Longitudinal Modeling and Aerodynamic Evaluation of Morphing Aircraft with Symmetric Folding Wing Tips

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Abstract: To study the change of aerodynamic characteristics of the spanwise adaptive wing with the folding angle, a kind of folding wing tip morphing aircraft, the modified Cessna 550 aircraft with shape memory alloys as hinge actuators, is considered. A novel modeling scheme is proposed based on the variable aerodynamic parameters about the folding angles. This kind of modeling scheme can explicitly account for changes in aerodynamic properties resulting from folding motion of symmetrical tip of the wing. To explore how folding motion will influence the aerodynamic performances of the aircraft, the computational fluid dynamics (CFD) is employed to build aircraft models and obtain aerodynamic coefficients under different folding angles of the wing tip. The aerodynamic parameters about the folding angles are specified through the curve fitting for obtaining the numeric nonlinear models. The taking-off, maneuvering and landing performances under different folding angles are analyzed to select the best morphing strategy and obtain the best aerodynamic performance. Longitudinal steady stability analysis is presented to validate the feasibility of the proposed morphing strategy.

Key words: morphing aircraft; aerodynamic evaluation; dynamic modeling; longitudinal modeling; symmetric folding wing tips

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0 Introduction

The folding wing-tip morphing aircraft at a conceptual level has proven to possess excellent advantages over the conventional fixed-wing aircraft and has been of great significance and interest to aircraft designers since the concept "morphing" is proposed. The morphing concept is originated from bionics, for example, eagles or gulls will fold their wings to dive when hunting. It is the inspiration for the NASA Spanwise Adaptive Wing (SAW) Project, which aims to explore the applications of new mate rials on the aircraft to replace complex mechanical, hydraulic, and electric actuators. As an important representative of large-scale morphing aircraft, the folding wingtip morphing aircraft is characterized by high maneuverability, high flight efficiency, and high endurance, which is an important development direction for future military and civil application^[1-3]. The most obvious feature of the folding wingtip morphing aircraft is that it contains symmetrical foldable wingtips, and the folding motion of the wingtips also provides the main source of lift and thrust. In different application scenarios, it can adaptively change its configuration to meet the demand of multiple missions and obtain the best aerodynamic per-

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formances.

Given the unique structural design and complex working condition of the folding wing tip morphing aircraft, its system dynamics model has the characteristics of strong coupling, multi-variable and strong nonlinear. Particularly, when the system's center of mass (CM) and moment of inertia change due to structural changes, the system parameters will change indeterminately and cause dynamic mutations, which brings a series of challenges to modeling and control of morphing aircraft^[4]. Hence, the process of modeling is a balance between realistic description of flight physics and simplified control of the model.

The existing literature on morphing aircraft mainly focuses on the aerodynamic design of the morphing structure^[5-9], modeling methodology^[10-12], aerodynamic analysis^[13-18] and flight controller design^[19-24]. There are two key problems in the study of folding wingtip morphing aircraft: (1) How to design a reasonable autonomous deformation strategy to achieve a flexible, smooth and autonomous change of aircraft wingtip structure, so as to achieve the desired structural deformation; (2) how to realize the stability control and system output tracking control in the folding process when the folding angle of wingtip changes. Therefore, it is necessary to establish a folding wingtip morphing aircraft model and analyze its aerodynamic characteristics under different wingtip folding angles, which is also the basic problem in the study of morphing aircraft. In Ref. [5], the shape memory alloy (SMA) spars were employed to enhance aeroelastic performance, and the aerodynamic performances of the composite smart wings were investigated. In Ref. [25], a two-degree-of-freedom (DOF) mechanism was designed and fabricated for morphing wing applications, and modification was done on a well-known model of SMA as well as the mechanism. To improve the trajectory-planning efficiency of morphing aircraft, Ref. [26] proposed an offline optimization method based on multi-fidelity kriging (MFK) modeling, which was more efficient than trajectory optimization. In the nascent field of micro unmanned aerial vehicles, large and rapid changes in wing geometry are achievable, resulting in difficulties in dynamic modeling of such aircrafts. To address the modeling problems of morphing aircraft, the modeling of a camber-morphing airborne wind energy system was proposed in Ref. [27], which focused on the coupled aeroelastic and flight dynamics of the aircraft and on the reduced-order structural and aerodynamic model of the morphing wing. In Ref. [28], the longitudinal nonlinear models of morphing aircraft were linearized, and the longitudinal linear parameter varying model of the morphing aircraft in the wing-folding process was obtained. Moreover, deformations of aircraft configurations can cause difficulties in modeling, and bring out the stability problems of the longitudinal lateral aircraft dynamics.

In addition, when the wingtips are folded, the aircraft itself needs control to ensure that the desired deformation state can be achieved within the desired time. More importantly, the change of aerodynamic shape will also lead to uncertainty changes in the center of mass, moment of inertia and other factors of the aircraft. This will make the original flight control system unable to apply to the stability control of morphing aircraft. Therefore, to guarantee the aircraft stability during the morphing process and deal with problems brought by mass distribution, center of mass shift or various inertia tensors, a number of control strategies were proposed, like sliding mode control^[29-30], adaptive control^[31], robust control^[32] and other control methods.

Although there are many studies on longitudinal modeling and controller design of morphing aircraft, few of them focus on aircraft models with deformed structure characteristics, and other open problems still exist, especially various wing configurations and the performance evaluation in different flight processes such as taking-off, maneuvering, or landing. In this paper, to explore the morphing effects on aircraft, the performance evaluation of the morphing aircraft is studied in different flight processes, as well as a modeling methodology is pro-

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posed. The main contributions of this paper are as follows:

(1) Based on the SAW Project, a class of folding wingtip morphing aircraft is studied. Solid-Works is employed to build the 3D aircraft model. The aerodynamic parameters under different folding angles with CFD are obtained to explore aerodynamic changes during the morphing process, which is essential to study how to achieve optimal aerodynamic performance through structural deformation of morphing aircraft in different flight stages.

(2) A modeling method containing structural deformation characteristics is proposed for a kind of folding wing tip aircraft based on the wing deformation parameters, to explore the differences between the folding wing tip morphing aircraft and the conventional fixed-wing aircraft. Then, the nonlinear dynamic model about the folding angle is established for the morphing aircraft, which lays the foundation of the control design and analysis.

(3) The aerodynamic performances of the folding wing tip morphing aircraft are studied at different folding angles in takeoff, maneuver, and landing stages, including the running distance, takeoff time, and acceleration time. This is useful for selecting the best morphing strategy to obtain better aerodynamic properties.

