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Non-cooperative Space Target Estimation Algorithm Without Prior Information Dependence Based on Temporal Line of Sight Constraint

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Abstract: Under single-satellite observation, the parameter estimation of the boost phase of high-precision space non-cooperative targets requires prior information. To improve the accuracy without prior information, we propose a parameter estimation model of the boost phase based on trajectory plane parametric cutting. The use of the plane passing through the geo-center and the cutting sequence line of sight (LOS) generates the trajectory-cutting plane. With the coefficient of the trajectory cutting plane directly used as the parameter to be estimated, a motion parameter estimation model in space non-cooperative targets is established, and the Gauss-Newton iteration method is used to solve the flight parameters. The experimental results show that the estimation algorithm proposed in this paper weakly relies on prior information and has higher estimation accuracy, providing a practical new idea and method for the parameter estimation of space non-cooperative targets under single-satellite warning.

Key words: motion parameter estimation; estimation of impact point; infrared early warning; boost phase modeling; trajectory database construction

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0 Introduction

With the development of long-range precision strike weapons, modern warfare puts higher demands on countries to warn and intercept enemy long-range weapons. The space-based infrared early warning system can locate and track the active segment tail flame of the detection missile, which is conducive to the early warning and interception of the incoming missile. Because the position and velocity vectors of the ballistic missile at the flameout point determine most of the ballistic parameters, whether the space-based early warning system can capture the target during this period is related to whether the anti-missile system can be ready to intercept in time. The space-based infrared observation system uses the infrared sensor to track space non-cooperative targets reliably and stably and provides key target indication functions[1-2]. The boost phase of the space non-cooperative targets produces some easily observed phenomena that are easily detected by the satellite sensors in high orbits[3-5]. The motion characteristics of the boost phase of the space non-cooperative targets are complex. In particular, the motion state parameters of the burnout point have a decisive influence on the entire trajectory and the impact point. Therefore, how to obtain relatively accurate ballistic data is one of the focus of many researchers[6]. Under the single-star passive observation condition, without the support of prior information, the parameter estimation of the active segment of the ballistic missile is an incomplete ob-

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servation problem, which is a hot spot and a difficult problem in the research direction of parameter estimation. The parameter estimation of the boost phase of space non-cooperative targets mainly includes general motion modeling and trajectory template methods.

The method based on general motion modeling (such as profile-free model (PFM))[7-8] does not consider prior trajectory template information. Scholars worldwide have carried on more relevant research on the PFM method. Based on the study of the PFM method, Liu et al.[9] proposed a multi-model trajectory reasoning method, and by using the selection method, constructed a general net acceleration model, which can accurately determine the types of boost levels. Li et al.[10] studied the motion model of tracking a maneuvering target and the relationship between the models, proposed the constant-acceleration (CA) model or more precisely nearly-constant-acceleration model, and emphasized the basic assumptions and ideas of the model. Liu et al.[11] modeled the motion of the boost phase of the space non-cooperative targets based on research on the single model.

The abovementioned method for estimating parameters of the boost phase based on general motion does not require the support of priori information, but under the condition of passive observation of a single satellite, without the support of priori information, the parameter estimation of the boost phase of the space non-cooperative targets is incomplete. Moreover, according to the constrained statistical inference method in statistics[12], it is difficult for the motion model to calculate the true value of the active segment parameter from the observed value, so the observation problem can be regarded as a non-identifiable or weakly recognized problem of the motion model.

Based on the trajectory template (such as profile-dependent model (PDM)) method[13-15], the prior information on the boost phase of the space non-cooperative targets has been used to achieve high-precision parameter estimation of the boost phase. Jilkov et al.[16] discussed the different methods to improve modeling accuracy when studying the motion modeling of the boost phase of space non-cooperative targets, and based on different trajectory assumptions, proposed some improved models, including the gravity turn model and the model of flight dynamics, which was in a more complex sense. Danis et al.[17] used the thrust acceleration of the boost phase of the space non-cooperative targets to characterize the motion characteristics of the space non-cooperative targets, and established a template library that takes into account the thrust acceleration, thereby reducing the difficulty of template library construction. Benavoli et al.[18] proposed a geometric constraint that estimated trajectory parameters through template matching and other methods, improving the traditional modeling method and final estimation accuracy. The abovementioned parameter estimation methods of the boost phase based on the trajectory template improve the accuracy to a certain extent, but they rely on a higher degree of prior information.

In this paper, we address the parameter estimation problem of non-cooperative targets, specifically mid-range and long-range ballistic missiles, utilizing data from single-satellite early warning systems with angle measurements but lacking ranging observations. Our proposed algorithm offers a solution tailored to the limitations of such systems, introducing a parameter estimation model for the boost phase of ballistic missiles. Leveraging trajectory plane parametric cutting, our approach mitigates strong prior condition dependence and enhances algorithm efficiency. The main contributions of this paper lie in the introduction of a novel parameter estimation model and the empirical validation through trajectory simulation and experimental analysis. By elucidating these contributions and outlining the structure of our article, we provide readers with a clear roadmap of our work and its significance in the field of non-cooperative target parameter estimation.

1 Materials and Methods

1.1 Process of the ballistic plane parametric cutting method

Generally, the PDM method needs to traverse all the trajectory-cutting planes in the height interval and estimate the boost phase parameters for each alternative cutting trajectory. When the height interval of the starting point and the endpoint is large, there are too many alternative cutting trajectories, and the calculation accuracy often depends on the accuracy of the estimation of the starting point height and the endpoint height. For this reason, this paper proposes to use the plane passing through the geo-center and the cutting time sequence line of sight (LOS) to establish the parametric cutting geometry model of the ballistic plane. The algorithm flow is shown in Fig.1.

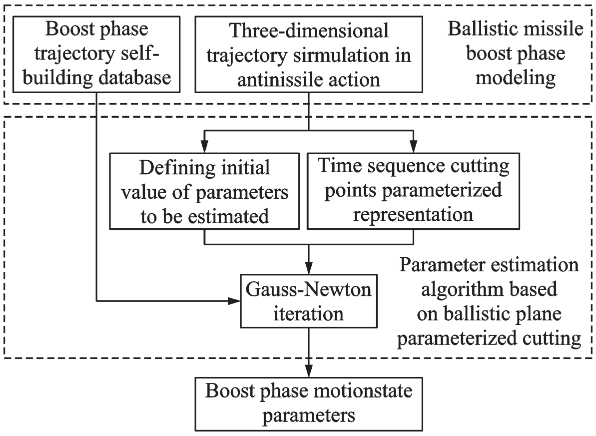

Fig.1 Flow chart of the boost phase parameter estimation based on the parametric cutting of the ballistic plane

1.2 Geometrical principles of the parametric cutting method for ballistic plane

A ballistic cutting plane is an analytical plane that intersects the trajectory of a ballistic missile. It is used to represent and parameterize the missile's path during its boost phase. The geocentric rectangular fixed coordinate system is one of the most widely used coordinate systems in space-based early warning. It is usually used as the basic coordinate system when estimating ballistic parameters. Its definition is as follows: The origin (O_c) is the center of the Earth; the O_cZ_c axis coincides with the Earth's rotation axis, pointing from the origin to the north pole of the Earth; the O_cX_c axis points from the origin to the intersection of the prime meridian and the equator; the O_cY_c axis is in the equatorial plane and forms a right-hand spiral coordinate system with the O_cX_c axis and O_cZ_c axis.

