Two-Step Iterative Optimization of Satellite Selection Algorithm Based on PDOP Contribution

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(Received 18 May 2023; revised 1 August 2023; accepted 5 October 2023)

Abstract: Global navigation satellite system (GNSS) is an important navigation sensor for the required navigation performance (RNP) operation. The positioning accuracy of GNSS determines whether the civil aircraft can meet the RNP flight requirements. Due to the limited computing power of airborne receivers, the calculation delay may reduce the positioning accuracy. To meet the high precision requirements, this error caused by calculation delay can no longer be ignored. This paper proposes a two-step iterative optimization of satellite selection algorithm based on the position dilution of precision (PDOP) contribution. It can effectively reduce the calculation delay and improve the positioning accuracy under the RNP operation. The simulation shows that the method has better real-time performance than the traditional algorithm, which is of great significance for ensuring the flight safety of civil aircraft.

Key words: global navigation satellite system (GNSS); multi-GNSS positioning; fast satellite selection; position dilution of precision (PDOP)

CLC number: TN925 Document code: A Article ID: 1005-1120(2023) S2-0040-08

0 Introduction

Required navigation performance (RNP) has become the navigation method of civil aircraft used in various countries today. Global navigation satellite system (GNSS) is an important sensor under RNP, and its positioning accuracy directly affects flight safety.

As the most important phases in the flight process, the approach and landing phases have high requirements under the RNP flight standard especially at the plateau airport^[1-2]. The requirements in the LPV200 (Localizer performance with vertical guidance-200 feet) phase which involved the approach and landing phases are shown in Table 1^[3].

With the development of GNSS in various countries, more and more satellites can participate in the positioning solution, and the number of visible satellites can reach 30^[4-5]. More satellites means more computation for each solution and more com-

Table 1	Navigation	performance	requirements	in
	LPV200			

Parameter	Value/m
Accuracy (horizontal)	16
Accuracy (vertical)	4
Alarm level (horizontal)	40
Alarm level (vertical)	35

putation for receiver integrity detection^[6-7]. The satellite receiver chips have low power and limited capability. The increase in the number of satellites leads to an increase in solution time, resulting in positioning errors caused by delays. Under RNP requirements, the error caused by the delay of GNSS position calculation cannot be ignored.

Selecting some satellites for positioning solution can effectively control the computation of solution. It can control the solution time and reducing errors caused by delay^[8]. With the increase of the number of satellites, the improvement of the geo-

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How to cite this article: DAI Yuting, YIN Haotian, LAI Jizhou, et al. Two-step iterative optimization of satellite selection algorithm based on PDOP contribution[J]. Transactions of Nanjing University of Aeronautics and Astronautics, 2023, 40(S2): 40-47.

http://dx.doi.org/10.16356/j.1005-1120.2023.S2.006

metric configuration is gradually reduced, and the improvement of the positioning accuracy is gradually reduced^[9-10]. Therefore, visible satellite with better configuration will be chosen for position calculation, for which the account the geometric configuration, calculation delay and chip performance can all be taken into account.

For the real-time satellite selection problem in multi-constellation scenarios, when the number of receiver channels is small, several methods were proposed^[11-12] to select four stars for positioning calculation. However, four satellites can no longer meet the accuracy requirements of RNP. For multiconstellation situation, Ref.[13] proposed a fast rotating partition satellite selection method, Ref. [14] proposed a novel fast satellite selection algorithm. The core idea of these methods is to obtain better satellite geometry according to the characteristics of satellite geometry. The satellite selection strategy is to select a zenith satellite with a high elevation, and the azimuth of the remaining visible satellites are evenly distributed. This method is easy to implement and works well under some visible satellite distributions, but not for others, and the dilution of precision (DOP) value fluctuates greatly. The calculation efficiency is high but the positioning accuracy cannot be guaranteed.

