Optimal Design of a Tilting Prop Bevel Gear Spoke Plate Structure with High Power Density

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Abstract: To meet the urgent need for the lightweight design of gears under complex conditions in helicopter transmission systems, a design optimization method is proposed for a tilting prop bevel gear spoke plate structure with a high power density. Based on the variable density method considering stress constraints, topology optimization and reconstruction of the bevel gear spoke plate structure are performed, which yield a novel tilting prop bevel gear spoke plate structure with a high power density, differing from the traditional configuration. Introducing an evolution factor and the Markov chain based on the traditional particle swarm optimization (PSO) algorithm, an intelligent and advanced switching delayed PSO (SDPSO) algorithm is developed. The SDPSO algorithm can adaptively select switching strategies and delay information, and it is employed for the size optimization of a tilting prop bevel gear spoke plate structure. After optimization, the mass of the bevel gear spoke plate is reduced by 19.24%, and the maximum von Mises stress of all the operating conditions is reduced by 7.27%. Additionally, the stress distribution of each operating condition becomes more uniform, which demonstrates the structural advantages of the designed bevel gear spoke plate and the superiority of the proposed optimization method.

Key words: particle swarm optimization (PSO) algorithm; topology optimization; size optimization; bevel gear spoke plate

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0 Introduction

With the continuous development of helicopter technology, the demand for power transmission through bevel gears has greatly increased. Consequently, the load-bearing capacity of gears has increased, and the issue of structural mass has become more prominent. In complex working conditions, the demand for high power density gear design becomes more urgent. For the design of gear, particular attention is given to the configuration of the spoke plate structure. The spoke plate plays a key role in transferring loads within the gear train, ensuring proper stress distribution and preventing excessive stress concentration. This, in turn, enhances the durability and lifespan of the gear train. Furthermore, the overall quality of the gear is significantly influenced by the quality of the spoke plate. Consequently, the design of the gear spoke plate directly impacts the performance of the gear. The term of "high power density" in gear transmission refers to the gear's ability to transmit greater power while utilizing less mass within a specific transmission system. To achieve high power density, the structural optimization plays a crucial role. By relying on reliable and efficient optimization algorithms, it can identify the optimal structural form that meets the design requirements under given constraints. Therefore, for the design of gear, there has been

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widespread concern about structure optimization.

Structural optimization can be divided into three categories: Topology, shape, and size optimizations. Topology optimization realizes optimal structural efficiency by finding the material distribution pattern under a given design domain and boundary conditions[1]. It does not rely on the initial configuration or designer experience and can greatly expand the design space. Advantageously, it provides a systematic and scientific design idea and a method for structural conceptual design. Scholars worldwide have conducted extensive research on the application of topology optimization in engineering. For example, Xing et al.[2] introduced a topology optimization technology based on the variable density method into the external support structure of an aero-engine under multiple loading conditions. They obtained a lightweight support structure that met the strength and vibration requirements. Liu et al.[3] used the variable density method based on solid isotropic material with penalization interpolation to topologically optimize a 1/18 cyclic symmetric simplified fan structure model. They greatly reduced the structural mass while meeting the strength and stiffness requirements. Song et al.[4] used bi-directional evolutionary structural optimization based on self-adaption random sampling strategy to topologically optimize the fan disc of an aero-engine, combined with shape optimization. They obtained a novel high-performance fan disc structure. Wang et al.[5] used the topology optimization method to remove the redundant material of spur gears and realized the lightweight design of gears while meeting the mechanical performance requirements. Dai[6] used the OptiStruct software to design gear structures, introduced a multi-objective topological optimization theory, and enhanced the gear stiffness and the ability to suppress external incentives. He et al.[7] conducted modal simulation analysis and stress-strain simulation analysis on a simplified cup-shaped component of a certain type of gear using software such as Abaqus, and they topologically optimized the bottom of the cup-shaped component according to the finite element results. Finally, they proposed a lightweight design scheme for the component.

Shape and size optimization commences from the structural details under the determined topological configuration, and the shape and size of the internal/external boundaries of the structure are optimized according to the actual engineering requirements to improve the structural performance. Extensive research has been conducted on size optimization. Sun et al.[8] used a modern bare-bones multi-objective particle swarm optimization (PSO) algorithm to optimize the basic parameters of spiral bevel gears, improved the contact performance of tooth surfaces, and proposed a basic parameter optimization design method for bevel gears. Wang et al.[9] used commercial software OptiStruct and Abaqus to perform topology and size optimizations for aviation gears and innovatively proposed a lightweight design method for gear spoke plate structures. Dai[10] conducted size optimization of the gear spoke plate structure design with the root stress as the constraint condition, effectively reducing the mass of the gear while satisfying the maximum equivalent stress of the gear root and reducing the cost. Xia et al.[11] employed design space decrease collaborative optimization to study planetary gear reducers and obtained a small volume planetary gear reducer that significantly improved the performance of aircraft electromechanical actuators. Luo et al.[12] introduced multidisciplinary design optimization into bevel gear design, established a multidisciplinary design optimization mathematical model for bevel gears, and obtained a small volume bevel gear using a multi-objective genetic algorithm while reducing the loading noise.