As shown in Fig.2, let (X, Y, Z) be the three-dimensional coordinates of the ballistic plane in the geocentric rectangular fixed coordinate system[19]; then the ballistic cutting plane in the geocentric rectangular fixed coordinate system is $aX+bY-Z=0$, with (a, b) as the parameters of the ballistic cutting plane.

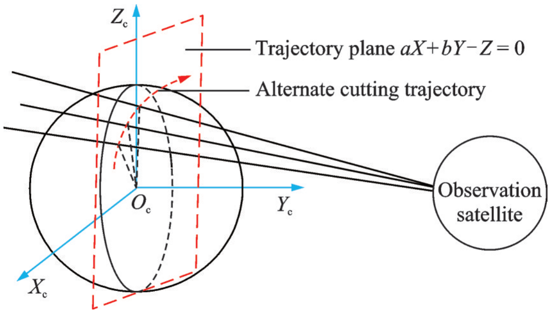

Fig.2 Schematic diagram of parametric ballistic plane cutting geometry

At the moment of observation (t_i), if the unit vector of the observation line of sight extracted from the infrared warning image is $(u_i, v_i, w_i)^{\mathrm{T}}$, and the coordinates of the cutting point of the observation line of sight on the ballistic cutting plane under the fixed coordinate system is $(X_i, Y_i, Z_i)^{\mathrm{T}}$, then we have

$$\begin{cases}\dfrac{X_i-X_s}{u_i}=\dfrac{Y_i-Y_s}{v_i}=\dfrac{Z_i-Z_s}{w_i}\\ aX_i+bY_i-Z_i=0\end{cases}\tag{1}$$

where (X_s, Y_s, Z_s) is the coordinate of the GEO early warning satellite in the ground-fixed coordinate system, and then we have

$$\begin{cases}X_i=\dfrac{-[b\cdot(Y_s-X_s\cdot v_i/u_i)-Z_s+X_s\cdot w_i/u_i]}{a-w_i/u_i+b\cdot v_i/u_i}\\ Y_i=\dfrac{-[a\cdot(X_s-Y_s\cdot u_i/v_i)-Z_s+Y_s\cdot w_i/v_i]}{b-w_i/v_i+a\cdot u_i/v_i}\\ Z_i=\dfrac{-[a\cdot(X_s-Z_s\cdot u_i/w_i)+b\cdot(Y_s-Z_s\cdot v_i/w_i)]}{a\cdot u_i/w_i+b\cdot v_i/w_i-1}\end{cases}\tag{2}$$

Eq.(2) is the parametric cutting model of the ballistic plane[20], which takes a and b as the estimated parameters of the ballistic cutting plane and converts all the motion estimation parameters of the boost phase into related functions of a and b. In this way, in the time sequence observed by the infrared sensor, the parametric cutting model of the ballistic plane can use the function containing parameters of a and b to express the coordinates of the ballistic cutting point. Therefore, the model can not only eliminate the dependence on prior information such as the height interval of the starting point and the endpoint, but also avoid the large amount of calculation when estimating the parameters of each alternative cutting trajectory. This is because the model does not need to traverse the cutting ballistic plane, and the estimation algorithm is more targeted, thus improving the accuracy of the parameter estimation in the boost phase of mid-range and long-range ballistic missiles.

1.3 Construction of the ballistic template library

This paper uses the prior design parameters of the conventional mid-range and long-range missiles and the prior constraint information of the active segment trajectory to characterize the characteristics of the active segment trajectory template.

At different stages, the mid-range and long-range missiles not only correspond to distinct dynamic characteristics but also different trajectory characteristics. The trajectory is taken as a piecewise quartic polynomial function in this paper, and the fitting model established is as follows

$$\begin{cases} x_i^{(k)} = l_1^{(k)} + l_2^{(k)} t_i + l_3^{(k)} t_i^2 + l_4^{(k)} t_i^3 + l_5^{(k)} t_i^4 \\ y_i^{(k)} = m_1^{(k)} + m_2^{(k)} t_i + m_3^{(k)} t_i^2 + m_4^{(k)} t_i^3 + m_5^{(k)} t_i^4 \end{cases} \tag{3}$$

where i is the serial number of the missile trajectory point, t_i the flight time of the missile, (x_i, y_i) the corresponding two-dimensional coordinate of the missile, and k the boost series of the missile. The judgment of each booster interstage can be determined according to the boost jump of the booster interstage and the different radiation characteristics of different booster stages. When solving the polynomial coefficients landm, the corresponding hierarchical solution method is also employed. The specific solution method for the coefficients of each boost level is as follows

$$\begin{cases} \boldsymbol{F}_x = [x_1, x_2, \cdots, x_N]^{\mathrm{T}} \\ \boldsymbol{F}_y = [y_1, y_2, \cdots, y_N]^{\mathrm{T}} \\ \boldsymbol{l} = [l_1, l_2, l_3, l_4, l_5]^{\mathrm{T}} \\ \boldsymbol{m} = [m_1, m_2, m_3, m_4, m_5]^{\mathrm{T}} \end{cases} \tag{4}$$

$$\boldsymbol{X} = \begin{bmatrix} 1, t_1, t_1^2, t_1^3, t_1^4 \\ 1, t_2, t_2^2, t_2^3, t_2^4 \\ \vdots \\ 1, t_N, t_N^2, t_N^3, t_N^4 \end{bmatrix} \tag{5}$$

The least-square principle of the linear model is used to solve the optimal coefficients $\boldsymbol{l}$ and $\boldsymbol{m}$, which are converted to solve the minimum problem

$$\min \| \boldsymbol{F}_x - \boldsymbol{X} \cdot \boldsymbol{l} \|, \min \| \boldsymbol{F}_y - \boldsymbol{X} \cdot \boldsymbol{m} \| \tag{6}$$

The final fitting residuals are $\mathrm{RSS}_x = \boldsymbol{F}_x - \boldsymbol{X} \cdot \hat{\boldsymbol{l}}$, $\mathrm{RSS}_y = \boldsymbol{F}_y - \boldsymbol{X} \cdot \hat{\boldsymbol{m}}$, where $\hat{\boldsymbol{l}}$ and $\hat{\boldsymbol{m}}$ are the final estimates of the coefficients $\boldsymbol{l}$ and $\boldsymbol{m}$, respectively.

Finally, all the calculated trajectory data of the missile's active phase in the direction of the plane are stored in the ballistic template library. The established ballistic template library contains conventional mid-range and long-range ballistic missiles, including ballistic missile trajectory templates of single-stage, two-stage, and three-stage rocket types. The format of the ballistic template library built in this paper is shown in Table 1.

