{K}\boldsymbol{p}(t)-\boldsymbol{C}\dot{\boldsymbol{p}}(t)+\boldsymbol{B}_u\boldsymbol{f}(t))= h(\boldsymbol{z}(t),\boldsymbol{f}(t)) \tag{3}$$

where $\boldsymbol{M}$, $\boldsymbol{K}$, $\boldsymbol{C}$ are the mass, stiffness and damping matrix. $\boldsymbol{B}_u$ is the excitation influence matrix and $\boldsymbol{H}_0$ the position matrix.

When the load is unknown, $\dot{\boldsymbol{z}}(t)$ and $\boldsymbol{y}(t)$ have two variables: $\boldsymbol{z}(t)$ and $\boldsymbol{f}(t)$. Using Taylor expansion, omit the high-order terms of Eqs.(2) and (3) retain the first-order polynomial, they can approximated written as linear equations, shown as

$$\begin{aligned}f^c(\boldsymbol{z}(t),\boldsymbol{f}(t))\approx{}& f^c(\boldsymbol{z}_{(k-1)|(k-1)},\hat{\boldsymbol{f}}_{k-1})+\\ &\nabla_z\boldsymbol{f}^c_{k-1}\cdot(\boldsymbol{z}(t)-\boldsymbol{z}_{(k-1)|(k-1)})+\\ &\nabla_f\boldsymbol{f}^c_{k-1}\cdot(\boldsymbol{f}(t)-\hat{\boldsymbol{f}}_{k-1})\end{aligned} \tag{4}$$

$$\begin{aligned}h(\boldsymbol{z}(t),\boldsymbol{f}(t))\approx{}& h(\boldsymbol{z}_{k|(k-1)},\hat{\boldsymbol{f}}_{k-1})+\\ &\nabla_z\boldsymbol{h}_k\cdot(\boldsymbol{z}(t)-\boldsymbol{z}_{k|(k-1)})+\\ &\nabla_f\boldsymbol{h}_k\cdot(\boldsymbol{f}(t)-\hat{\boldsymbol{f}}_{k-1})\end{aligned} \tag{5}$$

In order to identify the extended state vector and unknown load of the structure, the extended Kalman filter can be divided into a state update step, an excitation recognition step, and a measurement update step. These three steps are as follows

$$\boldsymbol{z}_{k(k-1)}=\boldsymbol{z}_{(k-1)|(k-1)}+\int_{t_{k-1}}^{t_k}\boldsymbol{f}^c(\boldsymbol{z}_{(k-1)|(k-1)},\hat{\boldsymbol{f}}_{k-1})\mathrm{d}t \tag{6}$$

$$\hat{\boldsymbol{f}}_k=\boldsymbol{J}_k(\boldsymbol{y}_k-\boldsymbol{h}_k(\boldsymbol{z}_{k|(k-1)},\hat{\boldsymbol{f}}_{k-1})+\nabla_f\boldsymbol{h}_k\cdot\hat{\boldsymbol{f}}_{k-1}) \tag{7}$$

$$\begin{aligned}\boldsymbol{z}_{k|k}={}&\boldsymbol{z}_{k|(k-1)}+K_k[\boldsymbol{y}_k-\boldsymbol{h}(\boldsymbol{z}_{k|(k-1)},\hat{\boldsymbol{f}}_{k-1})-\\ &\nabla_f\boldsymbol{h}_k\cdot(\hat{\boldsymbol{f}}_k-\hat{\boldsymbol{f}}_{k-1})]\end{aligned} \tag{8}$$

1.1 State update step

The recursive relationship of $\boldsymbol{z}_k$ can be written as

$$\boldsymbol{z}_k=\boldsymbol{z}_{k-1}+\int_{t_{k-1}}^{t_k}f^c(\boldsymbol{z}(t),\boldsymbol{f}(t))\mathrm{d}t+w \tag{9}$$

where w is the Gaussian white noise with variance Q and means 0.