Overall, topology optimization has been widely employed to design aviation structures and has yielded numerous innovative porous structures that revolutionize traditional configurations. However, the design of bevel gear spoke plates under multiple loading condition has not received sufficient attention from researchers, and it mainly relies on experience with traditional spoke plate design forms. Considerable opportunities exist for the topology optimization of bevel gear spoke plate structures, and the innovative spoke plate design needs to revolutionize the traditional design. In addition, although size optimization has been employed for the lightweight design of gears and related structures, research on intelligent algorithms suitable for gear size optimization is still insufficient. Advanced optimization algorithms are efficient and intelligent, and rarely used for the size optimization of bevel gear structures. Therefore, the mass reduction potential of bevel gear structures has not been fully explored.

In this study, a design optimization method is proposed for a bevel gear spoke plate structure with a tilting prop. First, based on the variable density method considering stress constraints, the topology optimization of the bevel gear spoke plate structure is performed, and a novel bevel gear spoke plate structure with a tilting prop is designed, revolutionizing the traditional configuration. The innovative design is characterized by its lightweight construction and uniform stress distribution, leading to a significantly high power density. During the reconstruction stage, the use of a structure in the form of a tilting prop is particularly advantageous in achieving an even higher power density. Then, an autonomous switching delayed PSO (SDPSO) algorithm is proposed and employed for the size optimization of the tilting prop bevel gear spoke plate structure, further enhancing the efficiency of the spoke plate structure.

1 Topology Optimization of Bevel Gear Spoke Plate

1.1 Variable density method considering stress constraint

In the past 30 years, topology optimization theory has rapidly developed, and a series of methods including homogenization, variable density, level set, and asymptotic structural optimization methods have been proposed[13-14]. Among them, the variable density method has the advantages of simple theory, convenient program implementation, and high computational efficiency. Moreover, it has been widely researched and applied in both academic and industrial fields. In this method, the structure is discretized into N_t units, the design and non-design domains are divided according to the design requirements, and a vector $\boldsymbol{x}$ is established that contains N density design variables, corresponding to the number of design domain units. The relationship between the design variable x_e and the material properties is expressed as a continuous variable density function. Each design variable x_e represents the state of the corresponding structural element, where x_e equal to 1 indicates that the element has material and 0 indicates that the element is empty material. Based on advanced and efficient optimization algorithms, the density design variable x_e of each element is optimized to make it approach 0 or 1, thereby controlling the presence or absence of materials and seeking the best load transfer path for the structure.

The mass minimization problem considering stress constraints can usually be expressed as

$$\begin{cases} \min\limits_{x} \quad m=\sum\limits_{e=1}^{N}\bar{x}_e v_e \\ \text{s.t.} \quad \sigma_e^k-\sigma_{\mathrm{L}}\leqslant 0 \quad k=1,2,\cdots,K \\ \qquad 0\leqslant x_e\leqslant 1 \quad e=1,2,\cdots,N \\ \boldsymbol{Ku}=\boldsymbol{f} \end{cases} \tag{1}$$

where m is the structural mass, $\bar{x}_e$ the density of the eth element, σ_e^k the stress of the eth element under the kth loading condition, v_e the volume of the eth element, σ_{L} the stress constraint for topology optimization, $\boldsymbol{f}$ the load vector for solving nodes, $\boldsymbol{K}$ the global stiffness matrix, and $\boldsymbol{u}$ the displacement vector of nodes based on the equilibrium equation.

In theory, the stress upper bound $\bar{\sigma}$ needs to be defined for all elements and the optimization problem with N_t stress constraints needs to be solved. However, due to the large computational complexity, a more convenient method needs to be found to replace it. To reduce the complexity of the optimization problem, the original design space can be divided into multiple cluster/set S. The stress of all the elements included in a cluster/set is represented by a single constraint. Thus, the maximum stress value of all the elements in the cluster/set S satisfies the following conditions

$$\max_{e\in S}\sigma_e\leqslant\bar{\sigma} \tag{2}$$

where σ_e is the mean value of von Mises stress of element e in the cluster/set S. Since stress maximization makes the stress formula non-differentiable, the

p-norm regularization technique can be used to approximate the maximum stress value

$$\max_{e\in S}\sigma_e=\|\boldsymbol{\sigma}_S\|_\infty\leqslant\|\boldsymbol{\sigma}_S\|_p=\left(\sum_{e\in S}\sigma_e^p\right)^{\frac{1}{p}} \tag{3}$$

where $\boldsymbol{\sigma}_S$ represents the stress values vector of all the elements in the cluster/set S and p the regularization factor. To improve the approximation accuracy, this study employs the normalized maximum value to approximate the stress values of each cluster/set. The regularization factor p is not fixed and adaptively changes during the iteration as follows

$$\lim_{i\to\infty} c_i\|\boldsymbol{\sigma}_S^i\|_p=\|\boldsymbol{\sigma}_S^i\|_\infty \tag{4}$$

where i represents the number of iterations.