To reduce the delay error and ensure that the GNSS positioning accuracy can meet the RNP requirements during the approach and landing phases, this paper proposes a two-step iterative optimization of the satellite selection algorithm based on the position dilution of precision (PDOP) contribution. In order to reduce the calculation delay, some satellites are selected to participate in the positioning calculation. The satellite with the highest PDOP contribution is selected to achieve the optimal positioning accuracy under the same calculation amount. The process of selecting the satellite with the highest PDOP contribution is repeated. The number of satellites to be solved is increased. And the PDOP value is reduced. Thus, the positioning accuracy under the RNP operation is finally achieved.

The structure of this paper is as follows: Section 1 describes the scheme of the satellite selection method. Section 2 shows the simulation results by comparison. Section 3 provides a brief summary of this paper.

1 Step-by-Step Satellite Selection Algorithm Based on Contribution

GNSS positioning accuracy is mainly affected by user range error (URE) and visible satellite geometry. Among them, URE is the pseudo range error caused by receiver equipment, tropospheric ionospheric error, satellite relativistic effect, and satellite ephemeris error. Geometric dilution of precision (GDOP) is the physical quantity used to represent the GNSS positioning accuracy by the geometric configuration of visible satellites^[15]. GDOP is the ratio of position error to ranging error.

The observation equation for GNSS positioning solution is

$$Z = HX + \epsilon \tag{1}$$

where Z is the pseudo range measurement vector; H the observation matrix; X the position and time estimate vector; and ϵ the observation error vector. The observation matrix H is formed as

$$H = \begin{bmatrix} \frac{X - X_{1}(t)}{S_{1}(t)} & \frac{Y - Y_{1}(t)}{S_{1}(t)} & \frac{Z - Z_{1}(t)}{S_{1}(t)} & 1\\ \vdots & \vdots & \vdots & \vdots\\ \frac{X - X_{i}(t)}{S_{i}(t)} & \frac{Y - Y_{i}(t)}{S_{i}(t)} & \frac{Z - Z_{i}(t)}{S_{i}(t)} & 1\\ \vdots & \vdots & \vdots & \vdots\\ \frac{X - X_{n}(t)}{S_{n}(t)} & \frac{Y - Y_{n}(t)}{S_{n}(t)} & \frac{Z - Z_{n}(t)}{S_{n}(t)} & 1 \end{bmatrix}$$

$$(2)$$

where *X*, *Y*, *Z* are the receiver position coordinates under the Earth-centered, Earth-fixed (ECEF) system; and $X_i(t)$, $Y_i(t)$, $Z_i(t)$ the position coordinates of the *i*th satellite under the ECEF system at time *t*. S_i is the pseudo range of the *i*th satellite at time *t*. Let $(H^T H)^{-1} = H_{HH}$. The elements on the H_{HH} main diagonal are $h_{11}, h_{22}, h_{33}, h_{44}$.

GDOP = $\sqrt{\text{trace}(H_{HH})} = \sqrt{h_{11} + h_{22} + h_{33} + h_{44}}$ (3) The PDOP can be expressed as

$$PDOP = \sqrt{h_{11} + h_{22} + h_{33}} \tag{4}$$

PDOP is only related to the position of the visible satellites involved in the calculation. It directly affects the GNSS positioning accuracy under the condition of constant pseudo range measurement error and clock error. When selecting visible satellites, a better PDOP is used as the satellite selection target.

The proposed algorithm is divided into two steps. First, four satellites are selected to form the basic selection-constellation. Second, based on the basic selection-constellation, the highest PDOP contribution satellite is added to the positioning solution. The specific algorithm is given as follows.

1.1 Basic selection-constellation composition

Assuming that the azimuth of the ith satellite is α_i , and the elevation is θ_i , then the observation matrix *H* can be rewritten as

	$\sin \alpha_1 \cos \theta_1$	$\cos \alpha_1 \cos \theta_1$	$\sin \theta_1$	1	
	÷	:	:	÷	
$H_n =$	$\sin \alpha_i \cos \theta_i$	$\cos \alpha_i \cos \theta_i$	$\sin \theta_i$	1	(5)
	÷	:	:	÷	
	$\sin \alpha_n \cos \theta_n$	$\cos \alpha_n \cos \theta_n$	$\sin \theta_n$	1	

It can be seen that the observation matrix is only related to the elevation and azimuth of the visible satellites. It means the PDOP value is related to the elevation and azimuth of the visible satellites. The satellite positioning solution requires at least four visible satellites to perform the calculation, so four satellites are used as the number of selected satellites for the basic selection-constellation.