The prior estimate of $\boldsymbol{z}_k$ is

$$\boldsymbol{z}_{k|(k-1)}=\boldsymbol{z}_{(k-1)|(k-1)}+\int_{t_{k-1}}^{t_k}\boldsymbol{f}^c(\boldsymbol{z}_{(k-1)|(k-1)},\boldsymbol{f}_{k-1})\mathrm{d}t \tag{10}$$

The prior estimation error $\tilde{\boldsymbol{z}}_{k|(k-1)}$ is

$$\begin{aligned}\tilde{\boldsymbol{z}}_{k|(k-1)}={}&\boldsymbol{z}_k-\boldsymbol{z}_{k|(k-1)}=\\ &(1+\Delta t\cdot\nabla_z\boldsymbol{f}^c_{k-1})\cdot\tilde{\boldsymbol{z}}_{(k-1)|(k-1)}+\\ &\Delta t\cdot\nabla_f\boldsymbol{f}^c_{k-1}\cdot\tilde{\boldsymbol{f}}_{k-1}+w\end{aligned} \tag{11}$$

The variance of $\tilde{\boldsymbol{z}}_{k(k-1)}$ is

$$\begin{aligned}\boldsymbol{P}^z_{k|(k-1)}={}&E(\tilde{\boldsymbol{z}}_{k(k-1)}\cdot(\tilde{\boldsymbol{z}}_{k(k-1)})^{\mathrm{T}})=\\ &Q+\left[1+\Delta t\cdot\nabla_z\boldsymbol{f}^c_{k-1}\quad\Delta t\cdot\nabla_f\boldsymbol{f}^c_{k-1}\right]\cdot\end{aligned}$$

$$\begin{bmatrix} P^{z}_{(k-1)|(k-1)} & P^{zf}_{k-1} \\ P^{fz}_{k-1} & P^{f}_{k-1} \end{bmatrix} \begin{bmatrix} (1+\Delta t\cdot\nabla_z \boldsymbol{f}^{c}_{k-1})^{\mathrm{T}} \\ (\Delta t\cdot\nabla_f \boldsymbol{f}^{c}_{k-1})^{\mathrm{T}} \end{bmatrix} \tag{12}$$

1.2 Incentive recognition step

To identify the load, we establish an equation about the load $\hat{\boldsymbol{f}}_k$ to be demanded and define

$$\tilde{\boldsymbol{y}}_k=\boldsymbol{y}_k-\boldsymbol{h}_k(\boldsymbol{z}_{k|(k-1)},\hat{\boldsymbol{f}}_{k-1})+\nabla_f \boldsymbol{h}_k\cdot\hat{\boldsymbol{f}}_{k-1}=\nabla_f \boldsymbol{h}_k\cdot\boldsymbol{f}_k+\tilde{\boldsymbol{e}} \tag{13}$$

where $\tilde{\boldsymbol{e}}=\nabla_z \boldsymbol{h}_k\cdot\tilde{\boldsymbol{z}}_{k(k-1)}+\boldsymbol{v}$, $\boldsymbol{v}$ is the Gaussian white noise with variance $\boldsymbol{R}$ and means $\mathit{0}$. We choose a weighting matrix $\boldsymbol{W}$ to make the least square estimation result is the best.

$$\hat{\boldsymbol{f}}_k=[(\nabla_f \boldsymbol{h}_k)^{\mathrm{T}}\cdot W\cdot\nabla_f \boldsymbol{h}_k]^{-1}(\nabla_f \boldsymbol{h}_k)^{\mathrm{T}} W\cdot\tilde{\boldsymbol{y}}_k \tag{14}$$

$$W=\tilde{\boldsymbol{R}}_k^{-1},\tilde{\boldsymbol{R}}_k=E(\tilde{\boldsymbol{e}}_k\tilde{\boldsymbol{e}}_k^{\mathrm{T}}) \tag{15}$$

$$J_k=[(\nabla_f \boldsymbol{h}_k)^{\mathrm{T}}\tilde{\boldsymbol{R}}_k^{-1}\nabla_f \boldsymbol{h}_k]^{-1}(\nabla_f \boldsymbol{h}_k)^{\mathrm{T}}\tilde{\boldsymbol{R}}_k^{-1} \tag{16}$$

$$P^f_k=[(\nabla_f \boldsymbol{h}_k)^{\mathrm{T}}\cdot\tilde{\boldsymbol{R}}_k^{-1}\cdot\nabla_f \boldsymbol{h}_k]^{-1} \tag{17}$$