The rest of this paper is organized as follows. In Section 1, the properties of the folding wing tip aircraft are introduced, as well as the quasi-steady hypothesis needed in aerodynamic analysis. Meanwhile, the modeling problem of the folding wing tip morphing aircraft is formulated. In Section 2, the aerodynamic analysis is conducted under different folding angles and angle of attack to explore aerodynamic changes during morphing process. In Section 3, the aerodynamic performances of the folding wing tip aircraft are evaluated from the prospective of taking off, maneuvering and landing. Moreover, the longitudinal steady stability is also analyzed in this section. Fig.1 illustrates the research procedures of this paper.



1 Overview of Folding Wing Tip Aircraft

In this section, the configuration of the folding wing tip aircraft is given. Then, the quasi-steady hypothesis is also given and will be used for aerodynamic analysis.

1.1 Geometric configurations of reference aircraft

Presently, the vast majority of morphing aircraft have focused on wing deformation. Folding wing-tips are applied to aircraft carriers to save occupied area. In addition to reducing occupied area, SAW Project achieved an 80% reduction in drive control structure mass due to the use of hinged SMA actuators. This enabled the outer wing segment to fold up and down in flight. Besides, the application of shape memory alloys has greatly improved the precise control of wings, thereby reducing or eliminating the rudder surfaces and corresponding control systems used in traditional aircraft maneuvering. All these can provide a new design idea for high subsonic and supersonic aircraft in the future. In 2017, NASA conducted 13 test flights using the SAW concept-based demonstrator Prototype Technology Evaluation Research Aircraft (PTERA), which demonstrated that folding the outer wing segments up and down during flight increases yaw stability and reduces rudder size and tail drag.

PTERA is an 11% scale model of the Boeing 737, with a total length of 3.66 m, a total takeoff

weight of 90.72 kg and a payload of 18.14 kg. The initial wing span was 3.44 m, and the outer wing segment of 38 cm could fold 75° up and down. However, in a subsequent flight test, the aircraft lost control and crashed after completing a roll turn while changing its attitude. NASA, which is still investigating and analyzing the incident, believes that the SMA drive technology has good application potential, will not stall SAW because of this minor setback, and will continue to conduct further research on the control law to facilitate future use of the foldable wing method.

Additionally, the SAW team proposes a followup project to validate the SMA folding wings in a supersonic aircraft. Ground trials are currently under way with a full-size F/A-18 fighter jet on a SMA driver with a larger driving force. In order to explore the aerodynamic characteristics and transformation law of handling characteristics of the SAW project, this paper adopts numerical simulation method to calculate and analyze the variation rules of aerodynamic characteristics and handling characteristics of adaptive wings with wing-tip folding angle, hoping to provide reference and program guidance for the overall aerodynamic design of similar aircraft in the future.

Remark 1 For confidentiality reasons, NASA has not disclosed the technical details of the SAW aircraft, and even the general aircraft properties such as the geometric parameters and airfoil parameters. Hence, to study the change of aerodynamic characteristics of the spanwise adaptive wing with the folding angle, the modified Cessna 550 aircraft with shape memory alloys as hinge actuators, is considered. As a new type of aircraft, we conduct some preliminary study using the aerodynamic simulation and focus on studying how this "morphing" of wingtip folding achieves better aerodynamic performance than traditional aircraft from the theoretical level, and to analyze from the theoretical level what morphing strategy should be adopted in each flight stage of this folding wingtip morphing aircraft.

The Cessna 550 aircraft with wing tip folding is shown in Fig.2 and the airfoil of the aircraft in Fig.3. The basic parameters of the Cessna 550 are described in Table 1.



Fig.2 Cessna 550 aircraft with wing tip folding at 30°



Table 1 Basic parameters of Cessna 550 aircraft

| Symbol | Physical meaning description | Value |
|--------------------------|---|-------|
| m | aircraft mass/kg | 3 655 |
| $l_{ m tip}$ | wingtip length/m | 0.3 |
| $S_{ m w}$ | Wing reference area/ m^2 | 31.83 |
| $b_{\rm main}$ | Single main wing length/m | 7.95 |
| b_{inner} | Inner wingspan length/m | 6.7 |
| Λ | Leading sweep angle/(°) | 12 |
| I_{xy}, I_{xz}, I_{yz} | Product of inertia/(kg•m ²) | 0 |

We should note that, this paper concentrates on how the folding wing tip motion will affect the aircraft and which morphing strategy we will adopt to acquire better aerodynamic performances than conventional aircraft. Due to confidential reasons and lack of data, we employ a "non-existent aircraft" to conduct our research with the assumption that both symmetric wing tips of the aircraft are foldable. The purpose of this paper is to give a specific longitudinal model of such aircraft and explore how folding wing tip motion will influence the aerodynamic performances of this aircraft than conventional fixed-wing aircraft through aerodynamic analysis. Fig.4 gives the specific geometric model of the modified Cessna 550 aircraft with symmetric foldable



Fig.4 Geometric model of Cessna 550

wing tips. Fig.4 shows the geometric configuration of the folding wing tip aircraft specified by the rudders, flaps, elevators, ailerons and the folding part marked, as well as the folding angle of the symmetric wing tip denoted by δ . The hinge is the SMA wires which will be heated or cooled to rotate the wing tip, instead of the mechanical hinge structure. Note that the SMA actuators dynamics are not considered in this paper, and are mentioned just to be consistent with NASA SAW aircraft.

Since the aircraft and the folding motion are symmetric, the lateral stability of the folding wing tip aircraft can be guaranteed. Hence, in this paper, since only the longitudinal dynamics of the aircraft are considered, we choose the elevator deflection δ_e and the engine opening δ_T as the input signal of this control system. Further, for enhancing the aircraft aerodynamic performances and for the control purposes, the folding angle δ may be taken as an auxiliary maneuver and added into the control vector, i.e., $u = [\delta_e, \delta_T, \delta]$, which will be discussed in Section 2.

1.2 Aerodynamic parameter acquisition

Aerodynamic parameters of morphing aircraft include aerodynamic coefficients and aerodynamic derivatives, are the basis of aircraft dynamics model. Therefore, obtaining aerodynamic parameters under different folding angles of wing tips is the premise of establishing aircraft mathematical model. In this paper, the computational fluid dynamics (CFD) method is used to conduct aerodynamic simulation of the wing tip of the aircraft at different folding angles.

CFD is an emerging interdisciplinary discipline based on classical fluid mechanics, numerical calculation methods and computer science. It is used to quantitatively describe the numerical solution of flow field distribution on space and time scale, and aimed to solve some problems that are difficult or impossible to conduct in wind tunnel.