1.2 Topology optimization and reconstruction

This study aims to minimize the mass of a bevel gear spoke plate. The topology optimization of the bevel gear spoke plate is performed using the variable density method considering stress constraints. Fig.1 displays the initial structure of the studied bevel gear, which mainly experiences meshing and centrifugal loads. In Fig.1, F_t is the tangential force, F_a the axial force, and F_r the radial force. The bevel gear comprises 84 teeth, and the mass of the spoke plate is 18.72 kg. The material of the gear is 9 310 steel, which has a density of 7.86 g/cm^3, yield strength of 940 MPa, safety factor of 1.5, Young's modulus of 200 GPa, Poisson's ratio of 0.316, and the maximum stress of the spoke plate is 599.73 MPa, as depicted in Fig.2.

This study only focuses on the optimization of the bevel gear spoke plate. Therefore, only the spoke plate is taken as the design domain. Furthermore, to fully enhance the performance potential of the bevel gear structure, the design domain is ex-

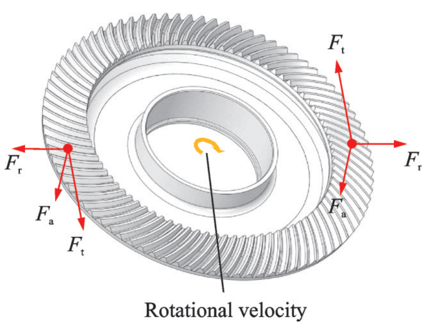

Fig.1 Initial structure of the bevel gear

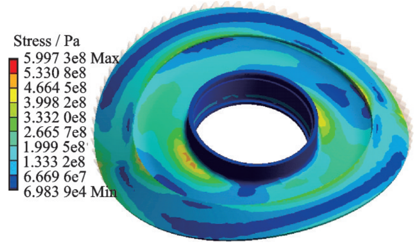

Fig.2 Stress contour plot of the initial gear spoke plate

panded. To improve the computational efficiency, a half cycle symmetric model of the bevel gear is established according to its geometric and load characteristics, as shown in Fig.3, where Φ_1 represents the outer diameter of the spoke plate of the bevel gear, Φ_2 the inner diameter of the gear tooth region under study, Φ_3 the inner diameter of the spoke plate, d_1 the total thickness of the spoke plate, and α the obtuse angle formed between the intersection line of the gear tooth region and the spoke plate and the horizontal line in the section.

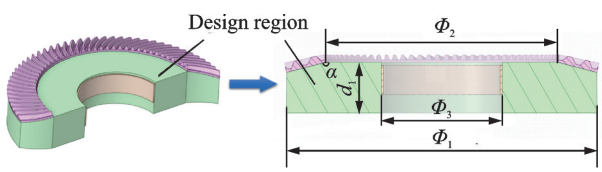

Fig.3 Half cycle symmetric model of the bevel gear with expanded design domain

In this study, the bevel gear consists of 84 teeth. The gear meshing loads are symmetrically distributed, with their position located at the center. At any given moment, at least one pair of teeth in symmetrical positions bear equal loads. Taking into account the load symmetry during the gear's actual operation, a total of 42 working conditions need to be considered. Specifically, each gear tooth can be treated as an independent working condition when subjected to loading. When the maximum von Mises stress of the structure under any loading condition exceeds the yield strength of the material, a risk of failure exists. For the design of the bevel gear spoke plate, the maximum von Mises stress of the structure under various loading conditions is a key evaluation index and it must be considered during optimization. However, the efficiency of topology optimization significantly decreases with increasing number of loading conditions. Therefore, to ensure both accuracy and efficiency in the topology optimization simulation, this study adopted a total of seven working conditions. This approach aimed to simplify the simulation of the actual working state of the bevel gear. In order to cover a range of working situations effectively, a working condition was set every six gear teeth, as illustrated in Fig.4. This strategy enables the simulation process to capture the comprehensive working characteristics of the gear system as much as possible. The selection of these working conditions is a careful balance between accuracy and efficiency, considering the limitations of simulation computing resources. This approach ensures that the simulation results not only accurately reflect the actual working state but also facilitate an efficient topology optimization analysis. Taking the maximum von Mises stress in the spoke plate under the above seven typical loading conditions as the constraint and the mass minimization of a certain bevel gear spoke plate as the optimization goal, the topology optimization of the bevel gear spoke plate is performed using the variable density method considering stress constraints.

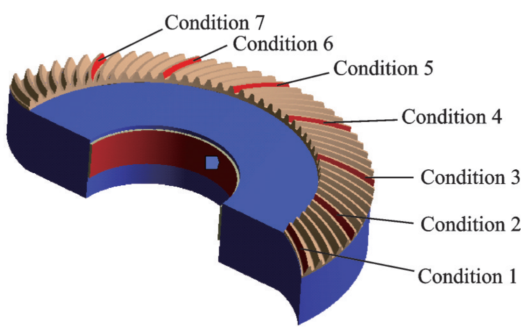

Fig.4 Seven typical working conditions

Fig.5 displays the optimized bevel gear spoke plate, which shows that the bevel gear spoke plate may exhibit a discontinuous and irregular shape after topology optimization, with localized areas prone to stress concentration. To ensure the validity and integrity of this structure, a detailed reconstruction and optimization are necessary. The model reconstruction is based on the topology form derived from the optimization results, taking into account the actual circumstances, and involves the parametric reconstruction of the gear spoke plate. The specific methods include three steps. (1) Post-processing and smoothing. The structure following topology optimization undergoes post-processing, utilizing smoothing techniques to reduce irregularities. This involves smoothing the structure, filling voids, and eliminating sharp geometry to achieve a more continuous and regular shape. (2) Material filling. Additional materials are introduced into more vulnerable parts to enhance the stability and integrity of the structure post-topology optimization. This can be achieved by adding support materials to critical areas or reinforcing structural connections. (3) Simulation testing. Finite element analysis tools are employed to evaluate the reconstructed structure. This step involves comprehensive simulation tests to ensure that the structure meets performance requirements.