1.3 Measurement update step

To determine the appropriate Kalman gain matrix $\boldsymbol{K}_k$. We obtain the posterior estimation error $\tilde{\boldsymbol{z}}_{k|k}$ is

$$\begin{aligned}\tilde{\boldsymbol{z}}_{k|k}&=\boldsymbol{z}_k-\boldsymbol{z}_{k|k}=(\boldsymbol{I}-K_k(\boldsymbol{I}-\\&\nabla_f \boldsymbol{h}_k\cdot\boldsymbol{J}_k)\nabla_z \boldsymbol{h}_k)\cdot\tilde{\boldsymbol{z}}_{k|(k-1)}+\boldsymbol{K}_k(\boldsymbol{I}-\\&\nabla_f \boldsymbol{h}_k\cdot\boldsymbol{J}_k)\nabla_f \boldsymbol{h}_k\cdot\boldsymbol{f}_k+\boldsymbol{K}_k(\boldsymbol{I}-\nabla_f \boldsymbol{h}_k\cdot\boldsymbol{J}_k)\cdot\boldsymbol{v}\end{aligned} \tag{18}$$

The posterior estimation variance is

$$\begin{aligned}P^z_{k|k}&=E(\tilde{\boldsymbol{z}}_{k|k}\cdot(\tilde{\boldsymbol{z}}_{k|k})^{\mathrm{T}})=(\mathrm{I}-L_k\nabla_z \boldsymbol{h}_k)\cdot P^z_{k|(k-1)}\cdot(\boldsymbol{I}-\\&L_k\nabla_z \boldsymbol{h}_k)^{\mathrm{T}}+L_k\cdot R\cdot L_k^{\mathrm{T}}\end{aligned} \tag{19}$$

where

$$L_k=\boldsymbol{K}_k(\boldsymbol{I}-\nabla_f \boldsymbol{h}_k\cdot\boldsymbol{J}_k) \tag{20}$$

To make the posterior estimate be the best estimate, $P^z_{k|k}$ should be the smallest, so we can get

$$\frac{\partial \boldsymbol{P}^z_{k|k}}{\partial \boldsymbol{K}_k}=0 \tag{21}$$

$$\boldsymbol{K}_k=\boldsymbol{P}^z_{k|(k-1)}(\nabla_z \boldsymbol{h}_k)^{\mathrm{T}}\tilde{\boldsymbol{R}}_k^{-1} \tag{22}$$

$$\boldsymbol{P}^z_{k|k}=\boldsymbol{P}^z_{k|(k-1)}-\boldsymbol{K}_k\cdot(\tilde{\boldsymbol{R}}_k-\nabla_f \boldsymbol{h}_k\cdot P^f_k\cdot(\nabla_f \boldsymbol{h}_k)^{\mathrm{T}})\cdot\boldsymbol{K}_k \tag{23}$$

$$P^{zf}_k=-P^z_{k|(k-1)}(\nabla_z \boldsymbol{h}_k)^{\mathrm{T}}\boldsymbol{J}_k^{\mathrm{T}}=-\boldsymbol{K}_k\cdot\nabla_f \boldsymbol{h}_k\cdot P^f_k \tag{24}$$

In the state update equation and the measurement update equation, different types of unknown parameters in the extended state vector have different matrix forms. For example, the mass matrix is in the form of an inverse matrix, and the stiffness matrix do not need to be inverted. At this time, if the unknown parameters seek the partial derivative of the equation, their partial derivative matrices will also be different. We give the extended Kalman filter method when the unknown parameter is stiffness coefficient.