Table 1 Self-built trajectory template database format

Missile serial number	Flight time/s	Fitting coefficient
Ⅰ	t …	$\boldsymbol{l}, \boldsymbol{m}$ …
Ⅱ	t …	$\boldsymbol{l}, \boldsymbol{m}$ …
…	…	…

The three-dimensional trajectory data of the active phase can be completely determined by combining the launch parameters of the ballistic missile with the trajectory template library information of the active phase. The parameters of the ballistic missile launching point include longitude (λ), latitude (φ), elevation (h), launching time (t_0), and launching azimuth angle (α_0) (i.e., firing angle). However, when the actual missile is launched, the same type of missile may change the angle of attack of the missile through different flight procedures and also change the shutdown time of all levels of boosters of the missile to control the ballistic missile to fly along the low trajectory or high trajectory. This changes the range and impact point of the missile to avoid the threat of antimissile devices as much as possible. Therefore, to reduce the difference between the actual active segment trajectory and the active segment trajectory of the template library, we introduce a dimensionless correction parameter (L). The coordinates of the missile launched into the plane can be expressed as follows

$$\begin{cases} x_k(t)=(1-1.5L)x(t) \\ y_k(t)=(1+L)y(t) \end{cases} \tag{7}$$

where $L\in[-0.25, +0.25]$, and $L=0$ corresponds to the trajectory with the maximum range.

1.4 Boost phase parameter estimation algorithm based on trajectory plane parametric cutting

1.4.1 Parametric ballistic plane cutting model

A quartic polynomial is used to conduct the ballistic fitting of the trajectory cutting point, i.e.

$$\begin{cases} X_i=l_0+l_1t_i+l_2t_i^2+l_3t_i^3+l_4t_i^4 \\ Y_i=m_0+m_1t_i+m_2t_i^2+m_3t_i^3+m_4t_i^4 \\ Z_i=n_0+n_1t_i+n_2t_i^2+n_3t_i^3+n_4t_i^4 \end{cases} \tag{8}$$

where l_i, m_i, n_i are the polynomial coefficients.

Let the fitted ballistic curve intersect the Earth sphere, and $(\bar{X}_i, \bar{Y}_i, \bar{Z}_i)^{\mathrm{T}}$; then, the obtained intersection point is the coordinate of the missile launching point. Let ε_1'' and ε_2'' be the pseudo-observation noise errors, and let the coordinates of the missile in the launching coordinate system be $(x_t(a, b, t_0), y_t(a, b, t_0), 0)$. Then, a certain trajectory (x_k, y_k) in the ballistic template library matches the trajectory of the missile, i.e.

$$\begin{cases} x_k(t_0)=x_t(a,b,t_0)+\varepsilon_1^p \\ y_k(t_0)=y_t(a,b,t_0)+\varepsilon_2^p \end{cases} \tag{9}$$

where $\varepsilon_j^p\sim N(0,R_j^p), j=1,2$. Owing to the nonlinearity in the conversion process, it is not suitable for the pseudo-observation noise to use the explicit expression of the original measurement noise. Therefore, in the actual simulation calculation, the pseudo-observation noise is often directly added to the original observation data, i.e.

$$\begin{cases} \boldsymbol{f}_1(t_0)=[x_k(t_0),y_k(t_0)]^{\mathrm{T}} \\ \boldsymbol{f}_2(a,b,t_0)=[x_t(a,b,t_0),y_t(a,b,t_0)]^{\mathrm{T}} \\ \boldsymbol{\varepsilon}=[\varepsilon_1^p,\varepsilon_2^p]^{\mathrm{T}} \end{cases} \tag{10}$$

Derived from Eqs.(9) and (10), the nonlinear mathematical model is established as

$$\boldsymbol{f}_1(t_0)=\boldsymbol{f}_2(a,b,t_0)+\boldsymbol{\varepsilon} \tag{11}$$

Let $\boldsymbol{f}_2(a, b, t_0)-\boldsymbol{f}_1(t_0, L)=\boldsymbol{f}(a, b, t_0, L)$, and then Eq.(11) can be expressed as

$$f(a,b,t_0)+\boldsymbol{\varepsilon}=0 \tag{12}$$

1.4.2 Model solving

This paper uses the Gauss-Newton iterative method to estimate the latitude and longitude of the launching point and the launching azimuth, and this is essentially a nonlinear least-squares estimation problem. Suppose the parameters that are to be estimated are $(a, b, t_0)^{\mathrm{T}}=\boldsymbol{\omega}$; let $F(\boldsymbol{\omega})=(f(\boldsymbol{\omega}, t_1), f(\boldsymbol{\omega}, t_2), \cdots, f(\boldsymbol{\omega}, t_N))^{\mathrm{T}}$, and suppose $\boldsymbol{J}(\boldsymbol{\omega})$ is the Jacobian matrix of $F(\boldsymbol{\omega})$ for $\boldsymbol{\omega}$

$$\boldsymbol{J}(\boldsymbol{\omega})=\nabla_\omega(F(\boldsymbol{\omega}))=\begin{pmatrix} \nabla_\omega(f(\boldsymbol{\omega},t_1)) \\ \nabla_\omega(f(\boldsymbol{\omega},t_2)) \\ \vdots \\ \nabla_\omega(f(\boldsymbol{\omega},t_N)) \end{pmatrix}_{N\times 4} \tag{13}$$

where $\nabla_\omega(\cdot)=(\partial f/\partial a, \partial f/\partial b, \partial f/\partial t_0)_{1\times 4}$. Set ω_n^{MLE} as the result of the nth iteration, and then

$$\omega_{n+1}^{\mathrm{MLE}}=\omega_n^{\mathrm{MLE}}+b_n \tag{14}$$

where b_n is the correction factor, and $b_n=(\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J})^{-1}\boldsymbol{J}^{\mathrm{T}}F(\omega_n^{\mathrm{MLE}})$.

To avoid the difficulty of convergence in the iterative process, we add the damping coefficient in the iterative process. Suppose the damping factor is χ, and then Eq.(14) becomes

$$\omega_{n+1}^{\mathrm{MLE}}=\omega_n^{\mathrm{MLE}}+\chi\cdot b_n \tag{15}$$

Therefore, the parametric ballistic plane cutting algorithm can be written in the pseudo-code shown in Table 2.

According to $(\hat{a}, \hat{b}, \hat{t}_0)^{\mathrm{T}}$ that are obtained, and the latitude and longitude of the launching point and the launching azimuth, the motion state parameters of the entire boost phase trajectory are calculated. Note that the accuracy of the initial value of the unknown also affects the speed of iterative convergence. Therefore, the initial ω_0^{MLE} setting steps are as follows:

(1) The first two images (i.e., the observation image data at t_1 and t_2), which are detected by the infrared sensor carried by the geosynchronous orbit early warning satellite, are extracted and expressed as the observation unit vectors $(u_1, v_1, w_1)^{\mathrm{T}}$ and