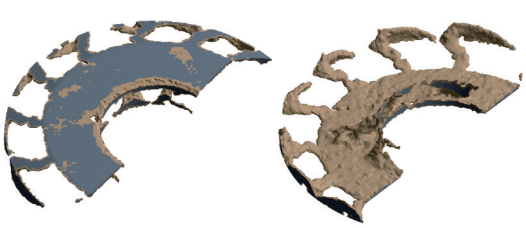

Fig.5 Bevel gear spoke plate after topology optimization

The combined application of these methods ensures that the topology-optimized bevel gear spoke plate not only exhibits superior performance but also maintains sufficient structural integrity. The reconstruction process takes into account the difficulties in machining and stress concentration. The methods mentioned above are employed to address these issues. It is important to address the potential risks present in the area below the gear teeth, as this region is relatively vulnerable to excessive stress. The topology optimization results (Fig.5) reveal substantial holes in this area, which further signifies its structural vulnerability. As a result, reasonable filling is applied to these holes during the reconstruction, with the adoption of elliptical holes. Furthermore, tangent curves and surfaces are employed to correct other areas. Ultimately, a parametric model of the spoke plate for the diagonal strut of the bevel gear is successfully reconstructed, as depicted in Fig.6.

The reconstructed bevel gear spoke plate com-

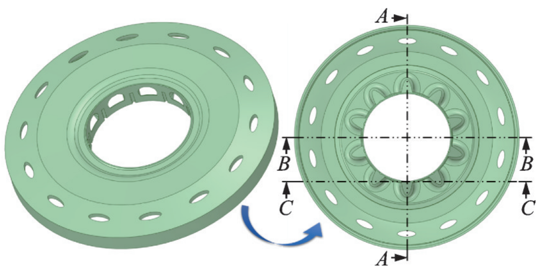

Fig.6 Parametric reconstruction model of the spoke plate of the tilting prop bevel gear

prises a single-layer structure with 10 oblique support columns at the bottom and 14 elliptical slots below the toothed area. Fig.7(a) presents the cross-sectional view of the spoke plate *A*-*A*. Surface 1 is the oblique section of the oblique support column, surface 2 is the cylindrical surface of the oblique support column, surface 3 is a rounded surface, surface 4 has a curvature radius of R_1 at the intersection of the line segment in the *A*-*A* cross-sectional view, and surface 5 has a curvature radius of R_2 at the intersection of the line segment in the *A*-*A* cross-sectional view. Additionally, H_1 represents the distance between the upper end surfaces of the cylindrical slot on the inner edge and surface 6, d_2 the distance between surface 6 and surface 10, and Φ_6 the diameter of the circle where the center of the elliptical slot is located. Fig.7(b) depicts the cross-sectional view of spoke plate *B*-*B*, with W_1 denoting the distance between surfaces 6 and 9 and W_3 (Fig.7(a)) denoting the distance between surfaces 6 and 7. H_2 represents the distance between the lower end surfaces of the inner edge through hole and surface 10, d_3 the distance between the highest point of the upper end surface of the thin plate and the lower end surface, and Φ_4 the inner diameter of the outer edge of the spoke plate. Surface 8 is connected to with the teeth of the gear and the positions of surfaces 6 and 10 are fixed. Fig.7(c) displays the cross-sectional view of spoke plate *C*-*C*, with the outer edge thickness represented by W_2, where Φ_5 represents the diameter of the cylindrical slot on the inner edge.

The reconstructed bevel gear spoke plate has a mass of 16.23 kg, which is 16.16% lower than that of the initial design. To validate the rationality of the reconstruction scheme and align with the requirements of engineering development, this paper opts to simulate the bevel gear under all working conditions. Specifically, each gear tooth is treated as an independent condition during loading, resulting in a total of 42 working conditions being considered. The aim of this comprehensive simulation calculation is to ensure the full consideration of the actual working state of the bevel gear. This process is undertaken to verify the rationality and feasibility of the proposed reconstruction scheme. Fig.8 displays the maximum von Mises stress of the reconstructed spoke plate under the 42 loading conditions. Condition 2 is the most dangerous loading condition, with a maximum von Mises stress of 587.26 MPa. Condition 12 has the highest safety factor, with a maximum von Mises stress of 496.87 MPa. The maximum von Mises stress exhibits large fluctuations under different loading conditions, with a high fluctua-