Fig.1 Tetrahedron with four satellites

As shown in Fig.1, the four points ABCD are the intersections of the unit circle with receiver position U as the center and the line connecting the four satellites and U. In the case of four satellites, the larger the volume of the tetrahedron ABCD, the smaller the DOP value. Therefore, in order to make the volume of the tetrahedron as large as possible, a satellite with a higher elevation is selected as the top satellite. Three satellites with evenly distributed azimuth among the satellites with a lower elevation angle are selected as the bottom satellite. The base constellation consists of one top satellite and three bottom satellites.

After obtaining the current position information of all visible satellites, the azimuth and elevation of each visible satellite relative to the satellite receiver are calculated according to the initial value of position iteration (usually the calculated position at the last moment). Assuming that the initial values of the receiver position iteration under the ECEF system are X, Y, Z. At time t, the elevation θ_i of the *i*th satellite is

$$\theta_i = \arcsin \frac{Z - Z_i(t)}{S_i(t)} \tag{6}$$

The azimuth α_i of the ith satellite is

$$\alpha_{i} = \arcsin \frac{X - X_{i}(t)}{S_{i}(t) \cos \theta_{i}} = \arccos \frac{Y - Y_{i}(t)}{S_{i}(t) \cos \theta_{i}}$$
(7)

The satellite with the largest elevation is selected from the visible satellites as the top satellite in the basic selection-constellation. Among the remaining visible satellites, the satellites with the elevation less than 35° are included in the set of candidate satellites. The combination which is composed of three satellites with azimuthal spacing closest to 120° in the set of candidate satellites is selected.

1.2 Satellite selection based on PDOP contribution

Based on the basic selection-constellation, more visible satellites are added to the basic selection-constellation to reduce DOP, which can improve positioning accuracy and GNSS integrity and meet the RNP requirements. The rest of the visible satellites are used as a candidate satellite set. Each time, the satellites with the greatest improvement in the DOP value are selected from the candidate satellites and added to the solution to improve the positioning accuracy. The method of satellite selection considering with DOP contribution is as follows.

When the number of satellites participating in the solution is n, the observation matrix is

$$H_{n} = \begin{bmatrix} \sin \alpha_{1} \cos \theta_{1} & \cos \alpha_{1} \cos \theta_{1} & \sin \theta_{1} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \sin \alpha_{i} \cos \theta_{i} & \cos \alpha_{i} \cos \theta_{i} & \sin \theta_{i} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \sin \alpha_{n} \cos \theta_{n} & \cos \alpha_{n} \cos \theta_{n} & \sin \theta_{n} & 1 \end{bmatrix} (8)$$

After adding a visible satellite, the observation matrix H becomes

$$H_{n+1} = \begin{bmatrix} H_n \\ h_{n+1} \end{bmatrix}$$
(9)

where

$$\boldsymbol{h}_{n+1} = \begin{bmatrix} \sin \alpha_{n+1} \cos \theta_{n+1} & \cos \alpha_{n+1} \cos \theta_{n+1} & \sin \theta_{n+1} & 1 \end{bmatrix}$$
(10)

Then there are n+1 visible satellites, the $H_{HH}^{(n+1)}$ value is

$$H_{HH}^{(n+1)} = (H_{n+1}^{T} H_{n+1})^{-1} = \left(\begin{bmatrix} H_{n}^{T} & h_{n+1}^{T} \end{bmatrix} \begin{bmatrix} H_{n} \\ h_{n+1} \end{bmatrix} \right)^{-1} = \left(H_{n}^{T} H_{n} + h_{n+1}^{T} h_{n+1} \right)^{-1}$$
(11)