2 EKF Method with Unknown Stiffness Coefficient

When the unknown parameter α is the stiffness coefficient, the measurement update equation and the state update equation of the system are linear functions of external excitation $\boldsymbol{f}_k$, and their coefficient matrices are constant matrices. So Eqs.(2, 3) can be rewritten as

$$f^c(\boldsymbol{z}_k,\boldsymbol{f}_k)=\begin{bmatrix}\dot{\boldsymbol{p}}_k\\ -\boldsymbol{M}^{-1}\boldsymbol{K}\boldsymbol{p}_k-\boldsymbol{M}^{-1}\boldsymbol{C}\boldsymbol{p}_k\\ \mathit{0}\end{bmatrix}+\begin{bmatrix}\mathit{0}\\ \boldsymbol{M}^{-1}\boldsymbol{B}_u\\ \mathit{0}\end{bmatrix}\cdot\boldsymbol{f}_k \tag{25}$$

$$\begin{aligned}\boldsymbol{y}_k=\boldsymbol{h}(\boldsymbol{z}_k,\boldsymbol{f}_k)&=H_0\boldsymbol{M}^{-1}(-\boldsymbol{C}\dot{\boldsymbol{p}}_k-\boldsymbol{K}\boldsymbol{p}_k)+\\&H_0\boldsymbol{M}^{-1}\boldsymbol{B}_u\boldsymbol{f}_k\end{aligned} \tag{26}$$

We can see that both the state update equation and the measurement update equation can be regarded as composed of two parts. One part is a nonlinear expression about $\boldsymbol{z}_k$ and the other part is a linear expression about $\boldsymbol{f}_k$. Because the mass coefficients of the structure are not in the extended state vector, their coefficient matrices are all constant matrices. These two coefficient matrices are the matrices obtained by calculating the partial derivative of the state update equation and the measurement update equation. Then Eqs.(25, 26) can be rewritten as

$$\boldsymbol{f}^c(\boldsymbol{z}_k,\boldsymbol{f}_k)=\bar{\boldsymbol{f}}^c(\boldsymbol{z}_k)+\nabla_f \boldsymbol{f}^c_{k-1}\boldsymbol{f}_k \tag{27}$$

$$\boldsymbol{h}(\boldsymbol{z}_k,\boldsymbol{f}_k)=\bar{\boldsymbol{h}}(\boldsymbol{z}_k)+\nabla_f \boldsymbol{h}_k\cdot\boldsymbol{f}_k \tag{28}$$

When the unknown parameter is stiffness coefficient, $\nabla_f \boldsymbol{f}^c_{k-1}$, $\nabla_f \boldsymbol{h}_k$ are constant matrices as

$$\nabla_f \boldsymbol{f}^c_{k-1}=\begin{bmatrix}\mathit{0}\\ \boldsymbol{M}^{-1}\boldsymbol{B}_u\\ \mathit{0}\end{bmatrix} \tag{29}$$

$$\nabla_f \boldsymbol{h}_k=\boldsymbol{H}_0\boldsymbol{M}^{-1}\boldsymbol{B}_u \tag{30}$$

The measurement update step, load identification step can be simplified as

$$\boldsymbol{z}_{k|k}=\boldsymbol{z}_{k|(k-1)}+\boldsymbol{K}_k[\boldsymbol{y}_k-\bar{\boldsymbol{h}}(\boldsymbol{z}_{k|(k-1)})-\nabla_f \boldsymbol{h}_k\cdot\hat{\boldsymbol{f}}_k] \tag{31}$$

$$\hat{\boldsymbol{f}}_k=\boldsymbol{J}_k(\boldsymbol{y}_k-\bar{\boldsymbol{h}}_k(\boldsymbol{z}_{k|(k-1)})) \tag{32}$$

Then we get the method of dynamic load identification when the unknown parameter vector does not contain the structural quality parameter at Table 1.

Table 1 EKF load identification method when unknown parameter is stiffness coefficient

(1) Given initial conditions

$$z_{0|-1} \text{ and } P^{z}_{0|-1} \text{ and } f_0$$

(2) Incentive recognition

$$\tilde{\boldsymbol{R}}_k = (\nabla_z \boldsymbol{h}_k)^{\mathrm{T}} \cdot \boldsymbol{P}^{z}_{k|(k-1)} \cdot \nabla_z \boldsymbol{h}_k + \boldsymbol{R}_k$$

$$\boldsymbol{J}_k = [(\nabla_f \boldsymbol{h}_k)^{\mathrm{T}} \tilde{\boldsymbol{R}}_k^{-1} \nabla_f \boldsymbol{h}_k]^{-1} (\nabla_f \boldsymbol{h}_k)^{\mathrm{T}} \tilde{\boldsymbol{R}}_k^{-1}$$

$$\hat{\boldsymbol{f}}_k = \boldsymbol{J}_k (\boldsymbol{y}_k - \overline{\boldsymbol{h}}_k (\boldsymbol{z}_{k|(k-1)}))$$

$$\boldsymbol{P}^{f}_k = [(\nabla_f \boldsymbol{h}_k)^{T} \cdot \tilde{\boldsymbol{R}}_k^{-1} \cdot \nabla_f \boldsymbol{h}_k]^{-1}$$

