

# Four-Position Initial Alignment Based on Forward and Reverse Filtering

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**Abstract:** In order to achieve fast and high accuracy of strapdown initial alignment, a four-position initial alignment algorithm based on forward and reverse filtering is proposed. The proposed scheme divides the whole alignment process into three stages: Coarse alignment, backtracking alignment and fine alignment. In the first stage, coarse alignment is performed, and data are collected and stored. In the second stage, the unscented Kalman filter(KF) is used for backtracking alignment. In the third stage, the KF is used for fine alignment. Four-position data are sampled in the whole process. The proposed scheme is used for initial alignment, and high precision alignment is achieved on the basis of ensuring fast alignment. The simulation results show that the alignment scheme can quickly complete the initial alignment and achieve a high alignment accuracy, and the sky misalignment error is within  $10''$ .

**Key words:** strapdown inertial navigation; initial alignment; forward and reverse filtering; four-position alignment

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## 0 Introduction

One of the key points in the research of the inertial navigation system is to improve the performance of initial alignment, including rapidity and accuracy. High accuracy and rapidity are important indicators of initial alignment. Generally, it takes a long time to achieve high alignment accuracy, so it is a key technique to deal with the contradiction between alignment accuracy and alignment rapidity. Usually, the way to improve the alignment accuracy is to reduce the interference and increase the alignment time. However, interference is often difficult to avoid, and increasing alignment time contradicts the rapidity requirement of alignment. Therefore, the optimal algorithm is designed to achieve fast alignment and improve the alignment accuracy of the inertial navigation system, which is the direction of scientific researchers have been trying to explore.

In recent years, with the continuous progress of computer technology, its storage capacity and

computing speed have been rapidly developed, which also lays a foundation for fast initial alignment based on the backtracking principle<sup>[1-2]</sup>. In initial alignment, navigation and positioning, reverse algorithm has been widely used<sup>[3]</sup>. Yan<sup>[4]</sup> applied the reverse navigation calculation to the initial alignment of the strapdown compass, designed a fast initial alignment method of the strapdown compass, and combined the forward initial alignment of the strapdown compass with the reverse dead reckoning by means of the auxiliary external velocity, thus realized the initial alignment and position navigation of the strapdown compass in the moving state. The alignment method of forward and reverse compass method based on data storage proposed in Ref. [5] can shorten the alignment time and improve the alignment accuracy.

In Ref. [6], forward and reverse alignment of Kalman filter (KF) was carried out. The forward and reverse navigation method was applied to the KF forward and reverse initial alignment method,

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which achieved good results and shortened the alignment time. The rapidity is one of the important indicators of the initial alignment of inertial navigation system. In order to further improve the rapidity of the alignment process, some scholars introduce the concept of backtracking alignment in the initial alignment of the inertial navigation system. In Ref. [7], backtracking alignment was introduced into the autonomous alignment of strapdown inertial navigation system (SINS) to improve the alignment speed.

KF technology is a practical tool for solving the parameter estimation of integrated navigation system, which is widely used in the scenes of multi-sensor and multi-information fusion<sup>[8-9]</sup>. Estimating and compensating the gyro drift of the rotating axis  $Z$ -axis is very important for the single-axis rotating inertial navigation system<sup>[10]</sup>. In this paper, multi-position alignment is designed, and  $Z$ -axis is taken as the rotating axis to improve the alignment speed and accuracy.

Initial alignment is one of the core technologies of inertial navigation system. The initial alignment process of inertial navigation system is divided into two stages: Coarse alignment and fine alignment. In the fine alignment stage, the misalignment angle error model is used to estimate and correct the misalignment angle using the navigation velocity error, and the accurate attitude matrix is obtained. Using the alignment scheme proposed in this paper, the coarse alignment algorithm based on inertial frame and the reverse unscented Kalman filter (UKF)

alignment algorithm are carried out through the stored data, and the fast alignment is achieved on the basis of ensuring the high-precision alignment effect. The algorithm of forward and reverse filtering technology in inertial navigation system proposed in this paper is of great significance for providing high precision navigation benchmark and evaluating real-time navigation performance.

## 1 Basic Principles Section

### 1.1 Alignment scheme based on forward and reverse filtering

In the initial alignment scheme based on forward and reverse filtering, data are collected and stored in the alignment process, and the alignment accuracy is improved and the alignment time is reduced by repeated use of data. The alignment scheme is shown in Fig.1.