Table 2 Algorithm steps

```
Algorithm: A parameter estimation model of the boost phase based on trajectory plane parametric cutting
Input: Initial value of the pending estimated parameter $\boldsymbol{\omega}$
Output: Final estimation $\boldsymbol{\omega}$
Iterative calculation process
(1) When $n=0$, set the initial value of the parameter to be estimated to $\omega_0$, and set the control limit $\psi$ and the damping factor $\chi=1$;
(2) Conduct the iterative calculation of equation $\omega_{n+1}^{\mathrm{MLE}}=\omega_n^{\mathrm{MLE}}+\chi\cdot b_n$;
(3) Calculate the square of the residuals and $\mathrm{RSS}(\boldsymbol{\omega}_{n+1})=F(\boldsymbol{\omega})^{\mathrm{T}}F(\boldsymbol{\omega})$;
(4) If $\left|\mathrm{RSS}(\boldsymbol{\omega}_{n+1})-\mathrm{RSS}(\boldsymbol{\omega}_{n+1})\right|<\psi$, turn to Step (5);
If $\mathrm{RSS}(\boldsymbol{\omega}_{n+1})<\mathrm{RSS}(\boldsymbol{\omega}_{n+1})$, set $n=n+1$, turn to Step (2);
If $\mathrm{RSS}(\boldsymbol{\omega}_{n+1})>\mathrm{RSS}(\boldsymbol{\omega}_{n+1})$, set $\chi=\chi/2$, turn to Step (2);
(5) Calculate the estimate of the parameter to be estimated based on the group of template trajectory: $\boldsymbol{\omega}=\boldsymbol{\omega}_{n+1}$.
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$(u_2, v_2, w_2)^{\mathrm{T}}$, respectively, under the fixed coordinate system. They intersect with the Earth surface to obtain two points P_1 and P_2. The Earth's center (O) and the points (P_1 and P_2) can determine a ballistic plane. Let the equation of this ballistic plane in the ground fixed coordinate system be $a_0X+b_0Y-Z=0$, and a_0 and b_0 calculated here can be approximately used as the initial values of the parameters a and b to be estimated, respectively.

(2) Estimate the latitude and longitude of the launching point of the ballistic missile according to the observation data of the starting point.

(3) Based on local prior weather information (including data on wind speed, atmospheric pressure, temperature, humidity, and other meteorological factors that affect the trajectory and timing of the target), estimate the release time of the ballistic missile from the launch point to the starting point, and estimate the time of the launch point.

2 Experiment and Analysis

2.1 Experimental data

2.1.1 Simulation parameter setting

The simulation observation platform is a single GEO early warning satellite, with its position at E128° and 0°, which always remains unchanged. The infrared sensor mounted on the satellite is an array gaze camera with a detection period of 10 s. Two kinds of mid-range and long-range ballistic missiles in the boost phase of flight are used as early warning observation targets, and their parameter settings and some geometric performance index parameters are shown in Tables 3 and 4, respectively.

Table 3 Simulation parameter settings of the missile

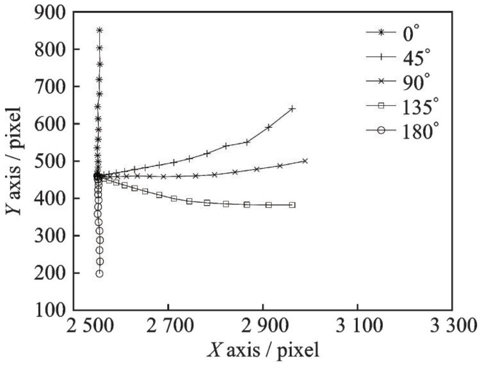
Parameter	Missile number	
	Ⅰ	Ⅱ
Ballistic coefficient	6 115	6 115
Booster series	2	2
Advance pulse (specific impulse)	300	250
Payload mass/kg	2 268	1 824
First-stage fuel mass/kg	38 640	31 904
First-stage total mass/kg	48 988	41 190
First-stage engine working time/s	65	55
Second-stage fuel mass/kg	19 401	15 956
Second-stage total mass/kg	21 669	17 528
Second-stage engine working time/s	65	55
Duration of interstage stalling/s	2	2
Burnout time/s	132	112
Launch longitude/(°E)	128	128
Launch latitude/(°N)	40	40
Launch height/m	0	0
Launching azimuth/(°)	45	45
Launching elevation/(°)	90	90

Table 4 Some geometric performance index parameters of space-borne sensors

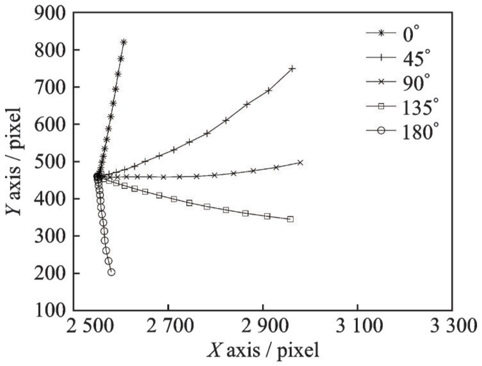
Parameter	Design value
Focal length/m	1
Warning image size	8 000×6 000
Display image size	512×512
Pixel size/(μm×μm)	30×30

2.1.2 Early warning image simulation

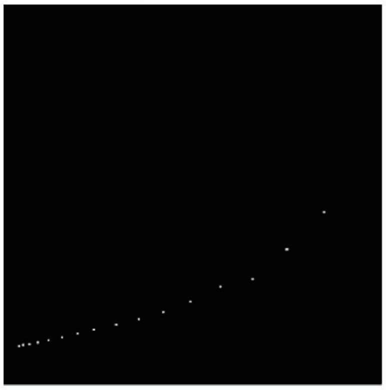
The imaging periods of Missile Ⅰ and Missile Ⅱ are set to 8 s and 6 s, respectively, and the number of missile trajectory points imaged in the boost phase of the two groups of missiles is 16. The 16 frames of images are superimposed to obtain the imaging track. The trajectory points of the missile at different launching azimuth angles are displayed in the same image plane, and the results are shown in Fig.3.



(a) Missile Ⅰ



(b) Missile Ⅱ



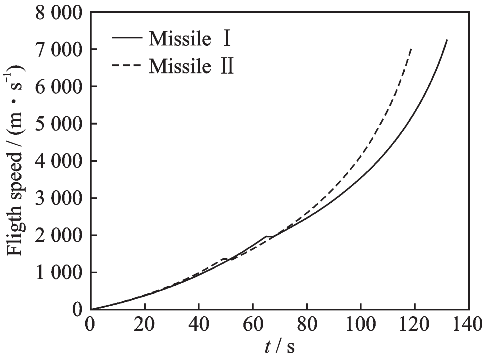
(c) Infrared warning image at angle of attack of 45°

Fig.3 Image plane trajectory of each set of missiles fired downward (different trajectories correspond to different azimuth angles)

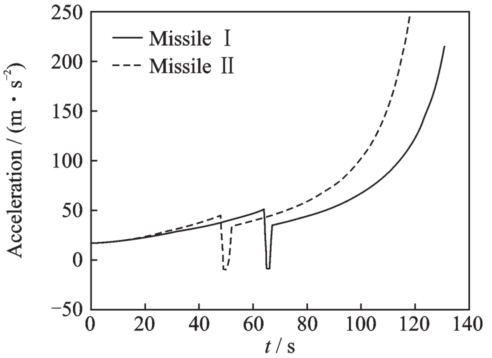
As shown in Fig.3, the image pixel coordinate of the imaging plane of the launching points of Missile Ⅰ and Missile Ⅱ remains unchanged under different launching azimuths, which is in agreement with the experimental design. Under the same launching azimuth, the image plane trajectory characteristics of Missile Ⅰ and Missile Ⅱ are relatively similar, indicating that if only the observation data provided by the early warning image are used, the motion characteristics of the boost phase of the ballistic missile cannot be accurately extracted. To achieve a more accurate parameter estimation of the boost phase, we require a high-precision boost phase motion model.