(3) Measurement update

$$\boldsymbol{K}_k = \boldsymbol{P}^{z}_{k|(k-1)} (\nabla_z \boldsymbol{h}_k)^{\mathrm{T}} \tilde{\boldsymbol{R}}_k^{-1}$$

$$\boldsymbol{z}_{k|k} = \boldsymbol{z}_{k|(k-1)} + \boldsymbol{K}_k [\boldsymbol{y}_k - \bar{\boldsymbol{h}}(\boldsymbol{z}_{k|(k-1)}) - \nabla_f \boldsymbol{h}_k \cdot \hat{\boldsymbol{f}}_k]$$

$$P^{z}_{k|k} = \boldsymbol{P}^{z}_{k|(k-1)} - \boldsymbol{K}_k \cdot (\tilde{\boldsymbol{R}}_k - \nabla_f \boldsymbol{h}_k \cdot \boldsymbol{P}^{f}_k \cdot (\nabla_f \boldsymbol{h}_k)^{\mathrm{T}}) \cdot \boldsymbol{K}_k \boldsymbol{P}^{zf}_k = -P^{z}_{k|k-1} (\nabla_z \boldsymbol{h}_k)^{\mathrm{T}} \boldsymbol{J}_k^{\ \mathrm{T}} = -\boldsymbol{K}_k \cdot \nabla_f \boldsymbol{h}_k \cdot \boldsymbol{P}^{f}_k$$

(4) Status update

$$\boldsymbol{z}_{(k+1)|k} = \boldsymbol{z}_{k|k} + \int_{t_k}^{t_{k+1}} \bar{\boldsymbol{f}}^{c}(\boldsymbol{z}_{k|k}) + \nabla_f \boldsymbol{f}^{c}_k \cdot \boldsymbol{f}_k \, \mathrm{d}t$$

$$\boldsymbol{P}^{z}_{(k+1)|k} = \boldsymbol{Q} + \begin{bmatrix} 1 + \Delta t \cdot \nabla_z \boldsymbol{f}^{c}_k & \Delta t \cdot \nabla_f \boldsymbol{f}^{c}_k \end{bmatrix} \begin{bmatrix} P^{z}_{k|k} & P^{zf}_k \\ P^{fz}_k & P^{f}_k \end{bmatrix} \begin{bmatrix} (1 + \Delta t \cdot \nabla_z \boldsymbol{f}^{c}_k)^{\mathrm{T}} \\ (\Delta t \cdot \nabla_f \boldsymbol{f}^{c}_k)^{T} \end{bmatrix}$$

3 Numerical Results

We first aim to identify the load applied on a five-degree-of-freedom slowly-varying-stiffness system, and also track its stiffness changes. The system is shown in Fig. 1.

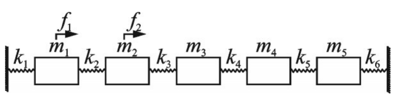


Fig.1 Five-degree-of-freedom slowly-varying-stiffness system

In Fig.1, $m_1 = m_2 = m_3 = m_4 = m_5 = 1$ kg, $k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = 200$ N/m. Supposing the damping of this system is proportional damping, and $C = \alpha M + \beta K$, $\alpha = 0.05$, $\beta = 0.02$. We make the stiffness coefficient k_4 of the structure slowly drops from 200 N/m to 120 N/m at 1.5 s.

Choose to observe the response of all measurement points as the measurement vector. Assuming that the estimates of stiffness coefficients k_1 and k_2 are very accurate, the remaining stiffness coefficients are used as unknown parameters and substituted into the extended state vector $\boldsymbol{z}$. Since the structure is filtered from static conditions, assuming that the initial response estimate is correct and the initial parameter estimates have a certain deviation, the initial expanded state vector can be set as $\boldsymbol{z}_{0|-1} = [\mathrm{zeros}(1, 10),\ 120, 220, 160, 180]^{\mathrm{T}}$.