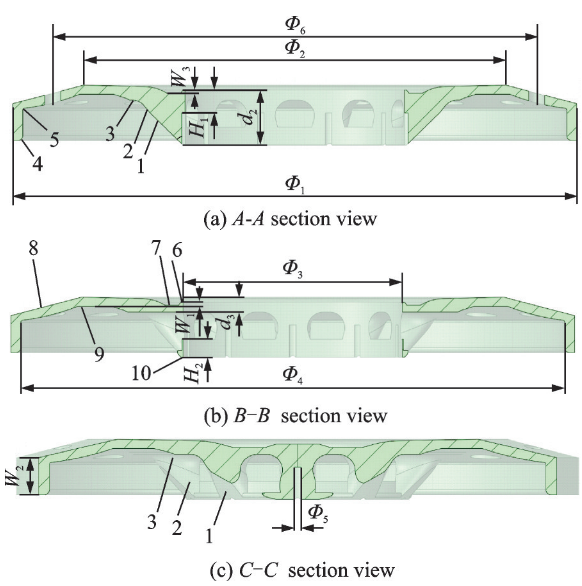

Fig.7 Section view of the parametric reconstruction model of the tilting prop bevel gear spoke plate

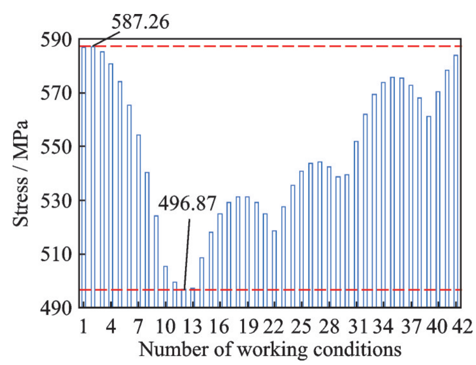

Fig.8 Maximum von Mises stress of the bevel gear spoke plate under 42 loading conditions after topology reconstruction

tion amplitude of up to 90.39 MPa.

Fig.9 displays the von Mises stress contour plot of the bevel gear spoke plate under condition 2 after the topological reconstruction. The stress concentration problem in the bevel gear spoke plate is quite severe, and the maximum stress is located at the edge of the spoke plate hole. Therefore, the design of the reconstructed bevel gear spoke plate needs to be further improved.

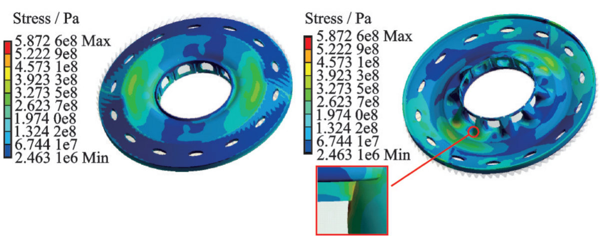

Fig.9 Stress contour plot of the bevel gear spoke plate under condition 2 after topology reconstruction

2 Size Optimization of Bevel Gear Spoke Plate

2.1 SDPSO algorithm

Domestic and foreign scholars have developed numerous optimization methods and their improved versions, including genetic algorithm (GA), marine predator algorithm (MPA), PSO algorithm, and surrogate models based optimization[15-17]. In comparison to traditional optimization algorithms like GA, the PSO algorithm has garnered considerable attention due to its advantages, such as simple parameters, ease of improvement, straightforward structure, easy implementation, rapid convergence speed, and wide applicability. Consequently, the PSO algorithm has found extensive use in addressing engineering problems. In contrast to the surrogate model algorithm, the PSO algorithm typically excels in global search problems. Thanks to its robust parallelism and global search capabilities, the algorithm can swiftly identify potential solutions. By establishing a surrogate model of the objective function, the surrogate model based optimization reduces the number of calls to the actual objective function in the iterative process, thereby significantly enhancing computational efficiency, especially in cases that the calculation cost of the objective function is high. It is important to note that the calculation accuracy of the surrogate model based optimization hinges on the quality of the model, and constructing the surrogate model may introduce some errors, potentially leading to reduced accuracy. Considering the global nature of the research problem, the requirement for calculation accuracy, and the broad applicability of the PSO algorithm in engineering problems, coupled with the relatively shorter time consumption in size optimization compared to topology optimization, this study ultimately opts to utilize the PSO algorithm for optimization.

The PSO algorithm is a type of stochastic optimization method. It simulates the aggregation behaviors of birds and fishes to enable particles to find the global optimal solution[18]. In the PSO algorithm, the velocity and position of the ith particle are represented by $\boldsymbol{v}_i(t)=(v_{i1}(t),v_{i2}(t),\cdots,v_{id}(t))$ and $\boldsymbol{x}_i(t)=(x_{i1}(t),x_{i2}(t),\cdots,x_{id}(t))$, respectively. Here, d and t represent the dimensionality and the number of iterations of the optimization problem, respectively. The updated equations for the velocity and position of each particle are

$$\boldsymbol{v}_i(t+1)=w\boldsymbol{v}_i(t)+c_1r_1(\boldsymbol{p}_i(t)-\boldsymbol{x}_i(t))+c_2r_2(\boldsymbol{p}_g(t)-\boldsymbol{x}_i(t)) \tag{5}$$

$$\boldsymbol{x}_i(t+1)=\boldsymbol{x}_i(t)+\boldsymbol{v}_i(t+1) \tag{6}$$

where c_1 and c_2 are the acceleration coefficients, r_1 and r_2 the two random numbers in the interval [0, 1], w is the inertia weight, $\boldsymbol{p}_i$ the historical best position of the ith particle, and $\boldsymbol{p}_g$ the historical best position of all particles.