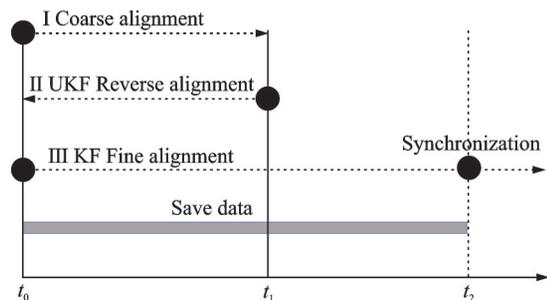


Fig.1 Alignment scheme based on forward and reverse filtering

In Fig.1,  $t_0$  is denoted as the initial time of alignment. The alignment scheme is divided into three stages. The work content of each stage is shown in Table 1.

**Table 1 Algorithmic process**

Phase	Description
The first stage	During the period $[t_0, t_1]$ , the coarse alignment is performed, and the data are collected and stored.
The second stage	After the coarse alignment is completed, the attitude at time $t_1$ is obtained. According to the stored data, the reverse UKF filtering is used to calculate the attitude at time $t_0$ , and the data are collected and stored at the same time.
The third stage	The KF algorithm is used for fine alignment, and data are collected and stored at the same time. The fine alignment calculation and sensor data acquisition reach synchronization at time $t_2$ .

In the first stage, the alignment algorithm based on inertial frame is used for coarse alignment. After coarse alignment, the sky misalignment angle is large and SINS error model is nonlinear, so the nonlinear filtering method is used to solve navigation. In the second stage, the UKF algorithm is selected for reverse alignment. The reason is that the UKF algorithm is simpler than the extended Kalman filter (EKF) algorithm, and the UKF algorithm does not need to solve the Jacobian matrix of the higher-order vector equation. In addition, if the nonlinearity of the system is weak, the EKF algorithm can be used. The more nonlinear the system, the more obvious the advantage of UKF algorithm. In this experiment scheme, the effect of UKF algorithm is better. After UKF reverse alignment, the misalignment angle error becomes a small angle, and SINS error model can be approximated as a linear equation. In the third stage, KF is used for fine alignment, which has higher accuracy and less calculation.

This section describes the self-alignment scheme based on forward and reverse filtering, discusses the methods adopted in the three stages, and analyzes the feasibility of the application of these three methods, which will provide important reference for the subsequent research.

$$\mathbf{C}_n^{n'} = \begin{bmatrix} \cos \alpha_y \cos \alpha_z - \sin \alpha_y \sin \alpha_x \sin \alpha_z & \cos \alpha_y \sin \alpha_z + \sin \alpha_y \sin \alpha_x \cos \alpha_z & -\sin \alpha_y \cos \alpha_x \\ -\cos \alpha_x \sin \alpha_z & \cos \alpha_x \cos \alpha_z & \sin \alpha_x \\ \sin \alpha_y \cos \alpha_z + \cos \alpha_y \sin \alpha_x \sin \alpha_z & \sin \alpha_y \sin \alpha_z - \cos \alpha_y \sin \alpha_x \cos \alpha_z & \cos \alpha_y \cos \alpha_x \end{bmatrix} \quad (5)$$

From time  $t_1$  to time  $t_0$ , the SINS attitude, velocity and position update process should adopt the inverse inertial navigation system (INS) solution model. The earth rotation angular velocity  $\boldsymbol{\omega}_{ie}^n$ , the navigation system rotation angular velocity  $\boldsymbol{\omega}_{en}^n$  relative to the earth system, the gyroscope output  $\boldsymbol{\omega}_{ib}^b$ , the accelerometer output  $\mathbf{f}_{sf}^b$  and the gravitational acceleration  $\mathbf{g}^n$  should be inverted and substituted into Eqs.(1, 2). The attitude error equation and velocity error equation of SINS in reverse navigation can be obtained. Combining the position error, zero deviation of gyroscope and zero deviation of accelerometer, and ignoring the gravity error term, we can ob-

## 1.2 SINS reverse filtering model

After the first stage of coarse alignment is completed, the sky misalignment angle error is large, and the error model of strapdown inertial navigation system is nonlinear, so the reverse alignment of the second stage can be carried out only after obtaining the nonlinear error model of the system.