2.1.3 Hierarchical and segmented trajectory simulation of the boost phase of ballistic missile

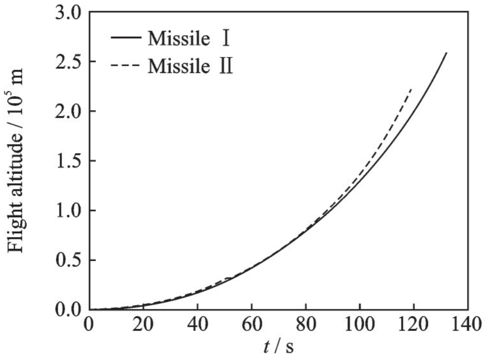
To advance the motion characteristics of the boost phase accurately, in the missile trajectory simulation process, we divide the boost phase into the vertical riser, the procedure turn section, the gravity turning section, and the interstage extinguishing section. The design parameters of the missile include the program attack angle, fuel mass ratio, the effective jet velocity of fuel, the initial total mass of the missile, the number of booster stages, and the quality of boost fuel at all levels. The shutdown time of each booster is calculated by simulation, and the fourth-order Runge-Kutta numerical analysis method is used to calculate the entire boost phase trajectory of the missile in the shooting plane. Moreover, the launching point parameters of the ballistic missile are set, specifically including the longitude of the launching point, latitude of the launching point, height of the launching point, launching time, and launching azimuth. The trajectory simulation results are shown in Figs.4(a—f).



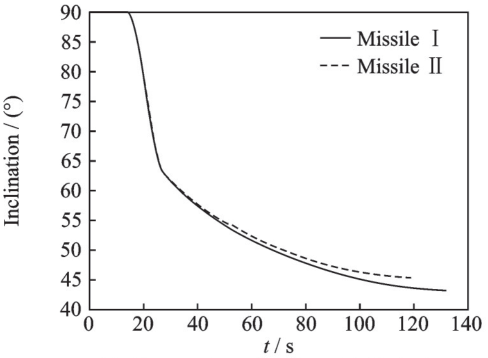
(a) Changes in flight speed with flight time



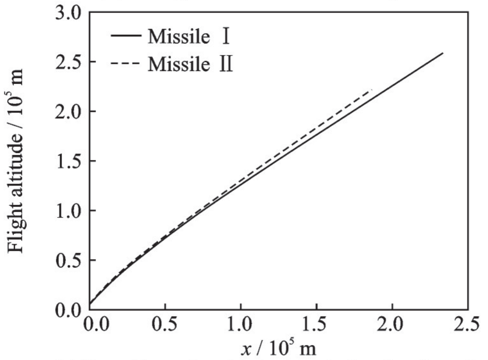
(b) Changes in acceleration with flight time



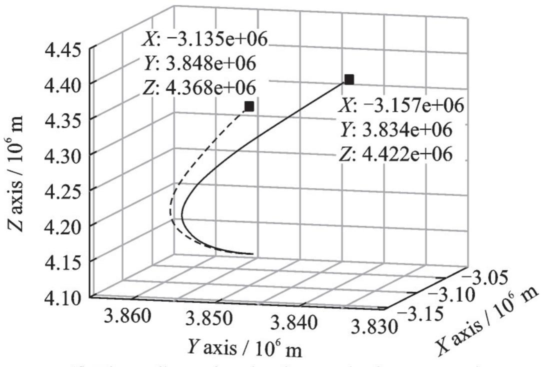
(c) Flight altitude changes with flight time



(d) Changes in inclination with flight time



(e) Two-dimensional trajectory in the shooting plane



(f) Three-dimensional trajectory in the geocentric right-angled fixed coordinate system

Fig.4 Simulation results of the hierarchical and segmented trajectory in the boost phase of the ballistic missile

As shown in Fig.4(a), during the boost phase of the flight, the speed of the two missiles keeps increasing and decreasing for a short time during the inter-stage flameout. At the end of the first stage boost, the speed of Missile Ⅰ reaches 1 971 m/s, and the speed of Missile Ⅱ reaches 1 573 m/s. At the end of the second stage boost, the speed reaches the maximum value of the boost phase. Speeds of Missile Ⅰ and Missile Ⅱ reach 7 263 m/s and 6 275 m/s, respectively. Fig.4(b) shows that the acceleration of the missile gradually increases in the first stage of boost until the flameout stage between stages jumps and becomes smaller rapidly. Then, the acceleration rapidly increases after the second stage of boost ignition starts, with the acceleration rate of change being much larger than that in the first stage. This is because after the first stage boost is completed, the first stage fuel exhaustion and the first stage cabin abandonment greatly reduce the total mass of the ballistic missile. Fig.4(c) shows that the missile flight altitude increases with the flight time, reaching the maximum altitude of the boost phase at the burnout point. As shown in Fig.4(d), the ballistic inclination remains unchanged in the vertical ascent section, and the ballistic inclination decreases rapidly in the program turning section. After entering the gravity-turning section, the ballistic inclination continues to decrease, and the rate of change is relatively gentle. In the two-dimensional trajectory in Fig.4(e), in the vertical ascent section, the highest elevations of Missile Ⅰ and Missile Ⅱ are 1 994.55 m and 1 416.30 m, respectively, and the corresponding first-stage booster lateral displacements are 31.442 km and 23.252 km, respectively. In Fig.4(f), the two sets of projectiles meet the outer and incremental constraints in the basic ballistic constraints of the boost phase; that is, the two sets of projectiles are on the outside of the Earth, and the height changes of each point on the ballistic curve meet the strict incremental relationship.

2.1.4 Simulation of ballistic template library

Because the dynamic characteristics of the missiles at different stages are different, the trajectory characteristics of ballistics are also different. Therefore, this paper establishes a ballistic template library based on the booster motion model of ballistic missiles with multi-stages, in which the thrust acceleration considers the multi-stage boost characteristics and inter-stage flameout. In this case, the drag acceleration adopts a rigorous drag model and considers the influence of the flight altitude on air resistance, and the acceleration of gravity considers the influence of the Earth's rotation. The trajectory data of the boost phase of the missile launched into the plane are stored in the ballistic template library. The boost phase trajectory template library data are combined with the ballistic missile launch parameter information to determine the three-dimensional ballistic data of the boost phase. However, when the actual missile is launched, the same type of missile may change the missile's angle of attack through different flight procedures and change the shutdown time of the missile's boosters at all levels. Thus, the missile is enabled to fly along a low or high trajectory, thereby changing the missile's range and landing point to achieve the goal of avoiding the threat of anti-missile devices as much as possible. Therefore, to reduce the difference between the actual trajectory of the boost phase and the trajectory of the template library, we introduce a dimensionless correction parameter.