Although the PSO algorithm has been widely used to solve complex structural optimization problems[19-20], it is prone to local optima and premature convergence in high-dimensional and multi-modal optimization problems[16,21-22]. The author proposed the SDPSO algorithm[23] based on an adaptive switching strategies and delay information. The SDPSO algorithm aims (1) to evaluate the evolution factor, determine the inhomogeneous Markov chain using the probability transfer matrix, and automatically update it during iteration and (2) to adaptively select the inertia weight, acceleration coefficient, and delay information by combining the evolution factor and Markov chain and then adjust the particle velocity.

In the SDPSO algorithm, the updated equations for the velocity and position of each particle can be expressed as

$$\begin{aligned}\boldsymbol{v}_i(t+1)=&w(t)\boldsymbol{v}_i(t)+\\&c_1\big(\xi(t)\big)r_1\Big(\boldsymbol{p}_i\big(t-\tau_1\big(\xi(t)\big)\big)-\boldsymbol{x}_i(t)\Big)+\\&c_2\big(\xi(t)\big)r_2\Big(\Big(\boldsymbol{p}_g\big(t-\tau_2\big(\xi(t)\big)\big)\Big)-\boldsymbol{x}_i(t)\Big)\end{aligned}\tag{7}$$

$$\boldsymbol{x}_i(t+1)=\boldsymbol{x}_i(t)+\boldsymbol{v}_i(t+1)\tag{8}$$

where $c_1\big(\xi(t)\big)$ and $c_2\big(\xi(t)\big)$ are the acceleration coefficients, and $\tau_1\big(\xi(t)\big)$ and $\tau_2\big(\xi(t)\big)$ the delay constants. These parameters can be determined according to the nonhomogeneous Markov chain $\xi(t)(t\geqslant 0)$.

The value of the Markov chain $\xi(t)$ can be 1, 2, 3, or 4, representing four states in the optimization search process: Convergence, exploitation, exploration, and jumping-out. The value of $\xi(t)$ can be first selected based on the evolution factor E_f, shown as

$$\xi(t)=\begin{cases}1 & 0\leqslant E_\mathrm{f}<0.25\\2 & 0.25\leqslant E_\mathrm{f}<0.5\\3 & 0.5\leqslant E_\mathrm{f}<0.75\\4 & 0.75\leqslant E_\mathrm{f}<1\end{cases}\tag{9}$$

where the E_f is defined as follows

$$E_\mathrm{f}=\frac{D_\mathrm{g}-D_{\min}}{D_{\max}-D_{\min}}\tag{10}$$

where $D_{\min}$ and $D_{\max}$ represent the minimum and maximum average distances between the ith particle and the other particles in the swarm, respectively, and D_g represents the average distance between the global optimal particle and the other particles in the swarm. Then, the $\xi(t)$ value can be randomly adjusted using the probability transition matrix $\boldsymbol{\Pi}^{(t)}=\begin{pmatrix}0.9 & 0.1 & 0 & 0\\0.05 & 0.9 & 0.05 & 0\\0 & 0.05 & 0.9 & 0.05\\0 & 0 & 0.1 & 0.9\end{pmatrix}$.

Moreover, the tendency of the inertia weight w is the same as that of the evolution factor E_f. It is defined as

$$w(t)=0.5E_\mathrm{f}+0.4\quad\forall E_\mathrm{f}\in[0,1]\tag{11}$$

Table 1 illustrates the automatic selection strategy for the acceleration coefficients c_1 and c_2 and delay constants $\tau_1\big(\xi(t)\big)$ and $\tau_2\big(\xi(t)\big)$. The initial values of c_1 and c_2 are set as 0.9. $\lfloor\bullet\rfloor$ represents the nearest integer less than or equal to the variable, and rand is a uniformly distributed random number between 0 and 1.

Table 1 Automatic selection strategy of acceleration coefficients and delayed constants

State	Mode	c_1	c_2	$\tau_1\big(\xi(t)\big)$	$\tau_2\big(\xi(t)\big)$
Convergence	$\xi(t)=1$	2	2	0	0
Exploitation	$\xi(t)=2$	2.1	1.9	$\lfloor t\cdot\text{rand}\rfloor$	0
Exploration	$\xi(t)=3$	2.2	1.8	0	$\lfloor t\cdot\text{rand}\rfloor$
Jumping-out	$\xi(t)=4$	1.8	2.2	$\lfloor t\cdot\text{rand}\rfloor$	$\lfloor t\cdot\text{rand}\rfloor$

2.2 Size optimization and analysis

To verify the engineering practicality of the proposed SDPSO algorithm and further reduce the mass of the bevel gear spoke plate, size optimization of the reconstructed model is performed. Based on the parameter characteristics of the reconstructed model, the spoke plate parameters R_1, R_2, Φ_4, W_1, W_2, W_3, H_1, and H_2 are chosen as the design variables. Since the reconstructed model is a uniquely shaped spoke plate, the stress varies across different working conditions, as explained in Section 1.2. Consequently, the maximum von Mises stress of the bevel gear spoke plate under seven typical loading conditions $\sigma_{k,\max}$ (k represents the condition number) are selected as constraints. The minimum mass of the bevel gear spoke plate is chosen as the optimization objective. Finally, the mathematical model for the size optimization of the bevel gear spoke plate is established as