Remark 2 In the preliminary study stage, we only obtain the basic overall layout parameters of the aircraft, and it is difficult to conduct wind tunnel test to obtain the aerodynamic data and derivatives which are necessary for the construction of flight dynamics model. Then, we compare two aerodynamic simulation methods: CFD and data compendium (DATCOM), the calculation results of these two methods have a high coincidence, and the errors meet the requirements of engineering estimation accuracy. Especially, with the development of CFD, it is a common research method to solve aerodynamic coefficients of aircraft with high availability based on CFD method, and the calculation accuracy is higher than other aerodynamic simulation software to some extent. As mentioned in Ref.[33], one of the most important applications of CFD is the conceptual design and early analysis and design stage of aircraft research and development. In order to improve the accuracy of aerodynamic simulation analysis, the near wall mesh is encrypted. And for facilitating simulation analysis, this paper closes the engine runner path and simplifies the overall shape of the Cessna 550.

The following steps are given for aerodynamic solution using CFD:

(1) A 3D model of the aircraft(Fig.5) wing is built. The folding angle of aircraft wingtip is taken as 30° for example, and the three-dimensional model of aircraft wing is established in SoildWorks.

(2) The aerofoil grid model is established. The3D wing model built in SolidWorks is imported into



Fig.5 3D model of aircraft wing in Solidworks

Ansys software. After Boolean operation, "inlet" surface is set in the same direction as the incoming flow, and other surfaces are set as "outlet". In Workbench 18.0, the unstructural mesh division of folded wingtip morphing aircraft is carried out. The boundary layer is refined by using the inflation method, and the mesh density at the boundary is finer than that at other places, and the ". mesh " file is generated at last. The generated aircraft wing grid model is shown in Fig.6.



Fig.6 Aircraft wing grid model generated in Ansys

(3) The aerodynamic calculation of the 3D model of the wingtip is carried out. The Laminar flow model suitable for solving viscous fluid problems is selected as the SST model among the two path models, and the incoming flow velocity is set for different angles of attack.

Considering that the equations of incompressible flow and moderately compressible flow are solved and more stable, the SIMPLE algorithm is selected to redefine the lift coefficient, drag coefficient and moment coefficient. The convergence value of lift coefficient, drag coefficient and moment coefficient of each angle of attack at a certain folding angle can be obtained through multiple numerical calculations. To ensure the accuracy of aerodynamic simulation and the approximation of aerodynamic coefficients is good, the near-wall mesh is encrypted. The y^+ value of the body fitting grid is controlled at about 30. The growth ratio from the wall grid to the outside is controlled at about 1.2, and the whole machine adopts unstructured grid. Seven different folding angles of the grid are total of 28 million. In this simulation, the time term is fully implicit based on Newton's method. In terms of spatial discretization, Piecewise linear method is used for variable reconstruction, Roe-FDS difference scheme and Venkatakrishnan limitter are used for convection terms, and SST model is used for turbulence model.

1.3 Quasi-steady hypothesis

Generally, the structural changes in the chord camber, the relative airfoil thickness and twist, the wing span, and the sweep angle, more or less, will all lead to changes in the aerodynamic performances of the morphing aircraft. Therefore, morphing is an unsteady dynamic process for aircraft flight control systems, with both structural and aerodynamic properties varying in different aircraft configurations. Hence, how to deal with such problems remains a significant problem in the morphing aircraft research.

Some existing works^[6,34] have proved that the configurations of morphing aircraft change slowly, such that the unsteady dynamic morphing process influences the aerodynamic parameters of the aircraft weakly. Therefore, the morphing process of the aircraft can be considered as a quasi-steady process, and the morphing aircraft under different folding angles during the folding process can be seen as a series of the conventional fixed-wing aircraft with different configurations.

2 Aerodynamic Analysis and Modeling

Based on the aerodynamic parameters of the aircraft, the aerodynamic analysis is conducted for the morphing aircraft under different folding angles. The aerodynamic forces and moments of the aircraft vary with the geometrical shape, flight attitude, speed, and atmospheric density of the aircraft. It is the basis to analyze the aircraft flight performance. Moreover, a dynamic modeling method is proposed for the folding wing tip aircraft to formulate a nonlinear model about the folding angle, which transforms the changes of the aerodynamic parameters into the functions of the folding angle.

2.1 Aerodynamic analysis of the morphing aircraft

This section focuses on the aerodynamic analysis of the folding wing tip morphing aircraft, to seek the best morphing strategy for the aircraft under different flight conditions. Different from those of the conventional aircraft, the morphing aircraft's, large motions of the wing segments make the rigid body approximation of the aircraft inadequate, and both the momentum of inertia and center of mass are functions of time, that is, the conventional rigid body motion equations will not be applicable.

However, the influence of the morphing motion will be reflected in the change of aerodynamic coefficient and force, that is, the aerodynamic coefficient and force will be the function of morphing parameters. In this paper, the origin of the body coordinates $o_b x_b y_b z_b$ is set at the center of mass of the morphing aircraft in the wing-extended configuration. Fig. 7 gives the definition of the body coordinates $o_b x_b y_b z_b$ and the ground coordinates $o_g x_g y_g z_g$. tained and calculated by CFD. It should be noted that the morphing process is dynamic and unsteady, while the existing literature has proved that for the morphing aircraft with a large scale and slow deformation, the calculation deviations of the aerodynamic parameters under such an unsteady process are small in the mean and can be ignored. Therefore, the morphing process of the aircraft can be regarded as a number of continuous quasi-steady morphing states. Under any static quasi-steady state, the morphing aircraft can be regarded as a conventional aircraft with the corresponding configurations (wing tip upwards at a certain angle) and the aerodynamic parameters of the aircraft at each operating point can be calculated by CFD.

We define the flight height $H = 2\,000$ m and Mach number Ma = 0.4, with the angle of attack $\alpha \in [-4^\circ, 8^\circ]$ and the folding angles $\delta \in [0^\circ, 60^\circ]$. We establish the modified Cessna 550 aircraft model in CFD with the wing tip folding at several folding angles and calculate corresponding aerodynamic parameters. The lift, drag, pitch moment coefficients and the polar curve are given in Figs.8—11 and the aerodynamic coefficients under $\alpha = 6^\circ$ are in Fig.12.