The first degree of freedom is applied a load of $f_1 = \sin(5\pi t) + 2\sin(2\pi t)$, the second degree of freedom is applied a load of $f_2 = \sin(6\pi t)$. Add 1% and 5% white noise to the measurement vector respectively. Then we use the extended Kalman filter method proposed to track the unknown parameter (stiffness coefficient k) of the structure, and identify the unknown load acting on the structure at the same time. The unknown parameter identification results are shown in Figs. 2—5, the unknown load identification results are shown in Figs.6 and 7.

Record the value of the unknown parameter identified by the method before the parameter change (t=1.4 s) and after the parameter change (t=5 s) relatively, and compare with the real value. The results are shown in Table 2.

The data before and after the parameter changes in Table 2 show that at a noise level of 1%, the error of the EKF method to identify stiffness parameters is under 1.5%; at a noise level of 5%, the error of the EKF method to identify stiffness parame-

Table 2 Stiffness parameter identification result error

Noise level	Stiffness parameter	Before parameter change			After parameter change		
		Identification value/(N·m^{-1})	Real value/(N·m^{-1})	Error/%	Identification value/(N·m^{-1})	Real value/(N·m^{-1})	Error/%
1%	k_3	198.8	200	0.6	199.3	200	0.35
	k_4	199.2	200	0.4	118.4	120	1.33
	k_5	199.6	200	0.2	198.9	200	0.55
	k_6	199.8	200	0.1	199.2	200	0.40
5%	k_3	187.2	200	6.40	194.4	200	2.80
	k_4	191.8	200	4.10	118.8	120	1.00
	k_5	198.7	200	0.65	197.5	200	1.25
	k_6	201.0	200	0.50	195.6	200	2.20

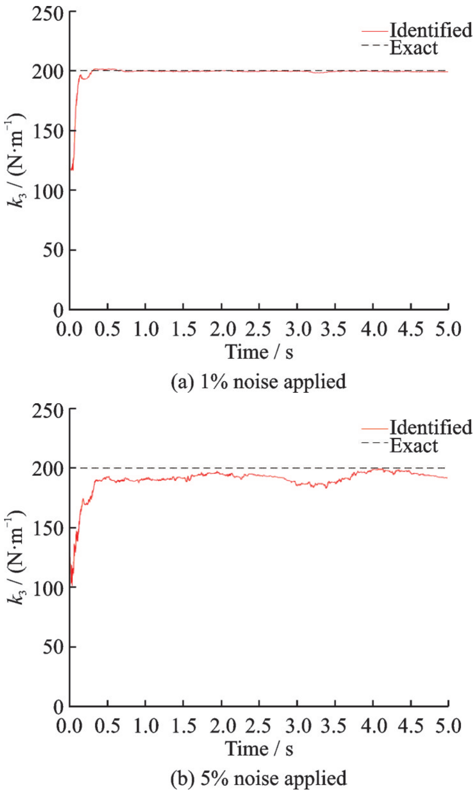


Fig.2 Identification results of stiffness coefficient k_3

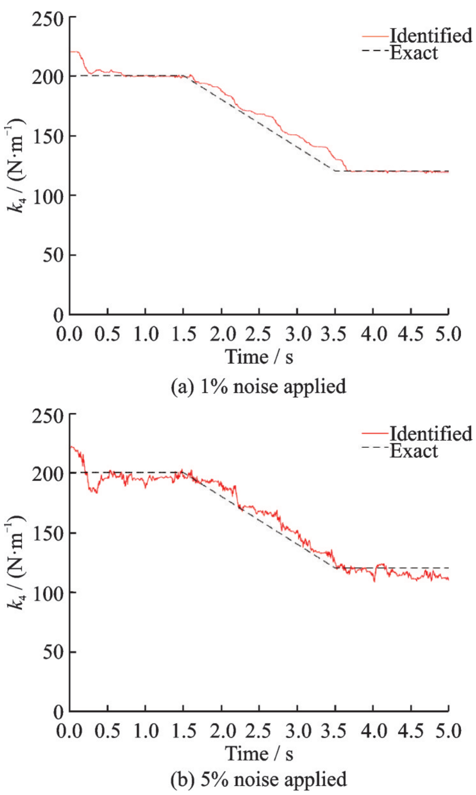


Fig.3 Identification results of stiffness coefficient k_4

ters is controlled within 6.5%. The data show that the EKF method leads to better identification accuracy for the parameter recognition of variable stiffness structures.

