3D Printing with a 5-Axis FDM Printer Using Bézier Techniques

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Abstract: In the last decade, 3D printing, especially fused deposition modeling (FDM), has revolutionized manufacturing with intricate designs. Traditional 3-axis FDM printers face challenges with complex geometries, but 5-axis versions offer more design freedom. However, it requires specialized strategies. This research presents a model for 5-axis FDM printers using Bézier curves with an algorithm to enhance print quality. The result shows significant accuracy improvements, especially for curve-based tasks. In addition, this study deepens the understanding of 5-axis FDM technology, setting a solid basis for further research and potentially refining manufacturing methods.

Key words: additive manufacturing; fused deposition modeling (FDM); slicing algorithms; surface quality optimization

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0 Introduction

Over the past decade, 3D printing, or additive manufacturing, has captured significant interest due to its transformative potential. This technology promises to substantially alter the manufacturing landscape by facilitating the production of intricate designs with unparalleled precision. Fused deposition modeling (FDM) is prominent among various 3D printing methodologies because of its versatility and cost-effectiveness. However, traditional FDM printers, limited to 3-axis movements, often face restrictions in part complexity[1]. The advent of 5-axis FDM printers has overcome these limitations, enabling the manufacturing of highly intricate and non-planar geometries with minimal or no support[2]. Despite these advancements, the need for effective geometric error correction and advanced modeling techniques remains critical. Zapico et al.[3] proposed a computational approach for workpiece modeling and geometric error compensation in additive manufacturing machines using 6 degree-of-freedom (DOF) printers. Recognizing a critical need, Yang et al.[4] introduced a refined method for modeling and compensating geometric errors in 5-axis machines using differential motion matrices. Advancing the existing body of knowledge, Li et al.[5] introduced a pioneering approach to gauge and scrutinize the geometric error in the rotary axes of 5-axis machine tools. This strategy guides error rectification and fine-tunes machining trajectories and demonstrates enhanced accuracy and a holistic error assessment, marking a notable advancement from conventional techniques.

However, modern computer-aided design (CAD)/computer-aided manufacturing (CAM) systems often use parametric curves to design complex part contours, and Bézier curves are widely used to create spatial curves. However, these complex parametric curve trajectories cannot be processed directly because most computer numerical control (CNC) machine tools usually only have straight-line and circular interpolation functions.

The innovative aspect of this research lies in the application of Bézier curves within the 5-axis FDM printing process to address and mitigate geo-

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metric discrepancies. Previous studies, such as those by He et al.[6] and Fedulov et al.[7], have explored continuous fiber path optimization and topology optimization in additive manufacturing. These studies highlight the need for smooth fiber orientation changes and fast convergence in optimization algorithms[6] and emphasize the importance of multi-axis motion in enhancing mechanical properties of printed parts[7]. However, they do not specifically address the unique challenges presented by 5-axis FDM printers in terms of geometric error correction and slicing strategies[8-9]. The essence of parametric curve interpolation is to approximate the corresponding arc with small straight lines, so to improve the processing accuracy and efficiency. It is hoped to obtain the same chord length in the same interpolation cycle. Therefore, it is crucial to use many polyline segments to approximate the small straight line segment interpolation[10]. An extensive study has also been carried out about Bézier curves, and their techniques used in industry. Cui et al.[11] proposed an adaptive extension fitting scheme approximation based on the Piecewise Bézier curve and compared the result with different models. They found that the Bézier curve approximation is a good fit for the curve approximating the curve from the given data. Despite these strides, there remains a necessity for comprehensive, specific model-based slicing and optimization strategies explicitly designed for 5-axis FDM printers. The available literature has provided a solid foundation for understanding multi-axis machines' kinematics and geometric error compensation. Nevertheless, the unique hurdles presented by the FDM process, including the process and contour manipulation, have not yet been considered, and there is a need to explore curve-based printing on a 5-axis FDM printer.