Fig.7 Coordinate axis definition



The original data of the aerodynamic parameters of the morphing aircraft in this paper are ob-





Fig.9 Pitch moment coefficient $C_{\rm m}$ under different folding angles



Fig.10 Drag coefficient $C_{\rm D}$ under different folding angles



Fig.11 Polar curves under different folding angles



Fig.12 Aerodynamic coefficients with respect to δ under $\alpha = 6^{\circ}$

Remark 3 In order to explore the aerodynamic characteristics of morphing aircraft with different wingtip folding angles within the range of large envelope, we conduct aerodynamic simulation on the whole aircraft with different altitudes, flight Mach numbers and wingtip folding angles, and obtain the lift coefficient, drag coefficient and pitching moment coefficient. In addition, the angle of attack range of the aircraft is selected as small angle of attack. When the folding angle of the wing is 0°, the aircraft is a conventional fixed-wing aircraft configuration with flat wing. Due to space limitation, lift coefficient, drag coefficient and pitching moment coefficient generated by the wingtip with the flight condition of flight height H = 2000 m, and Mach number Ma=0.4, with angle of attack $\alpha \in [-4^\circ, 8^\circ]$ and the folding angles $\delta \in [0^\circ, 60^\circ]$ are given in this paper for analysis. In order to reduce the amount of CFD simulation calculation, half mode was used for grid division and simulation calculation. In order to improve the analysis accuracy of aerodynamic simulation, the near-wall mesh is encrypted. Then, based on such accuracy of aerodynamic simulation, we give the performance analysis of the folding wing tip aircraft during the whole morphing process to evaluate how the folding motion will influence the system states from the perspective of taking off, landing and maneuvering.

Moreover, from the polar curve in Fig.11, we can see the lift-to-drag ratio of the aircraft has the maximum value under the folding angle $\delta = 0^{\circ}$, and the lift-to-drag ratio gradually decreases as the folding angle increases. Therefore, compared with the traditional fixed-wing aircraft, the morphing aircraft has different aerodynamic characteristics during the cruise phase. It can reduce the lift-drag ratio by folding the wing tip to obtain a higher sprint speed to adapt to various needs and complete tasks that the traditional aircraft cannot complete.

To specify the aerodynamic parameters $L(\delta)$, $D(\delta)$ and $M(\delta)$ in the nonlinear model, the specific expression of aerodynamic coefficients $C_{\rm L}$, $C_{\rm D}$ and $C_{\rm m}$ about the folding angle δ can be expressed by

$$\begin{cases} L(\delta) = QS_{w}C_{L}(\alpha, \delta_{e}, q, \delta) \\ D(\delta) = QS_{w}C_{D}(\alpha, \delta) \\ M(\delta) = QS_{w}c_{A}C_{m}(\alpha, \delta_{e}, q, \delta) \end{cases}$$
(1)

where aerodynamic coefficients $C_{\rm L}$, $C_{\rm D}$ and $C_{\rm m}$ are

$$\begin{cases} C_{\mathrm{L}}(\alpha, \delta_{\mathrm{e}}, q, \delta) = C_{\mathrm{L}_{a=0}} + C_{\mathrm{L}_{a}}\alpha + C_{\mathrm{L}_{a}}\delta_{\mathrm{e}} + C_{\mathrm{L}_{q}}q \\ C_{\mathrm{D}}(\alpha, \delta) = C_{\mathrm{D}_{a=0}} + C_{\mathrm{D}_{a}}\alpha + C_{\mathrm{D}_{a'}}\alpha^{2} \\ C_{\mathrm{m}}(\alpha, \delta_{\mathrm{e}}, q, \delta) = C_{\mathrm{m}_{a=0}} + C_{\mathrm{m}_{a}}\alpha + C_{\mathrm{m}_{a'}}\delta_{\mathrm{e}} + C_{\mathrm{m}_{q}}q \end{cases}$$

$$(2)$$

It can be seen in Eq.(2) that the aerodynamic coefficients are not explicit functions of the folding angle δ . However, the basic coefficients are all explicit functions of δ . To develop the explicit expressions of the aerodynamic parameters above, aerodynamic analysis with CFD will be presented to obtain an adequate relationship between the aerodynamic coefficients and the morphing term δ . Fig.13 presents the $C_{L_{a=0}}$, $C_{D_{a=0}}$ and $C_{m_{a=0}}$ with regard to the folding angle δ through curve fitting.



 $[C_{L_{a=0}}, C_{D_{a=0}}, C_{m_{a=0}}, C_{L_a}, C_{L_{a_*}}, C_{L_q}, C_{D_a}, C_{D_{a^2}}, C_{m_a}, C_{m_{a_*}}, C_{m_q}]$ where $c_{i1}(i=1, 2, \dots, 10)$ and $c_{i2}, (i=1, 2, \dots, 10)$ denote some constant terms, respectively, for example, $c_{01} = -0.0012195$ and $c_{02} = 0.6106$. Based on this, substituting Eqs.(3, 4) into Eqs.(1, 2), we can get

$$\begin{bmatrix} L(\delta) = QS_{w} [c_{01}\delta + c_{02} + (c_{31}\delta + c_{32})\alpha + (c_{41}\delta + c_{42})\delta_{e} + (c_{51}\delta + c_{52})q] \\ D(\delta) = QS_{w} [c_{11}\delta + c_{12} + (c_{61}\delta + c_{62})\alpha + (c_{71}\delta + c_{72})\alpha^{2}] \\ M(\delta) = QS_{w}c_{A} [c_{21}\delta + c_{22} + (c_{81}\delta + c_{82})\alpha + (c_{91}\delta + c_{92})\delta_{e} + (c_{101}\delta + c_{102})q] \end{bmatrix}$$
(5)

where Q and S_w denote the dynamic pressure and the reference area of the wing span, respectively; δ_e is the elevator deflection angle; α the angle of attack; and q the pitch angular rate. Next, based on the equations above, we will give the nonlinear models with regard to the folding angle δ of the morphing aircraft.

(3) Mathematical modeling

(2)Aerodynamic parameters specification

It can be seen from Fig.13 that it has great performance in both the fitting error and accuracy, which implies that $C_{L_{a=0}}$, $C_{D_{a=0}}$ and $C_{m_{a=0}}$ have a strict linear relationship with the folding angle δ . Based on the curve fitting, there exists

$$\begin{cases} C_{\text{L}_{s=0}} = -0.002 \, 195\delta + 0.610 \, 6\\ C_{\text{D}_{s=0}} = -0.000 \, 057 \, 07\delta + 0.033 \, 91 \\ C_{\text{m}_{s=0}} = 0.000 \, 746 \, 1\delta + 0.095 \, 43 \end{cases}$$
(3)

Similarly, we can get aerodynamic coefficients $C_{\rm L}$, $C_{\rm D}$ and $C_{\rm m}$ as

$$\begin{cases} C_{\text{L}_{a}} = 0.093\ 08, C_{\text{L}_{a}} = 0.010\ 12\\ C_{\text{L}_{q}} = -0.000\ 258\ 9\delta + 0.151\ 5\\ C_{\text{D}_{a}} = -6.889\text{E} - 07\delta + 0.004\ 558\\ C_{\text{D}_{a}} = 1.993\text{E} - 07\delta + 0.000\ 222\ 7\\ C_{\text{m}_{a}} = 7.325\text{E} - 05\delta - 0.021\ 33\\ C_{\text{m}_{a}} = -0.027\ 61\delta - 0.012\ 69\\ C_{\text{m}} = 4.357\text{E} - 05\delta - 0.291 \end{cases}$$
(4)