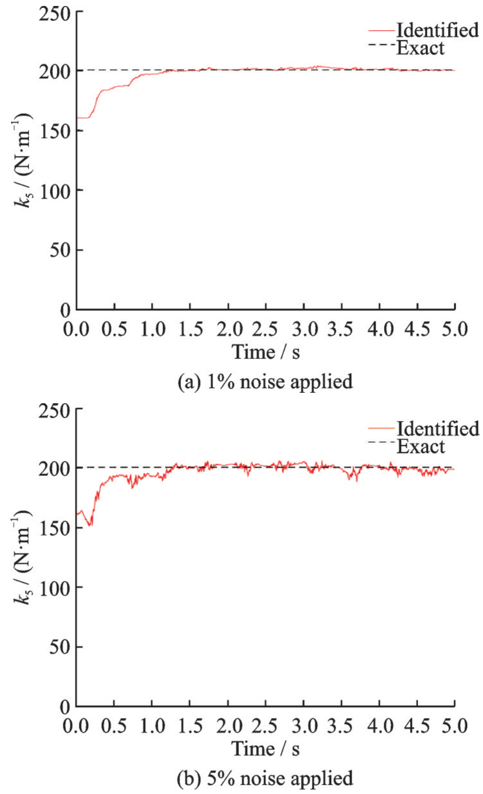


Fig.4 Identification results of stiffness coefficient k_5

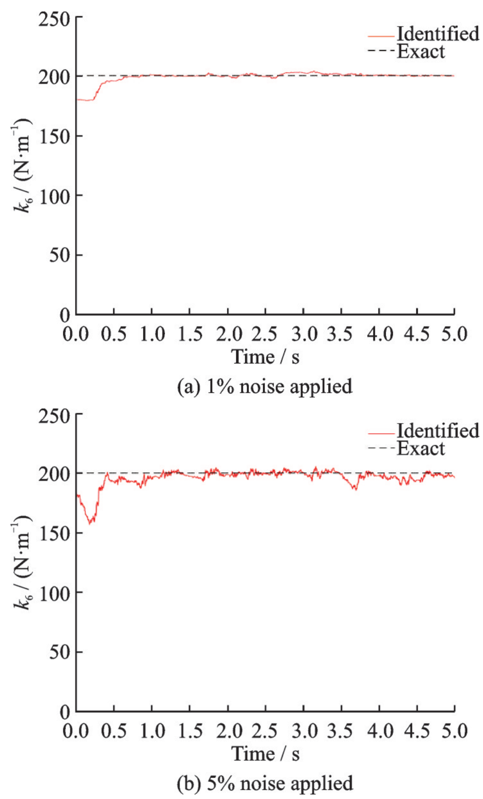
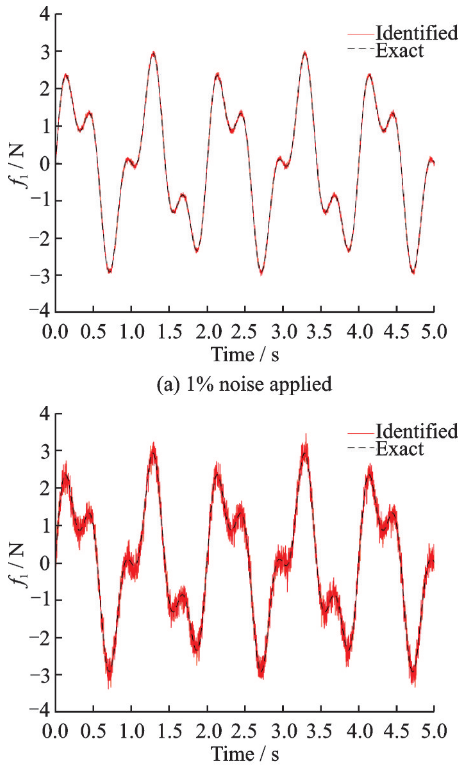


Fig.5 Identification results of stiffness coefficient k_6

Similarly, the recognition results of the method on the load under different noise conditions are shown in Table 3.