According to the Sherman-Morrison formula, we can get

$$H_{HH}^{(n+1)} = (H_n^{\mathrm{T}} H_n + h_{n+1}^{\mathrm{T}} h_{n+1})^{-1} = (H_n^{\mathrm{T}} H_n)^{-1} - \frac{(H_n^{\mathrm{T}} H_n)^{-1} h_{n+1}^{\mathrm{T}} h_{n+1} (H_n^{\mathrm{T}} H_n)^{-1}}{1 + h_{n+1} (H_n^{\mathrm{T}} H_n)^{-1} h_{n+1}^{\mathrm{T}}} = H_{HH}^{(n)} - \frac{H_{HH}^{(n)} h_{n+1}^{\mathrm{T}} h_{n+1} H_{HH}^{(n)}}{1 + h_{n+1} H_{HH}^{(n)}} = H_{HH}^{(n)} - \Delta h_{n+1}$$
(12)

Therefore, the effect on the GDOP value after adding visible satellites can be derived from the Δh_{n+1} elements on the diagonal. The larger the value of the Δh_{n+1} elements on the main diagonal, the smaller the value of the $H_{HH}^{(n+1)}$ elements on the main diagonal, and the smaller the PDOP value. By calculating the Δh_{n+1} matrix of each satellite, the satellite with the highest PDOP contribution is obtained. Add this satellite is to the solve satellite composition.

Two-step satellite selection is performed for the visible satellites at a certain moment, and the satellites selected in the two steps are shown in Fig.2 and Fig.3.



Fig.2 Step 1: Basic selection-constellation selection



Fig.3 Step 2: Satellite selection based on PDOP contribution

To sum up, the flow chart of the two-step iterative optimization of the satellite selection algorithm based on PDOP contribution is shown in Fig.4.

Step 1 Calculate the elevation and azimuth of each visible satellite according to the current position of each visible satellite.

Step 2 According to the elevation and azimuth of each visible satellite, select four satellites to form the basic selection-constellation.

Step 3 Calculate the PDOP contribution Δh_{n+1} of each visible satellite in the candidate satellite set in current selection-constellation.

Step 4 Select the candidate satellite with the largest PDOP contribution to join the satellite set for solving, and combine the selected satellite with the basic selection-constellation to form the selection-constellation.



Fig.4 Flow chart of Two-step iterative optimization of satellite selection algorithm based on PDOP contribution

Step 5 Determine whether the number of satellites n meets the requirements for satellite selection N. If so, go to the next step, otherwise return to Step 3.

Step 6 Output the satellite combination as the satellites for position solution.

2 Simulation

In order to verify the effect of the two-step iterative optimization of the satellite selection algorithm based on PDOP contribution, the following experiments were designed to simulate the typical RNP approach and landing phases. The simulation track was designed according to the real approach and landing track of the plateau airport Nyingchi Mainling Airport (ZUNZ, 94.39° east longitude, 29.62° north latitude, 2 948.9 m). The approach starting position was 94.435 296° east longitude, 29.333 897° north latitude, 3 951 m height. The approach starting heading angle was 95°. The approach starting ground speed was 200 m/s. The approach and landing time was 391 s. The simulation track was shown in Fig.5.

In order to simulate the computing ability of the airborne receiver chip, referring to the current aircraft commonly used receivers, this simulation used a single-core single-process to run the position-



Fig.5 ZUNZ approach and landing track

ing solution program. The simulation parameters of GNSS are shown in the Table 2 according to Ref.[3]

Table 2 GNSS simulation parameter settings

Parameter	Value
Number of GPS ephemeris satellites	31
Number of BDS ephemeris satellites	44
Number of selected satellites	12
Simulation satellite time	2022-7-20-0-0-0
Pseudo range error standard deviation/m	10.5
Shading angle/(°)	5

The proposed method was compared and analyzed with the fast rotating partition satellite selection method and all-satellite solution. The solution error and the calculated delay error were added to obtain the position error. The position error was divided into horizontal position error and vertical position error according to the standard. The horizontal direction error, vertical direction error and solution time are shown in Figs.6—8.