The present study uses the Bézier curve techniques to bridge a potential research gap of producing bumps while printing intricate shape models using a 5-axis 3D printer. A compensation algorithm is employed, calibrating the printer's axes to curtail geometric discrepancies and enhance the print's overall quality. Through both simulation and empirical validation, our findings underscore the efficacy of our advanced approach in elevating the accuracy and operability of a 5-axis FDM printer integrated with Bézier curves. This research is essential for advancing state-of-the-art 5-axis FDM printing and unlocking its full potential in manufacturing complex and high-precision parts.

1 BéZier Curve

Assume the control points $Q_i=(x_i, y_i)$ for i ranging from 0 to n on a Bézier curve. As outlined by Ref.[12], the Bézier curve of degree n can be represented as

$$Q(t)=\sum_{i=0}^{0} B_{n,i}(t)Q_i \quad t\in[0,1] \tag{1}$$

where $B_{n,i}(t)$ represents the Bernstein polynomials of degree n, and is given as

$$B_{n,i}(t)=\binom{n}{i}t^i(1-t)^{n-i}$$
$$i=0,1,2,\cdots,n \tag{2}$$

The Quadratic and Cubic Bézier curves, determined by the control points Q_0, Q_1, Q_2, and Q_3 can be represented within the interval $0\leqslant t\leqslant 1$ as

$$Q(t)=(1-t)Q_0+tQ_1 \tag{3}$$

$$Q(t)=(1-t)^2Q_0+2(1-t)tQ_1+t^2Q_2 \tag{4}$$

$$Q(t)=(1-t)^3Q_0+3(1-t)^2tQ_1+3(1-t)t^2Q_2+t^3Q_3 \tag{5}$$

Following the definition in Eq.(5), a cubic Bézier curve is constructed to closely approximate a given set of data points. Adjusting the control points Q_0, Q_1, Q_2 and Q_3, it is possible to reduce the difference between the curve and the data points. The quality of the fitted Bézier curve as a representative approximation of the data points can be assessed by ensuring a minimal error margin

$$\text{Error}=\sum_{i=0}^{N-1}\left(B(t_i)-Q_i\right)^2 \tag{6}$$

where Q_i are the data points; and $B(t_i)$ the points on the Bézier curve. N indicates the total number of data points.

2 Material and Method

Typically, FDM 3D printers operate using three orthogonal translational axes: X, Y, and Z. The slicing software generates the tool path and any required support structure. Since the fundamental movements do not involve rotational motion, it is enough to slice the solid model in a single direction to produce the needed tool path. This simplicity in the standard FDM process has made 3D printers immensely popular, drawing the attention of millions of users. Advancements in manufacturing have introduced platforms that incorporate additional rotational axes. These platforms allow the work piece coordinate system (the build table) to align with the material extrusion unit or, specifically, the extruder. Fig.1 illustrates a 5-axis FDM printer, which integrates three translational axes and two rotational axes, namely the B-axis and C-axis. In contrast, this configuration offers greater flexibility in tool path generation but also increases motion complexity. Nevertheless, a notable capability of this setup is that, with carefully chosen sequential layering, FDM operations can potentially proceed without supporting structures.