2.2 Simulation of parameter estimation for the boost phase of ballistic missiles

Firstly, we generate time-varying missile trajectory data through missile hierarchical segmented trajectory modeling. Then, the parameters of the early warning satellite and sensor are set, and the trajectory data of the missile in the Earth-solid coordinate system are input into the dense geometric imaging model. Finally, the trajectory of the ballistic missile in the warning image is output. The simulation flow is shown in Fig.5.

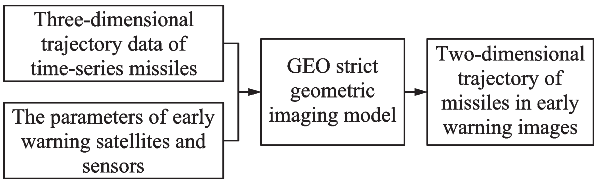


Fig.5 Flow chart of early warning image ballistic missile trajectory generation

To verify the superiority of the boost phase parameter estimation algorithm based on the parametric cutting of the ballistic plane (the parameter cutting method) proposed in this paper, we compare it with the trajectory plane ergodic cutting method (ergodic cutting), which requires certain prior information and low computational efficiency, and cannot meet the requirements of real-time early warning[21]. Besides, by designing the different longitudes of the missile launching point, the launching azimuth angle, and different numbers of observation points, we compare the parameter estimation performance of the two methods in different scenarios. Three simulation scenarios are designed, as shown in Table 5.

Table 5 Different simulation scenarios

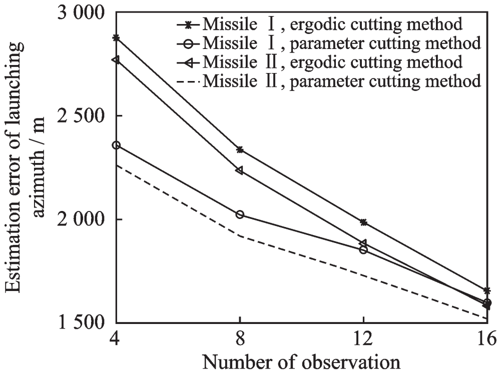
Simulation scenario	Latitude and longitude of the launching point	Launching azimuth	Number of observation point
Simulation scenario Ⅰ	Invariability	Invariability	Variation
Simulation scenario Ⅱ	Invariability	Variation	Invariability
Simulation scenario Ⅲ	Variation	Invariability	Invariability

2.2.1 Simulation of scenario Ⅰ

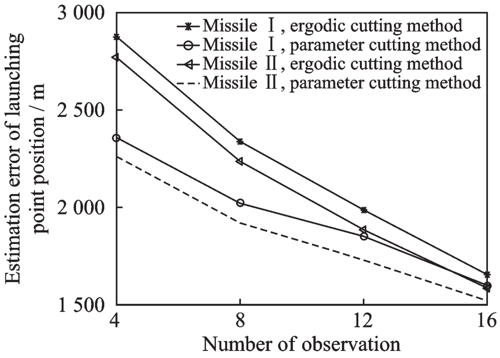
The longitude and latitude of the missile launching point are 128°E and 40°N, respectively. Moreover, when the launching azimuth is 45°, the number of observation points is taken as 4, 8, 12, and 16 according to different observation time intervals. In addition, the average error of the parameter estimation of the two groups of missiles in the boost phase under the corresponding number of observation points is calculated and counted, including the launching azimuth, launching point position, launching time, burnout point position, and burnout point speed. The statistical results of the simulation experiment are shown in Figs.6(a—e).

It can be seen from Fig.6 that under the same number of observation points, the parameter estimation accuracy of the boost phase of the parameter cutting method is slightly better than that of the ergodic cutting method. When the number of observation points reaches 16, the estimation accuracy of each parameter of the boost phase is the highest.

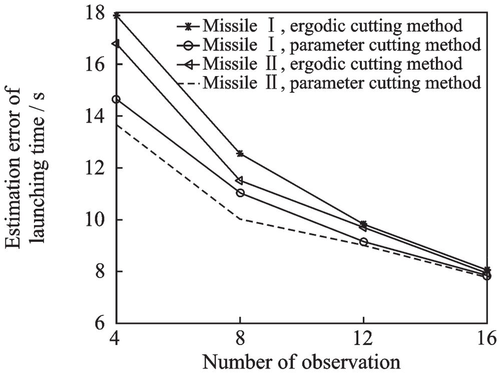
With the decrease in the number of observation points, the estimation errors of the boost phase parameters based on the two algorithms gradually increase, and the changing trends are relatively consistent. This is because the principles of the two estimation algorithms are ballistic plane cutting, and the boost phase parameter calculation is based on the extrapolation of the launching azimuth estimation and the launching time estimation. Further, the trend of the estimation error is relatively consistent. However, the estimation accuracy of the number of observation points, 16, is better than that of the number of observation points, 12. This is because the early warning detection situation is ideal when the number of observation points is 16. Moreover, the detection period almost runs through the entire boost phase, which makes it difficult to meet the requirement in actual detection. Therefore, in the experiment, the number of observation points of scenario Ⅱ and scenario Ⅲ is set to 12.



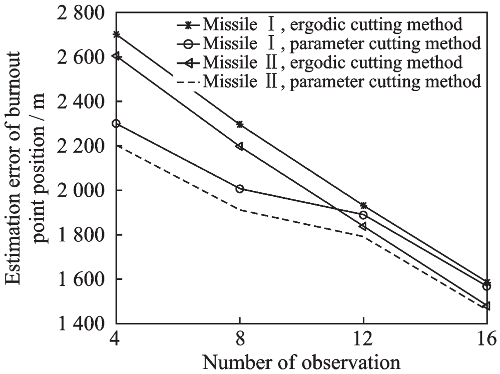
(a) Estimation error of the launching azimuth



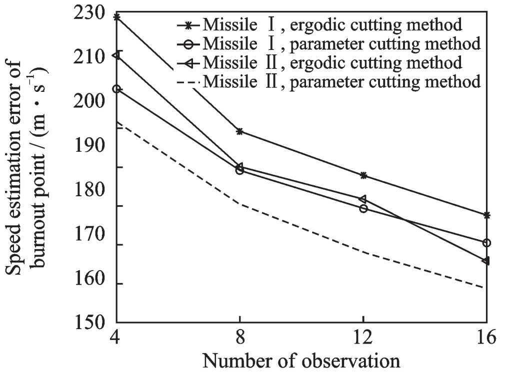
(b) Estimation error of the launching point position



(c) Estimation error of the launching time



(d) Estimation error of the burnout point position

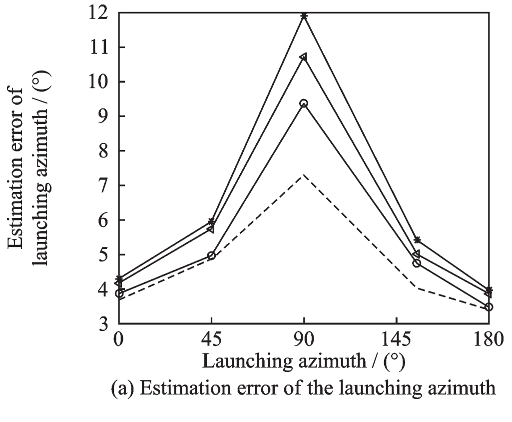


(e) Speed estimation error of the burnout point speed

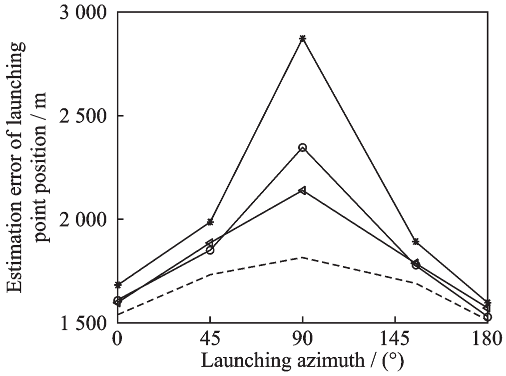
Fig.6 Average error of parameter estimation in the boost phase of Missile Ⅰ and Missile Ⅱ under the different number of observation points

2.2.2 Simulation of scenario Ⅱ

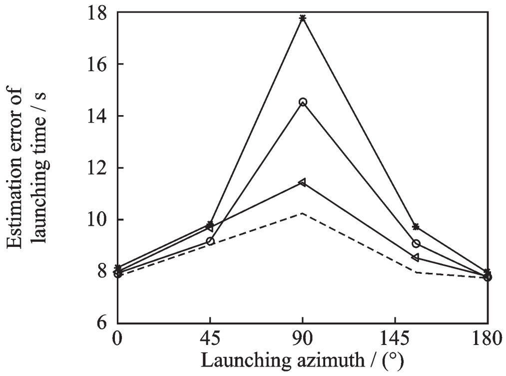
The longitude and latitude of the missile launching point are 128°E and 40°N, respectively. The number of observation points is 12, and the launching azimuth is 0°, 45°, 90°, 135°, and 180°, with the simulation experiment statistics shown in Figs.7(a—e).



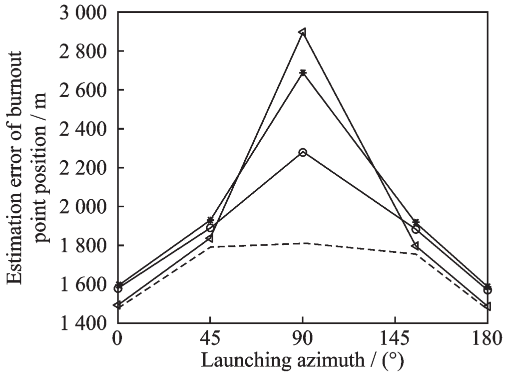
(a) Estimation error of the launching azimuth



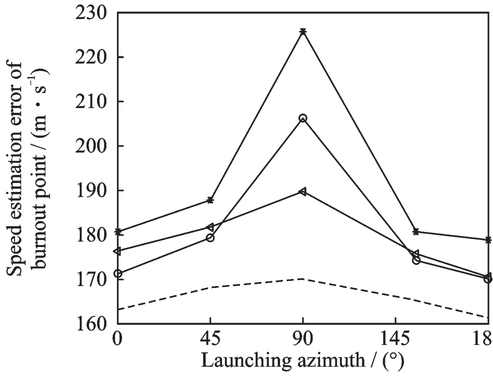
(b) Estimation error of the launching point position



(c) Estimation error of the launching time



(d) Estimation error of the burnout point position



(e) Speed estimation error of the burnout point