To simplify the expression of aerodynamic parameters mentioned above, some symbols are employed to replace the numeric expressions and they are expressed as

$$]^{\mathrm{T}} = [c_{01}, c_{11}, c_{21}, \cdots, c_{101}]^{\mathrm{T}} \delta + [c_{02}, c_{12}, c_{22}, \cdots, c_{101}]^{\mathrm{T}}$$

Based on the coordinate definition, similar to the modeling methodology of the conventional aircraft, the longitudinal nonlinear model of the morphing aircraft within the body frame $o_b x_b y_b z_b$ is

$$\begin{cases} \dot{V} = \frac{1}{m} T \cos \alpha - \frac{1}{m} D(\delta) - g \sin(\theta - \alpha) \\ \dot{\alpha} = -\frac{1}{mV} T \sin \alpha - \frac{1}{mV} L(\delta) + q + \\ \frac{1}{V} g \cos(\theta - \alpha) \\ \dot{\theta} = q \\ \dot{q} = \frac{1}{I_y} M(\delta) \\ \dot{h} = V \sin(\theta - \alpha) \end{cases}$$
(6)

where *m* is the mass of the aircraft; I_y , respectively the momentum of inertia; *V* and *h* are the airspeed and the flight height of the aircraft, respectively; α and θ are the angle of attack and the pitch angle, respectively; *q* is the pitching angular rate; and *T* the thrust and expressed by $T = T_{\delta t} \delta_t$. It should be noted that the aircraft engine offset angle is $\alpha_T = \beta_T = 0^\circ$.

Different from the mathematical model of the conventional fixed-wing aircraft, some parameters in Eq.(6), like the lift force *L*, drag force *D*, and pitching moment *M*, are not only related to the angle of attack α , elevator deflection angle δ_e or pitching angular rate *q*, but also functions of δ .

2.2 Longitudinal nonlinear models

Based on the above quasi-steady hypothesis, we know that the folding angle of the symmetric wing tip has a tremendous influence on the aerodynamic performances of the morphing aircraft. Moreover, the dynamic and unsteady folding process is taken as a set of conventional fixed-wing aircraft with different configurations. From the control perspective, we choose the folding angle as one system input, i.e., an auxiliary maneuver actuator. The longitudinal nonlinear model can be written as

$$\begin{cases} \dot{\boldsymbol{x}} = f(\boldsymbol{x}) + g(\boldsymbol{x})\boldsymbol{u} \\ \boldsymbol{y} = h(\boldsymbol{x}) \end{cases}$$
(7)

where $x = [V, \alpha, \theta, q, h]^{T}$ is the system state vector, and $u = [\delta_{e}, \delta_{T}, \delta]^{T}$ the control input f(x) and g(x) are nonlinear functions of the system state vector

$$f(\mathbf{x}) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{bmatrix}, g(\mathbf{x}) = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \\ g_{41} & g_{42} & g_{43} \\ g_{51} & g_{52} & g_{53} \end{bmatrix}$$

where

$$f_{1} = -\frac{\rho V^{2} S_{w}}{2m} (c_{12} + c_{62} \alpha + c_{72} \alpha^{2}) - g \sin(\theta - \alpha)$$

$$f_{2} = -\frac{\rho V^{2} S_{w}}{2m V} (c_{02} + c_{32} \alpha + c_{52} q) + q + \frac{g}{V} \cos(\theta - \alpha)$$

$$f_{3} = q, f_{4} = \frac{\rho V^{2} S_{w} C_{A}}{I_{y}} (c_{22} + c_{82} \alpha + c_{102} q)$$

$$f_{5} = V \sin(\theta - \alpha)$$

$$g_{11} = 0, g_{12} = \frac{\cos \alpha}{m} T_{\delta_{T}}$$

$$g_{13} = -\frac{Q S_{w}}{m} (c_{11} + c_{61} \alpha + c_{71} \alpha^{2})$$

$$g_{21} = -\frac{\rho V^2 S_w}{2mV} C_{L_{s_i}}, g_{22} = -\frac{\sin \alpha}{mV} T_{\delta_T}$$

$$g_{23} = -\frac{\rho V S_w}{2m} (c_{01} + c_{31}\alpha + c_{51}q)$$

$$g_{31} = g_{32} = g_{33} = 0$$

$$g_{41} = \frac{\rho V^2 S_w}{2I_y} C_{m_{s_i}}, g_{42} = 0$$

$$g_{43} = \frac{\rho V^2 S_w c_A}{2I_y} (c_{21} + c_{81}\alpha + c_{101}q)$$

$$g_{51} = g_{52} = g_{53} = 0$$

Compared with the conventional aircraft, the variable of the wing folding angle is added to the model of the nonlinear dynamics of the morphing aircraft described in Eq.(6), which describes the influence of the changes in aerodynamic characteristics caused by the wing folding on flight motion law of the aircraft during flight. Analysis of its influence can further affect the deformation strategy of folding. The essential characteristics of the morphing aircraft are to study the coupling effect of deformation and flight motion and to lay a theoretical foundation for the subsequent control research of variant aircraft.

3 Performance Analysis

In this section, based on the nonlinear model presented above, we give the dynamic responses of the aircraft with the fixed rudder and throttle. Moreover, with aerodynamic analysis in Section 2, we give the performance analysis of the folding wing tip aircraft during the whole morphing process to evaluate how the folding motion will influence the system states from the perspective of taking off, landing and maneuvering.

3.1 Longitudinal dynamic responses with fixed elevator and throttle

To evaluate how the folding wing tip motion influences the longitudinal system dynamics, we first calculate the system dynamic responses with the fixed elevator and throttle, i.e., keeping the elevator deflection angle and throttle opening constant. The initial state x_0 is set as $x_0 = [200, 4, 0, 0, 2000]^T$, and the folding angles of the wing tips fold from 0° to 60° within 60 s in a uniform speed. Fig.14 shows



the system dynamic responses of the aircraft during the folding process.

After the aircraft begins to fold, there is a large decrease in the angle of attack α , which is caused by the shift of the center of mass. With the wing tip folding up, the mass center of the aircraft is not coincident with the body frame origin, and it moves towards the negative direction of the x-axis but the lift force L does not encounter a significant decrease. Therefore, the aircraft will dive, resulting the decrease of both angle of attack α and the altitude h. Moreover, the decrease in the angle of attack α will result in the gradual decrease of the downward force moment. However, when the upward force moment produced by the rudder deflection exceeds the downward force moment of the whole aircraft, and the total moment of the aircraft becomes positive, leading to the pitch angle positive, and the speed V decreases.