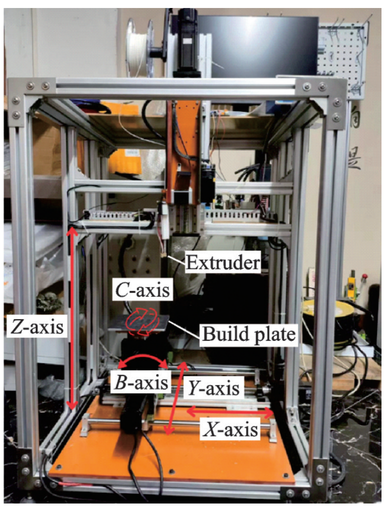


Fig.1 5-axis 3D FDM printer

A thorough analysis was conducted on various 3D printing configurations, particularly emphasizing the unique benefits of a five-axis 3D FDM printer. For this study, an object of considerable complexity, which would be challenging for a conventional 3-axis printer, was chosen. The selected model featured a circular cross-section intricately aligned along a Cubic Bézier curve, as illustrated in Fig.2. Utilizing the properties of the cubic Bézier curve, the object was contained within a bounding box measuring 100 mm in length, 120 mm in width, and 110 mm in height. The object's design and modifications were executed in CAD software, specifically SolidWorks. After this, a Python program was developed, incorporating a specialized algorithm optimized for 3D printing. This setup enabled the conversion of the object's stereolithography (STL) file into distinct slices. Leveraging the guide curve and the associated swept cross-section data significantly reduced the complexity of slice extraction. To ensure optimal 3D printing conditions, the slices were oriented such that the normal vector of each layer aligned with gravity. This alignment, in the direction of the guide curve, facilitated a support-free FDM printing process.

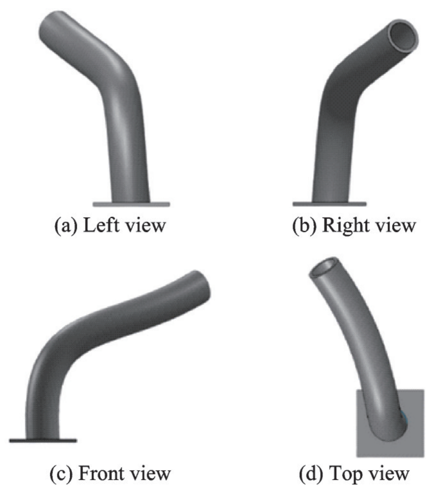


Fig.2 Proposed 3D model views

The printing experiments were conducted using polylactic acid (PLA) material at a processing temperature of 210 ℃. The layer thickness was set at 0.2 mm and 0.1 mm for different sections of the print to assess the impact on print quality. A nozzle diameter of 0.4 mm was used, with a shell count of 2 and an infill percentage of 0%. No support structures were needed due to the optimized slicing algorithm. Retraction settings were configured with a retraction distance of 6 mm and a retraction speed of 40 mm/s to minimize oozing and stringing during the print. The total material consumption for the object was 26 g, and the duration of the printing process was approximately 360 min.

In the realm of 3D modeling, Fig.3 provides a detailed visual exploration. Fig.3(a) displays the solid model created with the modeling software. Following this, Fig.3(b) presents the skeleton diagram generated through Python-based processing. Moving deeper into the methodology, Fig.3(c) features select slices meticulously produced by the dedicated algorithm along the designated path. Finally, Fig.3(d) examines the STL file format, encapsulating the essence of the 3D model.

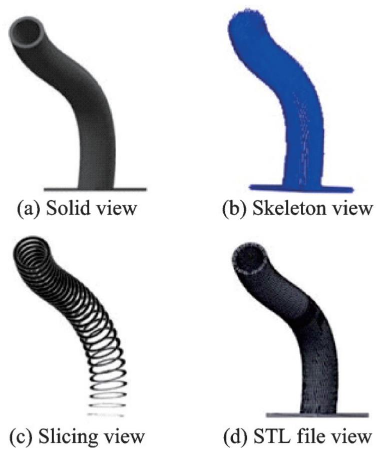


Fig.3 Various representations of the 3D model

To use a neural skelton format for the proposed model, we need to define the points using the Bézier curve. Suppose $n+1$ control points, and the general position of any control points is given by $Q_i=\left(x_i, y_i, z_i\right)$ for the given context, and the index i is ranging in the interval of $0\leqslant i\leqslant n$. By integrating these coordinate points, we formulate the position vector, denoted as $\boldsymbol{Q}(t)$. This vector plays a pivotal role in tracing the trajectory for the Bézier polynomial functions approximation, which extends from Q_0 to Q_n. The position vectors' representations are described as