The actual SINS calculation coordinate system  $n'$  and navigation coordinate system  $n$  can be converted by three-axis rotation, and the rotation angle is the platform error angle, which is denoted as a vector  $\boldsymbol{\alpha}^n = [\alpha_x \quad \alpha_y \quad \alpha_z]^T$ .

The nonlinear attitude error equation and velocity error equation of SINS are

$$\dot{\boldsymbol{\alpha}}^n = \mathbf{C}_\omega^{-1} [(\mathbf{I} - \mathbf{C}_n^{n'}) \tilde{\boldsymbol{\omega}}_{in}^n + \mathbf{C}_n^{n'} \delta \boldsymbol{\omega}_{in}^n - \mathbf{C}_b^{n'} \boldsymbol{\epsilon}^b] \quad (1)$$

$$\delta \dot{\mathbf{v}}^n = (\mathbf{I} - \mathbf{C}_n^n) \mathbf{C}_b^{n'} \tilde{\mathbf{f}}_{sf}^b - (2\delta \boldsymbol{\omega}_{ie}^n + \delta \boldsymbol{\omega}_{en}^n) \times \mathbf{v}^n - (2\tilde{\boldsymbol{\omega}}_{ie}^n + \tilde{\boldsymbol{\omega}}_{en}^n) \times \delta \mathbf{v}^n + \delta \mathbf{g}^n + \mathbf{C}_n^n \mathbf{C}_b^{n'} \nabla^b \quad (2)$$

where  $\mathbf{C}_\omega$  is the Euler angle matrix;  $\mathbf{C}_n^{n'}$  the attitude transition matrix from  $n$  system to  $n'$  system. They are defined as

$$\mathbf{C}_\omega = \begin{bmatrix} \cos \alpha_y & 0 & -\sin \alpha_y \cos \alpha_x \\ 0 & 1 & \sin \alpha_x \\ \sin \alpha_y & 0 & \cos \alpha_y \cos \alpha_x \end{bmatrix} \quad (3)$$

$$\mathbf{C}_\omega^{-1} = \frac{1}{\cos \alpha_x} \begin{bmatrix} \cos \alpha_y \cos \alpha_x & 0 & \sin \alpha_y \cos \alpha_x \\ \sin \alpha_y \sin \alpha_x & c \alpha_x & -\cos \alpha_y \sin \alpha_x \\ -\sin \alpha_y & 0 & \cos \alpha_y \end{bmatrix} \quad (4)$$

tain the state equation of SINS in reverse integrated navigation system as

$$\begin{cases} \dot{\boldsymbol{\alpha}}^n = \mathbf{C}_\omega^{-1} [(\mathbf{C}_n^{n'} - \mathbf{I}) \tilde{\boldsymbol{\omega}}_{in}^n - \mathbf{C}_n^{n'} \delta \boldsymbol{\omega}_{in}^n + \mathbf{C}_b^{n'} \boldsymbol{\epsilon}^b] \\ \delta \dot{\mathbf{v}}^n = (\mathbf{C}_n^n - \mathbf{I}) \mathbf{C}_b^{n'} \tilde{\mathbf{f}}_{sf}^b + (2\delta \boldsymbol{\omega}_{ie}^n + \delta \boldsymbol{\omega}_{en}^n) \times \mathbf{v}^n + (2\tilde{\boldsymbol{\omega}}_{ie}^n + \tilde{\boldsymbol{\omega}}_{en}^n) \times \delta \mathbf{v}^n - \delta \mathbf{g}^n - \mathbf{C}_n^n \mathbf{C}_b^{n'} \nabla^b \\ \delta \dot{L} = -\frac{\delta v_N}{R_M + h} \\ \delta \dot{\lambda} = -\frac{\delta v_E}{R_N + h} \sec L - \frac{v_E}{R_N + h} \sec L \tan L \delta L \\ \delta \dot{h} = -\delta v_U \\ \dot{\boldsymbol{\epsilon}}^b = \mathbf{0} \\ \dot{\nabla}^b = \mathbf{0} \end{cases} \quad (6)$$

where  $\boldsymbol{\alpha}^n$  and  $\delta \mathbf{v}^n$  are the platform error angle and

the velocity error, the respectively; and  $\delta L$ ,  $\delta \lambda$  and  $\delta h$  the the latitude error, the longitude error and the height error, respectively.  $\epsilon^b$  is the gyroscope zero deviation;  $\nabla^b$  the accelerometer zero deviation. A 15-dimensional state vector is composed and denoted as  $x$ .