Fig.7 Average error of the parameter estimation in the boost phase of Missile Ⅰ and Missile Ⅱ at different launching azimuths

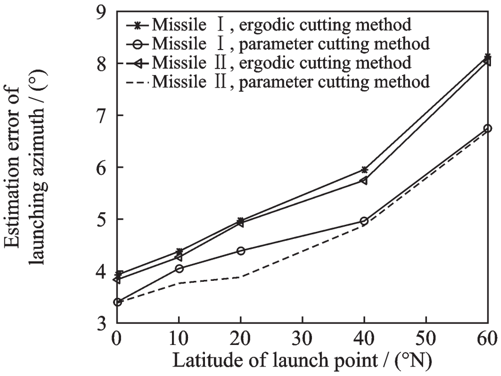
It can be seen from Fig.7 that when the launching azimuth is 90°, the estimation error of the parameters of Missile Ⅰ and Missile Ⅱ in the boost phase is the largest. When the launching azimuth rises from 0° to 90°, the estimation errors of the parameters in the boost phase based on the two algorithms gradually increase. When the launching azimuth rises from 90° to 180°, the estimation errors of the parameters of the boost phase based on the two algorithms gradually decrease. The changing trend in the parameter estimation error is still consistent with the changing trend of the shooting angle estimation error. This is because when the launching azimuth is close to 90°, the first few frames of the time sequence trajectory points in the boost phase of the missile in the launching plane almost overlap, so the motion characteristics in the boost phase cannot be accurately extracted.

When the launching azimuth is the same, the parameter estimation accuracy in the boost phase of the parameter cutting method is better than that of the ergodic cutting method. When the launching azimuth is 90°, the estimation error of the parametric cutting method is significantly lower than that of the ergodic cutting method. When the launching azimuths are 0° and 180°, the estimation errors of the two algorithms are relatively small. This is because the ergodic cutting method needs to use the traversal step length of the starting point and the endpoint height to determine all possible candidate ballistic cutting planes. In addition, when estimating the launching azimuth, the ergodic cutting method first takes the positions of the starting point and the target point as the initial value and obtains the position of the landing points through the iterative calculation of the boost phase, the free phase, and the re-entry of the trajectory. Then, it adjusts the launching azimuth according to the deviation between the landing position and the target point. It can be seen that the ergodic cutting method is highly dependent on the height of the starting point and the endpoint. When the shooting angle is close to 90°, the time sequence imaging points of the first few frames come closer, and the extracted starting point and the endpoint height are extremely inaccurate, so the accuracy of the estimation results obtained by the ergodic cutting method is greatly reduced. However, the parameter-cutting method proposed in this paper expresses the coordinates of the trajectory cutting point by cutting the ballistic plane coefficient. This can eliminate the dependence on a priori starting point and the endpoint height interval so that the estimation accuracy is less affected by the starting point and end point information.

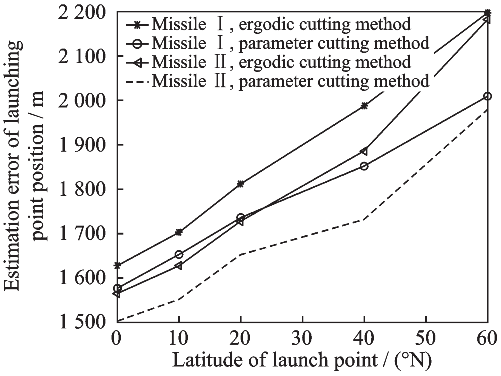
2.2.3 Simulation of scenario Ⅲ

When the launching azimuth of Missile Ⅰ and Missile Ⅱ is 45°, the number of observation points is set to 12. The statistical results of the simulation experiment are shown in Figs.8(a—e).

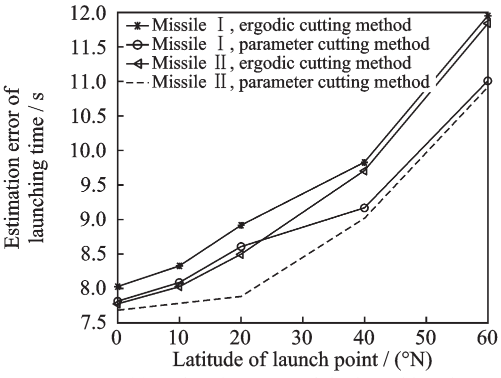
It can be seen from Fig.8 that when the latitude angle of the launching point is 0°, the estimation accuracy of each parameter of the boost phase of the two methods is the highest. When the latitude of the launching point increases from 0° to 60°, the estimation errors of the parameters in the boost phase based on the two algorithms gradually increase. This is because the longitude and latitude of the GEO early warning satellite are set to 128°E and 0°, respectively. Thus, when the launching azimuth of the missile is 45° and the launching point latitude is 0°, the GEO early warning satellite's observation of the missile is a vertical one, with the geometric conditions being poor.



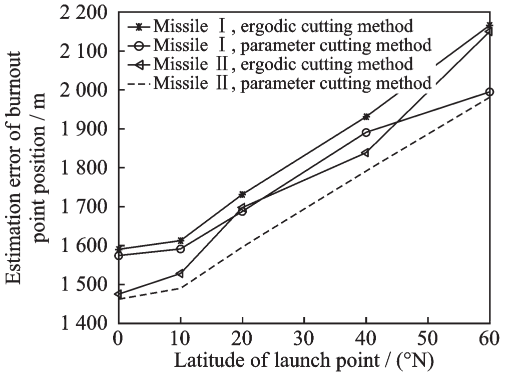
(a) Estimation error of the launching azimuth



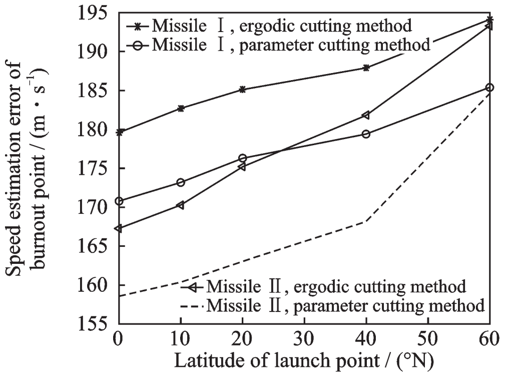
(b) Estimation error of the launching point position



(c) Estimation error of the launching time



(d) Estimation error of the burnout point position



(e) Speed estimation error of the burnout point

Fig.8 Average error of the parameter estimation in the boost phase of Missile Ⅰ and Missile Ⅱ at different launching point latitudes

When the observation latitudes are the same, the parameter estimation accuracy of the boost phase of the parameter cutting method is generally better than that of the ergodic cutting method. When the latitude of the launching point is 60°N, the estimation error difference between the two methods is the largest. This is because the latitude and longitude of the launching point and the target point also affect the estimation accuracy of the launching azimuth, and then affect the launching point and the height of the end point. Moreover, the change in the latitude of the launching point has the greatest impact on the launching point and the endpoint height. When the latitude of the launching point increases and approaches 60°N, the imaging point spacing of the next few frames is too large, and the end point of the boost phase even exceeds the imaging range. Thus, the obtained motion characteristics of the boost phase are inaccurate. As a result, the estimation accuracy of the ergodic cutting method, which depends on the endpoint height, becomes lower and lower.