The simulation data were analyzed. The hori-



Fig.6 Horizontal positioning error of each method



Fig.7 Vertical positioning error of each method



zontal position errors RMS, the landing phase and the average solution time results of the three methods are shown in Table 3.

Method	Horizontal posi- tion error RMS/m	Average solution time/ s
The proposed method	12.701	0.019 6
Fast rotating partition satellite selection	18.494	0.028 5
All satellite solution	16.945	0.041 6

Table 3 Simulation data analysis

In the flight descent phases, the vertical accuracy directly affects the flight safety and needs to be paid attention to. Therefore, the vertical position error of the transition descent phase and the approach and landing phase were calculated. The vertical position error RMS in descent phases of the three methods are shown in Table 4.

Table 4 Descent phase s	imulation data analysis	
Method	Vertical position error	
Method	RMS in descent phases/m	
The proposed method	7.259	
Fast rotating partition satellite	12.409	
selection	12.405	
All satellite solution	7.752	

The satellite selection method in this paper has better positioning accuracy and less solution time delay compared with the all-satellite solution and the fast rotating partition satellite selection.

Compared with the all-satellite solution, the horizontal position accuracy has been improved by 25.02%, improving positioning performance. At the same time, the vertical position accuracy is also improved in the descending phase. The solution time is reduced by 52.88%. The fast rotating partition satellite selection only considers the uniform distribution of the azimuth. The selected satellites have poor geometric configuration at some moments. Due to the large number of satellites involved in the positioning calculation, the calculation time is long during the whole-satellite calculation, resulting in a calculation delay error. After calculation, it can be seen that the satellite selection algorithm proposed in this paper is better than the other two algorithms in terms of solution time and positioning accuracy. It shows that the proposed algorithm can effectively reduce the receiver calculation delay and improve the positioning accuracy.

3 Conclusions

In order to solve the problem of too long solution time caused by too many satellites, a real-time satellite selection algorithm is designed. The first contribution of this paper is the establishment of a satellite's contribution evaluation method with PDOP as the indicator. The second contribution of this paper is the design of a two-step iterative rapid star selection strategy, which, based on the selection of the optimal basic selection-constellation, swiftly selects available stars that meet performance requirements through satellite contribution assessment. The preferred constellation are selected for positioning solution according to the PDOP contribution, to meet the high requirements of GNSS for civil aircraft under RNP. In this way, the positioning error caused by the long solution time is reduced, and the positioning accuracy is improved.

The simulation results show that this method can reduce the calculation delay error, improve the GNSS positioning accuracy, and ensure flight safety.

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Acknowledgements This work was supported by the National Key Research and Development Program of China (No. 2022YFB3904300) and Joint Funds of the National Natural Science Foundation of China (No. U2233215).

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Author contributions Mr. DAI Tingyu designed the study, complied the models, conducted the analysis, interpreted the results and wrote the manuscript. Mr. YIN

Haotain contributed data and conducted the analysis. Prof. LAI Jizhou, Prof. ZHANG Qieqie and Dr. LI Zhimin contributed to review and editing of the study. All authors commented on the manuscript draft and approved the submission

Competing interests The authors declare no competing interests.

(Production Editor: ZHANG Bei)

基于 PDOP 贡献度的分步式迭代优化选星算法

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摘要:全球导航卫星系统(Global navigation satellite system, GNSS)是民航所需导航性能(Required navigation performance, RNP)程序的重要导航传感器。GNSS的定位精度决定了民用飞机是否能够满足RNP飞行要求。由于飞机上接收机的计算能力有限,计算延迟可能会降低定位精度。为了满足高精度要求,计算延迟引起的误差不能再被忽视。本文提出了一种基于位置精度因子(Position dilution of precision, PDOP)贡献的两步迭代卫星选星算法优化,可以有效减小计算延迟并提高RNP程序下的定位精度。仿真结果表明,该方法在实时性方面优于传统算法,对确保民用飞机飞行安全具有重要意义。

关键词:全球卫星导航系统;多星座卫星定位;快速卫星选星;位置精度因子