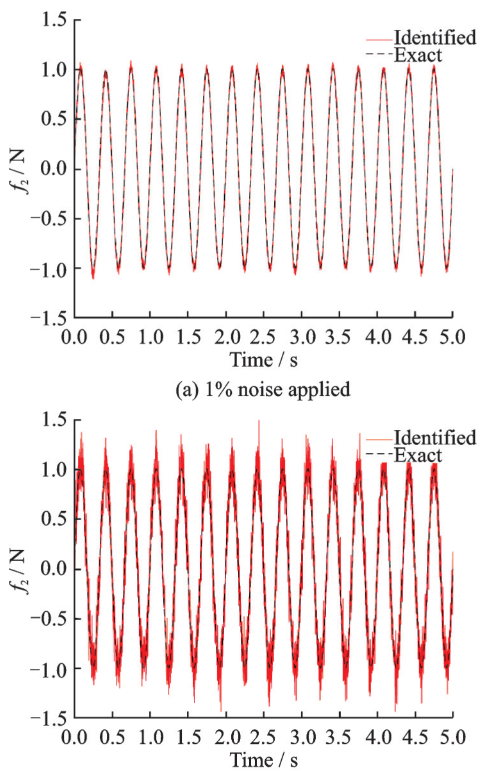
Table 3 shows that when the noise level is



(a) 1% noise applied

(b) 5% noise applied

Fig.6 Identification results of load f_1



(a) 1% noise applied

(b) 5% noise applied

Fig.7 Identification results of load f_2

1%, the relative error of the dynamic load identification result is less than 5%, and the correlation coefficient result is better; when the noise level is 5%, the relative error of the dynamic load identification result is less than 20%, the correlation coefficient result is better. So the algorithm has very good recognition accuracy when the noise is low, and the recognition accuracy when the noise is high still needs to be improved.

Table 3 Force identification results error

Noise level	Stiffness parameter	Relative error/%	Correlation coefficient/%
1%	f_1	2.64	99.97
	f_2	4.98	99.88
5%	f_1	10.24	99.32
	f_2	19.85	98.05

Tables 2 and 3 show that the method can accurately identify the unknown load on the slowly varying stiffness structure under the condition of noise interference, and track the change process of the stiffness parameter at the same time.

4 Conclusions

We propose a dynamic load identification method combined with the extended Kalman filter of the least square estimation to identify the unknown load acting on the time-varying structure and track the structural parameters of the time-varying system at the same time. We present the algorithm of dynamic load identification when the unknown parameter type is stiffness coefficient at first. Then we use a numerical example to verify the effectiveness and accuracy of the EKF method. The numerical results show that the proposed method can accurately identify unknown loads and structural parameters even when considering noise in the input data. The proposed method can have a wide range of applications in several domain such as design optimization, diagnostics, control and monitoring of vibrating structures.

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Author contributions Ms. **LI Yilin** designed the study, complied the models, conducted the analysis, interpreted the results and wrote the manuscript. Mr. **TANG Hongzhi** contributed to the discussion and background of the study. Dr. **JIANG Jinhui** guided the work and checked experimental results. All authors commented on the manuscript draft and approved the submission.

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基于扩展卡尔曼滤波的变刚度结构动载荷识别

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摘 要：引入扩展卡尔曼滤波(Extended Kalman filter，EKF)方法，结合最小二乘估计来识别作用在时变结构上的未知载荷，并实现对时变系统结构参数的识别。首先提出未知参数类型为刚度系数时的动载荷识别方法。然后通过一个五自由度缓变刚度结构仿真算例来验证方法的有效性和准确性。仿真结果表明，即使考虑输入数据中的噪声，该方法也能同时准确识别出未知载荷和结构参数。

关键词：扩展卡尔曼滤波；最小二乘估计；载荷识别；参数识别