$$\begin{aligned}&\min\quad m\\&\text{w.r.t.}\quad R_1,R_2,\Phi_4,W_1,W_2,W_3,H_1,H_2\\&\text{s.t.}\quad\begin{cases}0\leqslant R_1\leqslant 2.00\\0\leqslant R_2\leqslant 2.00\\520.00\leqslant\Phi_4\leqslant 540.00\\0\leqslant W_1\leqslant 5.00\\9.00\leqslant W_2\leqslant 30.00\\0\leqslant W_3\leqslant 7.40\\3.91\leqslant H_1\leqslant 22.00\\7.18\leqslant H_2\leqslant 22.00\\\sigma_{k,\max}\leqslant\sigma_m\quad k=1,2,\cdots,7\end{cases}\end{aligned}$$

where σ_m represents the stress constraint during size optimization as defined in this study.

During optimization, the particle number and maximum generation number of SDPSO are set as 20 and 50, respectively, with a total of 500 calculations. Fig.10 presents the convergence process of the size optimization of the bevel gear spoke plate.

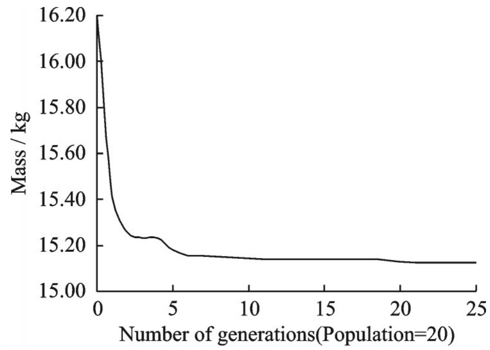

Fig.10 Convergence process for the size optimization of the bevel gear spoke plate

Fig.10 shows that the size optimization quickly and stably converges, and it yields the optimal design of the bevel gear spoke plate size that meets the stress constraints. Therefore, the proposed SDPSO algorithm is suitable for constrained practical engineering optimization problems and efficient for the size design optimization of a bevel gear spoke plate.

Table 2 presents the key geometric and performance parameters of the bevel gear spoke plate before and after size optimization. Fig.11 compares the bevel gear spoke plate structure scheme before and after size optimization. After the size optimization, the eight design parameters of the spoke plate obviously change. The mass of the plate decreases from 16.23 kg to 15.12 kg (6.84% decrease). The maximum von Mises stress decreases from 587.26 MPa to 556.13 MPa (5.30% decrease).

Table 2 Key parameters of the bevel gear spoke plate before and after size optimization

Parameter		Reconstruction scheme	Optimal scheme	Difference / %
Mass / kg		16.23	15.12	−6.84
Maximum von Mises stress / MPa	Condition 1	587.26	543.70	−7.42
	Condition 2	554.27	515.47	−7.00
	Condition 3	524.94	556.13	5.94
	Condition 4	531.37	521.92	−1.78
	Condition 5	540.81	552.11	2.09
	Condition 6	552.01	528.09	−4.33
	Condition 7	572.96	527.60	−7.92
	Maximum value	587.26	556.13	−5.30
Design variable / mm	R_1	1.00	1.05	5.00
	R_2	1.00	0.10	−90.00
	Φ_4	530.00	536.00	1.13
	W_1	1.91	0.46	−75.92
	W_2	19.33	9.43	−51.22
	W_3	5.39	7.39	37.11
	H_1	12.90	3.91	−69.69
	H_2	12.20	7.18	−41.15

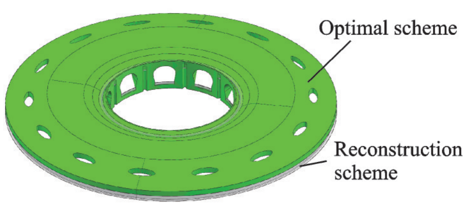

Fig.11 Comparison of bevel gear spoke plate structure schemes before and after size optimization

To further verify the rationality of the bevel gear spoke plate design scheme, a comparison of the maximum von Mises stress of the spoke plate under 42 different loading conditions before and after size optimization is presented in Fig.12. After size optimization, condition 16 becomes the most dangerous loading condition of the spoke plate, with a maximum von Mises stress of 556.13 MPa, and condition 7 exhibits the highest safety margin, with a maximum von Mises stress of 515.47 MPa. The fluctuation of the maximum von Mises stress under various loading conditions is significantly reduced, with a decrease of 40.66 MPa (55.02%) in the fluc-

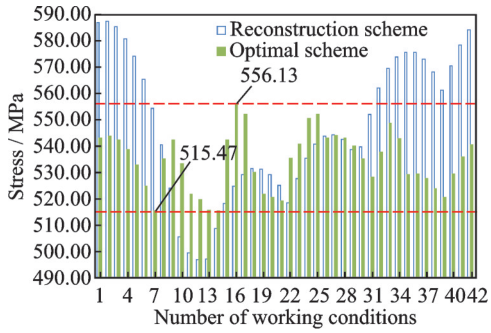

Fig.12 Comparison of the maximum stress of the spoke plate under 42 conditions before and after size optimization

tuation amplitude. Clearly, the size optimization makes the structure of the bevel gear spoke plate more reasonable.