The difference between the position at the initial time and the position solved by SINS is selected as the measurement, that is  $z = p_{\text{sins}} - p_0$ , where  $p_0$  represents the position at the initial time. The filtering model is

$$\begin{cases} \dot{x} = f(x) + g(x)w \\ z = Hx + v \end{cases} \quad (7)$$

where  $f(x)$  and  $g(x)$  can be established according to Eq. (6),  $H = [0_{3 \times 6} \quad I_{3 \times 3} \quad 0_{3 \times 6}]$ , and  $v$  is the measurement noise.

KF is a commonly used initial alignment method at present, but when the coarse alignment is completed, the error angle is still large, and the linear KF model cannot meet the alignment requirements of the nonlinear system. Therefore, this section intends to use UKF to solve the backward alignment problem in the case of large misalignment angle.

## 2 Simulation Experiment and Analysis

In order to verify the effectiveness of the proposed four-position alignment method based on forward and reverse filtering, a simulation platform is established for simulation verification and analysis.

Table 2 shows the parameter settings of inertial devices.

**Table 2 Inertial device parameter setting**

Error parameter type	Value
Zero deviation of gyroscope/ $(^\circ \cdot \text{h}^{-1})$	0.004 5
Gyroscope angle random walk/ $(^\circ \cdot \text{h}^{-\frac{1}{2}})$	0.000 1
Zero deviation of accelerometer / $\mu\text{g}$	8
Accelerometer velocity Random walk / $(\mu\text{g} \cdot \text{Hz}^{-\frac{1}{2}})$	1

The following three groups of experiments are designed for simulation analysis.

Group A: Based on the traditional KF four-position alignment, each position ( $0^\circ$ ,  $90^\circ$ ,  $180^\circ$ ,

$270^\circ$ ) was respectively collected for 75 s, a total of 5 min;

Group B: Four-position alignment based on forward and reverse filtering, each position was collected for 75 s, a total of 5 min;

Group C: Four-position alignment based on forward and reverse filtering, each position was collected for 60 s, a total of 4 min.

Figs.2—7 are the error curves of the east direction, north direction and sky direction after initial alignment of three experiments.

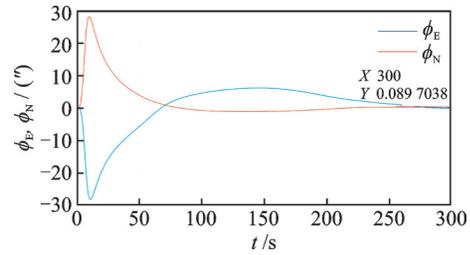


Fig.2 East-direction misalignment angle error and north-direction misalignment angle error in Group A

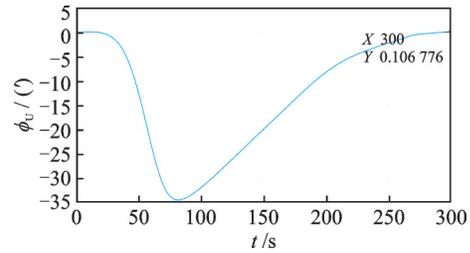


Fig.3 Sky-direction misalignment angle error in Group A

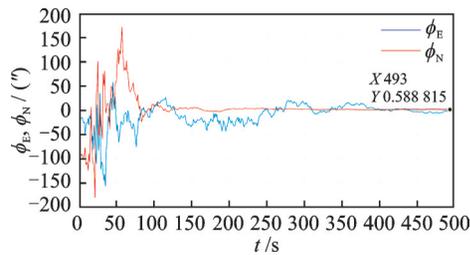


Fig.4 East-direction misalignment angle error and north-direction misalignment angle error in Group B