In conclusion, it is obvious that both the speed V and altitude h change violently during the folding process of the wing tip and need a long time to restore, which will possibly lead to destructive disasters. Hence, with the fixed elevator and throttle, the folding wing tip will make the whole aircraft unstable and an effective flight control system is required to guarantee a good flight quality in the process of wing tip folding.

3.2 Taking off performances

The taking-off process refers to the period that the aircraft takes off and rises to a certain altitude. The taking-off performance of the aircraft is evaluated by the taking-off distances and the taking-off time. Therefore, the taking-off time and distance are calculated for the folding wing tip aircraft under different folding angles of symmetric wing tips to evaluate the taking-off performance of the folding wing tip aircraft.

The taking-off distance d and taking-off time t can be divided into two parts: (1) The taxiing distance d_1 and time t_1 before taking-off, (2) the rising distance d_2 and time t_2 , and there exists $d = d_1 + d_2, t = t_1 + t_2$.

(1) Taxiing distance and time

The equations of motion of the taxiing period can be given as

$$\begin{cases} m \frac{\mathrm{d}V}{\mathrm{d}t} = T_{\mathrm{a}} - D - F \\ N = G - L \end{cases}$$
(8)

where *m* is the mass and *G* the gravity of the aircraft; T_a and *D* are the propulsive and the drag force, respectively; *N* is the supporting force and F = fN the friction with the friction coefficient *f*.

With $L = 0.5 \rho V^2 S_w C_L$ and $D = 0.5 \rho V^2 S_w C_D$, the equation above can be transformed as

$$\frac{1}{g}\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{T_{\mathrm{a}}}{G} - f - \frac{\rho V^2 S_{\mathrm{w}}}{2G} \left(C_{\mathrm{D}} - fC_{\mathrm{L}}\right) \quad (9)$$

where $C_{\rm L}$ and $C_{\rm D}$ are the lift and the drag coefficient, respectively; ρ is the air density; and $S_{\rm w}$ the reference area of the wing span. The liftoff speed $V_{\rm lof}$ can be written as

$$V_{\rm lof} = \sqrt{\frac{2G}{\rho S_{\rm w} C_{\rm L_{\rm lof}}}} \tag{10}$$

where $C_{L_{\text{LM}}}$ is the liftoff lift coefficient and can be generally calculated by $C_{L_{\text{LM}}} = (0.8 - 0.9) C_{L_{\text{MMX}}}$. Based on this, the taxiing distance d_1 and time t_1 are given as

$$d_{1} = \frac{1}{g} \int_{0}^{V_{\rm lot}} \frac{V \mathrm{d}V}{\frac{T_{\rm a}}{G} - f - \frac{\rho V^{2} S_{\rm w}}{2G} (C_{\rm D} - fC_{\rm L})}$$
(11)
$$t_{1} = \frac{1}{g} \int_{0}^{V_{\rm lot}} \frac{\mathrm{d}V}{\frac{T_{\rm a}}{G} - f - \frac{\rho V^{2} S_{\rm w}}{2G} (C_{\rm D} - fC_{\rm L})}$$
(12)

(2) Rising distance and time

The trajectory of the morphing aircraft in the rising period can be seen as a straight line. Based on the law of the conservation of energy, there exists

$$\frac{G}{2g}V_{\rm H}^2 + 15G = \frac{G}{2g}V_{\rm lof}^2 + (T_{\rm a} - D)d_2 \quad (13)$$

where $V_{\rm H}$ is the velocity when the aircraft rises to a certain altitude and can be denoted by $V_{\rm H} = 1.3 V_{\rm lof}$. Therefore, the rising distance d_2 and time t_2 can be given as

$$d_{2} = \frac{G}{T_{a} - D} \left(\frac{V_{H}^{2} - V_{lof}^{2}}{2g} + 15 \right)$$
(14)

$$t_2 = \frac{d_2}{V_{\rm av}} \tag{15}$$

where $V_{av} = 0.5(V_{lof} + V_{H})$. Thus, Figs.15—18 present the taking-off time $t = t_1 + t_2$ and distance $d = d_1 + d_2$ under the folding angle δ .



Fig.15 Liftoff velocity $V_{L_{\rm lof}}$ under different folding angles



Fig.16 Taking off time t under different folding angles



Fig.17 Liftoff lift coefficient under different folding angles



Fig.18 Taking off distance d under different folding angles

(3) Taking-off performance analysis

Figs.15—18 show the taking-off properties of the folding wing tip aircraft. We can see that with the folding angles increasing, the liftoff lift coefficient $C_{L_{bd}}$ decreases and the liftoff velocity $V_{L_{bd}}$ increases, which means it takes more time and longer distance for the aircraft to take off. Hence, it can be obtained that the larger folding angles of the symmetric wing tip will lead to an increase in taking off time and distance, i.e., leading to the worse takingoff performances. Therefore, with the wing tip keeping level, i.e., $\delta = 0^{\circ}$, the best taking-off performance can be achieved.

In summary, for the folding wing tip aircraft, in the taking-off phase, keeping the symmetric wing tip level can achieve the best taking off performances instead of keeping wing tip folding, with less taking off time and shorter distance.

3.3 Maneuvering performances

Low altitude penetration performance is vital for aircraft with variable configurations. For morphing aircraft, the maneuvering performances are evaluated by the maximum level speed $V_{\rm max}$ and the level flight acceleration time $t_{\rm a}$. Based on this, we will calculate $V_{\rm max}$ and $t_{\rm a}$ to find out how the folding angles of the symmetric wing tip influence the maneuvering performance of the aircraft.

The maximum level speed $V_{\rm max}$ can be calculated by

$$V_{\rm max} = \sqrt{\frac{2T_{\rm a}}{C_{\rm D}\rho S}} \tag{16}$$

and the level flight acceleration time t_a refers to the

accelerating and decelerating time from one level speed to another, and we employ this index to evaluate the maneuvering performance of the aircraft. In this paper, we consider that the morphing aircraft is subsonic and the Mach number is set as Ma = 0.4. Hence, the accelerating time denotes the time that the aircraft takes to accelerate from $0.7V_{\text{max}}$ to $0.97V_{\text{max}}$, and similarly, the decelerating time denotes the time that the aircraft takes to decelerate from V_{max} to $0.7 V_{\text{max}}$. The aircraft motion equations during the level straight flight process can be written as

$$\begin{cases} \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{T-D}{m} = \frac{1}{m}\Delta T \\ L = G \end{cases}$$
(17)

Integrating the equation above becomes

$$t = m \int_{V_0}^{V_1} \frac{\mathrm{d}V}{\Delta T} \tag{18}$$

where V_0 is the initial level speed; V_1 the final level speed after accelerating or decelerating process. Based on Eqs.(16, 18), the maximum level velocity V_{max} is shown in Fig.17 and the accelerating time *t* from V_0 to V_1 is given in Fig.18.