Fig.13 depicts the von Mises stress distribution map of the bevel gear spoke plate under typical loading conditions after size optimization. The von Mises stress distribution under various loading conditions becomes more uniform and the stress concentration problem improves after the size optimization. Moreover, the maximum von Mises stress remains relatively low and is mainly located at the end of the tilting prop.

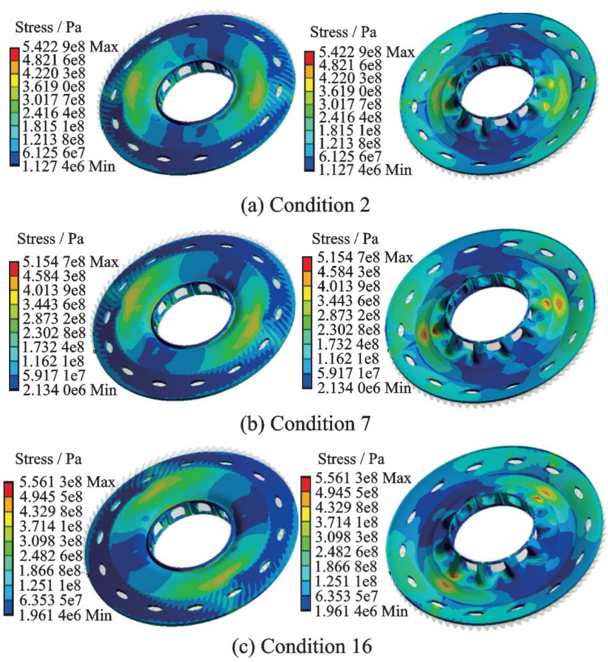

Fig.13 Stress contour plot of the bevel gear spoke plate under typical conditions after size optimization

To summarize, following topology optimization and size optimization, the mass of the bevel gear spoke plate is reduced from 18.72 kg to 15.12 kg, indicating a decrease of 19.24%. Additionally, the stress distribution in each working condition becomes more uniform. The maximum von Mises stress across all conditions decreases from 599.73 MPa to 556.13 MPa, representing a reduction of 7.27%. These changes successfully meet the material's strength requirements. The results illustrate that the proposed method possesses distinct advantages in optimizing the design of a high power ratio bevel gear spoke plate.

3 Conclusions

This study proposed a design optimization method for a tilting prop bevel gear spoke plate structure with a high power density. First, the topology optimization of the bevel gear spoke plate was performed based on the variable density method considering stress constraints. Then, an intelligent and advanced SDPSO algorithm was developed based on an adaptive jumping strategy and delayed information to improve the traditional PSO algorithm, and it was employed for the size optimization of a bevel gear spoke plate. The study conclusions were as follows:

(1) After the topology optimization of the bevel gear spoke plate, a novel tilting prop bevel gear spoke plate structure with a high power density was reconstructed, revolutionizing the traditional configuration. The spoke plate comprised a single-layer structure with 10 oblique struts at the bottom and 14 elliptical slots below the gear area. Compared to the initial scheme, the mass of the reconstructed spoke plate was reduced by 16.16% and the maximum stress was not increased, providing an important reference for the design of a bevel gear spoke plate with a high power density.

(2) After the size optimization of the bevel gear spoke plate, the mass of the spoke plate further decreased by 6.84%, the maximum von Mises stress decreased by 5.30%, the stress distribution became more uniform, the stress concentration problem improved, and the structural efficiency of the spoke plate improved. This indicates that the proposed SDPSO algorithm is applicable for con-

strained practical engineering optimization problems and is an intelligent and efficient oblique gear spoke size design optimization algorithm.

(3) After the topology and size optimizations, the mass of the novel bevel gear spoke plate reduced from 18.72 kg to 15.12 kg (19.24% reduction) and the stress distribution under all loading conditions became more uniform. The maximum von Mises stress for all conditions also decreased from 599.73 MPa to 556.13 MPa (7.27% decrease). Furthermore, it is worth noting that all conditions satisfactorily adhered to the material's strength requirements. This demonstrates the structural advantages of the designed tilting prop bevel gear spoke plate structure with a high power density as well as the advanced nature of the proposed bevel gear spoke plate design optimization method.

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高功率密度斜向支柱锥齿轮辐板结构设计优化

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摘要:为满足直升机传动系统对复杂工况下齿轮轻量化设计的迫切需求,提出了一种高功率密度斜向支柱锥齿轮辐板结构的设计优化方法。基于考虑应力约束的变密度法,对锥齿轮辐板进行了拓扑优化与重构,得到一种新颖的突破传统构型的高功率密度斜向支柱锥齿轮辐板结构。在传统粒子群优化(Particle swarm optimization, PSO)算法的基础上,引入马尔可夫链和进化因子,自适应地选择跳变策略和延迟信息,进而发展了智能先进的时滞跳变粒子群优化(Switching delayed PSO, SDPSO)算法,并将其应用于斜向支柱锥齿轮辐板结构的尺寸优化。优化后,锥齿轮辐板质量共减小了19.24%,所有工况的最大von Mises应力共减小了7.27%,各工况应力分布更加均匀,证明了所设计的锥齿轮辐板的结构优势和所提出的设计优化方法的先进性。

关键词:粒子群优化算法;拓扑优化;尺寸优化;锥齿轮辐板