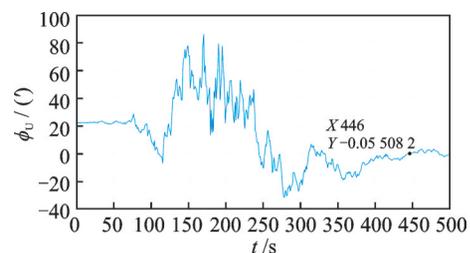


Fig.5 Sky-direction misalignment angle error in Group B

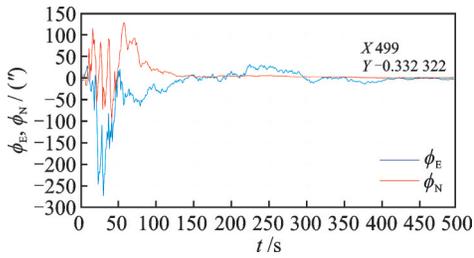


Fig.6 East-direction misalignment angle error and north-direction misalignment angle error in Group C

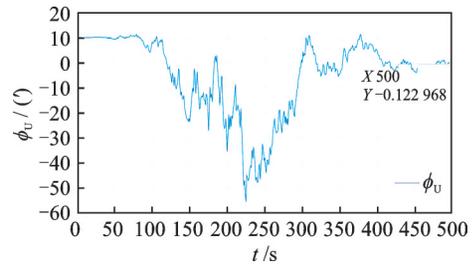


Fig.7 Sky-direction misalignment angle error in Group C

The accuracy of the misalignment angle in three directions of three group experiments A, B

and C is compared. Table 3 shows the alignment results of three groups of experiments.

**Table 3 Experimental alignment result**

Group	East-direction misalignment angle/( $''$ )	North-direction misalignment angle/( $''$ )	Sky-direction misalignment angle/( $''$ )
A	0.09	0.09	6.41
B	0.59	0.59	3.30
C	0.33	0.33	7.38

Both the east-direction and north-direction misalignment angles can ensure good alignment accuracy, so we pay more attention to the sky-direction misalignment angle.

Group A experiment are compared with group B experiment. The used alignment methods are different. The data collection time is 5 min. The four-position alignment scheme based on forward and reverse filtering is better than the traditional KF alignment accuracy.

Group B experiment are compared with Group C experiment. The alignment methods used are same, but the data collection time is different. The longer the data acquisition time, the higher the accuracy of the alignment results.

Group A experiment are compared with Group C experiment. The used alignment methods are different. The data collection time of Group A is 5 min, while the data collection time of Group C is 4 min. However, there is little difference in the alignment accuracy between the two groups, so the algorithm proposed in this paper has more advantages.

In summary, the initial alignment with the proposed scheme achieves high precision alignment on the basis of ensuring fast alignment. The effectiveness and feasibility of the proposed four-position alignment algorithm based on forward and reverse

filtering are illustrated.

### 3 Conclusions

This paper proposes a four-position alignment scheme based on forward and reverse filtering. The whole alignment process is divided into three stages, and four-position data are sampled in the whole process. On the basis of ensuring high precision alignment, fast alignment is achieved. Simulation results show that the proposed method can quickly complete the initial alignment and achieve high alignment accuracy, and the sky alignment error is within  $10''$ . The proposed scheme is of great significance for providing high precision navigation benchmark and navigation performance evaluation.

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**Author contributions** Mr. QIAO Wei conducted theoretical analysis, research models, experimental analysis, interpretation of results, and wrote the draft. Prof. ZENG Qinghua provided methods and ideas for theoretical analysis and experimental design. Dr. ZHAO Bin provided guidance for the design and analysis of the experiment. Mr. ZHU Xiaoling provided basis for theoretical analysis. All authors commented on the manuscript draft and approved the submission.

**Competing interests** The authors declare no competing interests.

(Production Editor: ZHANG Bei)

## 基于正逆向滤波的四位置对准

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**摘要:**为实现捷联惯导初始对准的快速性和高精度,提出一种基于正逆向滤波的四位置初始对准算法。与传统的先粗对准再精对准的两段式对准法不同,所提出的方案将整个对准过程分为3个阶段:粗对准、回溯对准、精对准。在第一阶段进行粗对准并采集存储数据;在第二阶段使用无迹卡尔曼滤波(Unscented Kalman filter, UKF)对系统状态进行回溯对准;在第三阶段采用KF进行精对准。整个过程中采样四位置数据。用所提方案进行初始对准,在保证快速对准的基础上,实现了高精度对准效果。仿真实验表明:该对准方案可以快速完成初始对准,并达到较高的对准精度,天向失准角误差在 $10''$ 以内。

**关键词:**捷联惯导;初始对准;正逆向滤波;四位置对准