In Fig.19, with the symmetric wing tip folding, the maximum level velocity gradually increases from 348 m/s to 368 m/s, which is resulted from the decrease of the aspect ratio and the wing span area. Thus, this leads to the decrease of the drag force D with constant propulsive force T, and the maximum level velocity will increase. Bigger maximum velocity under bigger folding angles will benefit the maneuvering properties of the aircraft.



Fig.19 Maximum level velocity V_{max}

However, in Fig. 20, we can see that with the wing tip folding, the accelerating time increases, i. e., it takes a longer time to accelerate. Yet, it cannot be derived that the folding wing tip aircraft has worse maneuvering performances because with wing tip folding, the maximum level speed V_{max} will also increase, leading to the changes in V_0 and V_1 . Therefore, it is not adequate to evaluate the maneuvering performances of the aircraft just from the accelerating time *t*. To get rid of the effects of V_{max} , we adopt another index: The accelerating time from 0.4 Mach to 0.6 Mach evaluate the maneuvering performances. Therefore, based on Eq.(18), the accelerating time from 0.4 Mach to 0.4 Mach to 0.6 Mach to 0.6 Mach is presented in Fig.21.







Fig.21 Accelerating time t from 0.4 Mach to 0.6 Mach

Different from the results in Fig.18, results in Fig.21 show that it will take shorter time to accelerate from 0.4 Mach to 0.6 Mach under a bigger folding angle, which exhibits a better maneuvering performance. Results in Fig.18 are not adequate in evaluating the maneuvering performances because it could be influenced by varying $V_{\rm max}$, which is vali-

dated in Fig.19. Moreover, we can see in Eq.(18) that the accelerating time is correlated with the varying ΔT . Based on the study in Section 3, the folding wing tip will lead to the decrease of the drag force D, with $\Delta T = T - D$, and we can see ΔT will increase and therefore the accelerating time will decrease with the wing tip folding.

In summary, for the folding wing tip aircraft, folding wing tip can enhance maneuvering performances, which makes it suitable in some specific missions. When maneuvering, the symmetric wing tips can fold to a certain configuration, such that the aircraft takes a shorter distance and less time to dive, which is superior to the conventional fixedwing aircraft.

3.4 Landing performances

In this subsection, we evaluate the landing properties under different folding angles of the wing tip to find the best landing strategy to obtain the best aerodynamic performances. The landing stage is divided into two stages: The deceleration stage and the taxiing stage. Similar to the taking-off period, the landing performances are evaluated by the landing time t_1 and the landing distances d_1 . The landing time and distance are

$$t_{\rm l} = t_{\rm d} + t_{\rm t}, d_{\rm l} = d_{\rm d} + d_{\rm t}$$

where t_d and d_d refer to the time and distance during the aircraft deceleration stage; t_t and d_t the time and distance during the taxiing stage. They can be denoted by

$$d_{\rm d} = \frac{L}{D_{\rm ad}} \left(\frac{V_{\rm H}^2 - V_{\rm ad}^2}{2g} + 15 \right) \approx K_{\rm ad} \left(\frac{V_{\rm H}^2 - V_{\rm ad}^2}{2g} + 15 \right)$$
(19)

$$t_{\rm d} = \frac{d_{\rm d}}{V_{\rm ad}} \tag{20}$$

and

$$t_{\rm t} = \frac{1}{g} \int_{0}^{V_{\rm o}} \frac{\mathrm{d}V}{\frac{T_{\rm a}}{G} - f - \frac{\rho V^2 S}{2G} (C_{\rm D} - fC_{\rm L})} \quad (21)$$

$$d_{t} = \frac{1}{g} \int_{0}^{V_{o}} \frac{V dV}{\frac{T_{a}}{G} - f - \frac{\rho V^{2}S}{2G} (C_{D} - fC_{L})}$$
(22)

where $D_{\rm ad}$ is the average drag force during the deceleration stage; $V_{\rm H}$ the instantaneous speed at the

safe altitude and there exists $V_{\rm H} = 1.3 V_{\rm td}$, here $V_{\rm td}$ represents the instantaneous landing speed and $V_{\rm td} = K_1 \sqrt{2W/(\rho SC_{\rm Ltd})}$, with $C_{\rm Ltd}$ being the lift coefficient when landing and K_1 being the correction coefficient. $V_{\rm ad}$ is the average speed during the deceleration stage and $V_{\rm ad} = 0.5(V_{\rm H} + V_{\rm td})$. $K_{\rm ad}$ is the mean lift-drag ratio. Assuming the friction coefficient f = 0.35 when braking, Figs. 22, 23 present the landing distance d_1 and landing time t_1 under different folding angles of the wing tip.



Fig.22 Landing distance under different folding angles



Fig.23 Landing time t_1 under different folding angles

Figs.22, 23 show that both the landing distance and time increase with the folding angle δ . As the wing tip folds, the reference area of the wing span decreases and thus leads to the decrease of the lift coefficient when landing. Hence, the instantaneous landing speed $V_{\rm td}$ increases due to $V_{\rm td} = K_1 \sqrt{2W/(\rho SC_{\rm Ltd})}$. In general, similar to taking-off case, with wing tip keeping plain, the best landing performance can be achieved, in a shorter distance and quicker time.

3.5 Longitudinal steady stability analysis

In this subsection, the longitudinal steady sta-

bility of the morphing aircrafts is analyzed to evaluate the ability to restore from the disturbed states to the steady states. For morphing aircraft, the folding motion of the symmetric wing tip will result in the CM shift along the x-axis of the body frame and the changes in both the wing span and the reference wing span area. Therefore, the stability issues of the whole aircraft should be considered during folding process of the wing tip.

Longitudinal stability refers to the ability to recover from the angle of attack disturbances, i.e., if the aircraft encounters an angle of attack perturbation $\Delta \alpha > 0$, the aircraft can generate a bow pitching moment $\Delta C_{\rm m}$, which satisfies $\Delta C_{\rm m} < 0$. Hence, for an aircraft with longitudinal steady stability, the following condition can be guaranteed

$$C_{\mathrm{m}_{s}} = \frac{\partial C_{\mathrm{m}}}{\partial \alpha} = \frac{\partial C_{\mathrm{m}}}{\partial C_{\mathrm{L}}} \cdot \frac{\partial C_{\mathrm{L}}}{\partial \alpha} = C_{\mathrm{L}_{s}} \cdot \frac{\partial C_{\mathrm{m}}}{\partial C_{\mathrm{L}}} < 0 \quad (23)$$

Based on the aerodynamic coefficients presented in Section 2, $C_{L_s} = 0.09308$, we can evaluate the steady stability through the sign of the term $\partial C_m / \partial C_L$. Therefore, the lift coefficient C_L with respect to the pitching moment coefficient C_m under different folding angles is given in Fig.24.