Finally, according to the application of the two algorithms in the parameter estimation of the space non-cooperative target in the propulsion phase under the observation of a single early warning satellite, and the analysis of the efficiency requirements of the non-cooperative target trajectory prediction algorithm, the efficiency of the two algorithms is further analyzed. Further, the average value of the maximum and minimum time spent by the algorithm is statistically analyzed through simulation experiments. The results are shown in Table 6. Compared with the time spent by the ergodic cutting algorithm, the time spent by the parametric cutting algorithm is significantly reduced. The average time of the parametric cutting algorithm is only 6.8 s, and the maximum time is 7.3 s. This is because the number of iterations in the parametric cutting algorithm is far less than that required by the ergodic cutting algorithm.

Table 6 Time consumption experimental results of different algorithms s

Algorithm	Maximum time consumption	Minimum time consumption	Average time
Ergodic cutting	15.212	11.335	13.171
Parametric cutting	7.346	6.143	6.784

3 Discussion

In the trajectory simulation of the boost phase of a ballistic missile, this paper constructs a hierarchical segmented motion model. The flight duration of the boost phase is 132 s and 122 s, and the flight transverse to the longitudinal ratio is close to 1∶1. The two sets of missiles in the simulation design meet the ballistic constraints of the boost phase of the mid-range and long-range ballistic missile, and meet the experimental expectations. The results verify the reliability of the segmented multi-staged booster model of the boost phase of the ballistic missile established in this paper.

In the parameter estimation of the boost phase of a ballistic missile, when a single GEO early warning observation is made for a mid-range and long-range missile launched from middle and low latitudes at any angle of firing, both the parameter estimation algorithm of the boost phase based on the ergodic cutting of the ballistic plane and the parameter estimation algorithm of the boost phase of the ballistic plane parametric cutting can realize the estimation of the parameters of the boost phase of the ballistic missile. Under the same number of observation points, launching azimuth, and launching point latitude, the boost phase parameter estimation algorithm based on ballistic plane parametric cutting generally has better estimation accuracy than the boost phase parameter estimation algorithm based on the ergodic cutting of the ballistic plane. When the missile launching azimuths are 0° and 180°, the number of observation points is set to 16, the launching point latitude angle is 0°, and the parameter estimation accuracy of the boost phase using the parametric cutting method is the highest. This is because the observation geometry in this scenario is ideal, and the observation data are complete. Especially in the low- and mid-latitude regions, the parameter estimation algorithm of the boost phase based on the parametric cutting of the ballistic plane can realize the parameter estimation of the boost phase of the mid-range and long-range ballistic missiles that need high precision.

4 Conclusions

The parameter estimation of the boost phase of a single geosynchronous orbit early warning satellite for a non-cooperative space target is studied. A parameter estimation algorithm based on ballistic plane parameter cutting is proposed. The LOS cutting point of the time series is represented by the ballistic plane coefficient. This method reduces the dependence on prior information and ensures the real-time reliability of parameter estimation. The experimental results show that the algorithm has a high estimation accuracy for the space target with any launching azimuth in the middle and low latitudes, and provides a practical and feasible method for the single-star early warning system.

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Author contributions Dr. **XIAO Hui** designed the study and compiled the models. Mr. **ZHU Chongrui** designed and conducted the experiments, performed data analysis, interpreted the results, and led the manuscript preparation. Mr. **LIU Xinqi** conducted the analysis and interpreted the results. Mr. **YU Yifan** completed the paper. Prof. **SHENG Qinghong** contributed to the discussion and background of the study. Mr. **YANG Rui** contributed to the computational modeling and simulation aspects. All authors commented on the manuscript draft and approved its submission.

Competing interests The authors declare no competing interests.

（Production Editor：ZHANG Huangqun）

基于时间视线约束的无先验信息依赖非合作空间目标估计算法

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摘要：在单星观测条件下，高精度空间非合作目标助推段参数估计需要先验信息。为了提高在无先验信息情况下的精度，提出了一种基于弹道平面参数切割的助推段参数估计模型。利用穿过地心的平面和切割序列视线(Line of sight, LOS)生成轨迹切割平面。直接以轨迹切割面系数作为待估计参数，建立了空间非合作目标运动参数估计模型，采用高斯-牛顿迭代法求解飞行参数。实验结果表明，本文提出的估计算法对先验信息依赖较弱，具有较高的估计精度，为单星预警下空间非合作目标参数估计提供了一种实用的新思路和新方法。

关键词：运动参数估计；落点估计；红外预警；升压相位建模；轨迹数据库建设