Fig.24 Longitudinal steady stability under different folding angles

It can be derived from Fig.24 that $\partial C_{\rm m}/\partial C_{\rm L} < 0$, i.e., the longitudinal steady stability of the morphing aircraft can be guaranteed. Moreover, with the folding angles of the wing tip increasing, the longitudinal steady stability will get better, which is resulted from the back-shift of the aerodynamic center (AC), as shown in Fig.25.

We should note that there exists another way to evaluate the steady stability of the aircraft. When



Fig.25 Airfoil aerodynamic center and CM

 $x_{\rm AC} > x_{\rm CM}$, the morphing aircraft is longitudinal steady stable. In Fig.24, when $\Delta \alpha > 0$, the increment of the lift force ΔL acting on the AC will generate a bow pitch moment, i.e., $\Delta M < 0$ and make the angle of attack decrease to maintain the stability of the whole aircraft. With the symmetric wing tip folding, the AC will shift back and thus the $x_{\rm AC}$ will increase to achieve a better longitudinal steady stability.

One of the main contributors to the pitching moment of the aircraft around the center of mass is the wing. The wing pitching moment is mainly generated by the lift force of the aerodynamic center. For steady stability, any change in the angle of attack must produce a moment that prevents the change, that is, the derivative of the pitching moment with respect to the angle of attack must be negative. Therefore, to obtain the ideal longitudinal steady stability, the aircraft wing can be folded up and down to achieve the purpose of changing the position of the aerodynamic center of the aircraft.

The above aerodynamic analysis and modeling of the morphing aircraft are important for establishing an accurate and efficient model to represent the dynamic characteristics of the morphing aircraft, which play key roles in studying control designs to achieve some desired system performances.

Remark 4 For the performance analysis, we first analyze the uncontrolled response of the longitudinal morphing aircraft dynamics under the given wing tip folding process (in Section 2.1). By analyzing the response of the folding wing tip morphing aircraft in the uncontrolled state, the influence of the folding wing tip process on the state and stability of the aircraft can be evaluated through the change of

the system state in some sense. Then, in order to analyze the global system performance of the folding wing tip morphing aircraft from taking-off to landing, based on the characteristics of this aircraft, the variation range of the wing tip folding angle is $\delta \in [0^{\circ}, 60^{\circ}]$, and the height variation range is $H \in [0, 2000 \text{ m}]$. The wing tip folding angle varies according to the speed requirements. Under the condition of increasing flight speed, the wingtip folding angle also increases, so as to verify the performance of the global system. Since the process of the morphing is complex and the change of wing tip folding angle has a great impact on the dynamic performance of the aircraft, we divide the whole flight process of the aircraft into three stages: Taking-off, maneuvering and landing, analyze the aerodynamic performance of the folding wing tip morphing aircraft in these three stages, and select different wing tip folding angles in different flight stages. To ensure the optimal aerodynamic performance of the aircraft at each flight stage, the longitudinal stability analysis of the aircraft is given from the perspective of the aircraft airfoil. Finally, it can be seen that the uncontrolled response analysis of the folding wingtip morphing aircraft is carried out at the equilibrium point, but its motion process is divided into different stages, and different performance indexes are selected to evaluate the aerodynamic performance of the system under different flight states when analyzing the overall performance of the aircraft, so as to obtain the optimal morphing strategy under different flight states.

4 Conclusions

Based on the geometry configuration of the modified Cessna 550 aircraft, a kind of folding wing tip morphing aircraft, a time-varying nonlinear dynamic model including the deformation characteristics of wing tip structure is established by CFD. It represents the longitudinal dynamics of the aircraft with the folding wing tips.

CFD has been employed to build the aircraft models and obtain the aerodynamic coefficients under different folding angles of the wing tip. Then the taking-off performance, maneuvering performance, and landing performance under different folding angles are analyzed, which lay a foundation for control of the morphing aircraft. The morphing aircraft dynamics are complex (e.g., strong nonlinearity, strong uncertainties, and strong coupling), especially when the morphing aircraft is in maneuvering flight, or when the morphing aircraft undergoes uncertain system faults or external disturbances. Moreover, the deformation of the wing tips may lead the aerodynamic characteristics of the morphing aircraft to completely change, not only including the variation of the system parameters but also the dynamic mutations.

The longitudinal modeling and the flight control method are two key areas of research with respect to morphing aircraft. This paper is the basic part of the research on folding wingtip morphing aircraft. In future, it is necessary to carry out stable tracking control on this model. We will focus on adaptive control of morphing aircraft with dynamic mutations due to wingtip folding motion, including adaptive fault-tolerant control and adaptive disturbance rejection of the morphing aircraft. For this topic, many research results have been done^[35-42], and it is helpful to study control of morphing aircrafts.

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Author contributions Dr. WEN Liyan designed the study, investigated related literature, conducted the analysis, interpreted the results and wrote the manuscript. Ms. REN Qiuping drew the image, complied the models and contributed to data collection and the analysis of the Cessna 550 aircraft. Mr. YANG Jie contributed to data collection and model components for the aircraft model. Mr. XIANG Zhengchao contributed to the discussion and background of this research. All authors commented on the manuscript and approved the submission.

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对称折叠翼尖变体飞行器的纵向建模与气动分析

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摘要:为研究展向自适应机翼的气动特性随翼尖折叠角的变化规律,本文以一种新型形状记忆合金驱动的折叠 翼尖变体飞行器-改进型Cessna 550为研究对象,提出了一种基于翼尖折叠角度的可变气动参数的纵向非线性动 力学建模方法。首先,为了探索翼尖折叠运动如何影响飞行器的气动性能,本文采用计算流体动力学建立了变 体飞行器的三维模型,并通过气动仿真获得不同翼尖折叠角度下的气动系数。通过曲线拟合得到了各气动参数 关于翼尖折叠角度的函数关系式,建立了关于翼尖折叠角度的变体飞行器非线性动力学模型。进而,基于所得 到的气动参数,对飞行器的起飞、机动和着陆3个阶段以及折叠过程中的纵向静稳定性进行了分析,得到了折叠 翼尖变体飞行器在不同的飞行状态下的最佳变体策略。最后,通过纵向稳定分析验证了所提变形策略的可 行性。

关键词:变体飞行器;气动评估;动态建模;纵向建模;对称折叠翼尖