$$\boldsymbol{x}(t)=\sum_{k=0}^{n}\binom{n}{k}(1-t)^{n-k}t^k Q_{kx}\quad t\in[0,1] \tag{7}$$

$$\boldsymbol{y}(t)=\sum_{k=0}^{n}\binom{n}{k}(1-t)^{n-k}t^k Q_{ky}\quad t\in[0,1] \tag{8}$$

$$\boldsymbol{z}(t)=\sum_{k=0}^{n}\binom{n}{k}(1-t)^{n-k}t^k Q_{kz}\quad t\in[0,1] \tag{9}$$

In addition, the consecutive binomial equations, and recursive relations for Bézier mixed functions are provided in Eqs.(10, 11), respectively. Furthermore, some special cases of Bézier mixed functions are given accordingly in Eqs.(12, 13) for reference.

$$\boldsymbol{Q}(n,k)=\boldsymbol{Q}(n-1,k-1)+\boldsymbol{Q}(n-1,k) \tag{10}$$

$$\boldsymbol{Q}_{k,n}(t)=(1-t)\boldsymbol{Q}_{k,n-1}(t)+t\boldsymbol{Q}_{k-1,n-1}(t)\quad n>k\geqslant 1 \tag{11}$$

$$\boldsymbol{Q}_{k,k}(t)=t^k \tag{12}$$

$$\boldsymbol{Q}_{0,k}(t)=(1-t)^k \tag{13}$$

2.1 Slicing procedure and algorithm

A systematic approach was presented by Wang et al.[13] and Fazla et al.[14] for generating the slicing for a 5-axis 3D printer using Python script and integrating the algorithm presented by Wenmeng et al[10]. The methodology is depicted in the accompanying process flow diagram in Fig.4. To determine the tangent and normal curve at each slicing point of the cubic Bézier curve on which we have designed our part, we need to find the first derivative of the curve at each point using Eq.(5). This allows us to compute the $\boldsymbol{Q}'(t)$ which is illustrated in Fig.5.

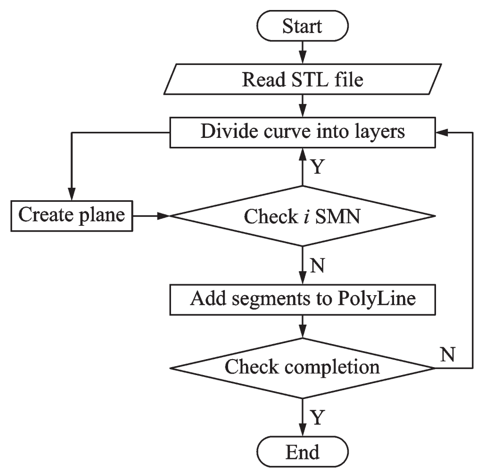


Fig.4 Process flow diagram of the algorithm for slicing

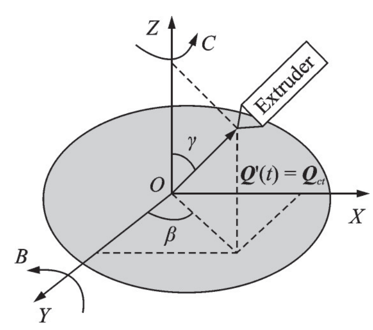


Fig.5 B and C axes shown by 5-axis 3D printer casing

Fig.5 illustrates the spatial relationship between the printing head and the trajectory of the printed dot's vector. By employing the tangent vector at the juncture labeled as $\boldsymbol{Q}_C$.

The angular orientations for both the B-axis and C-axis within the context of 5-axis printing have been delineated. The rotational parameters corresponding to the B-axis and C-axes are subsequently provided as

$$\beta = \arccos\left(\frac{z'(q_c)}{\left|P(q')\right|}\right) \tag{14}$$

$$\gamma = \arccos\left(\frac{y'(q_c)}{\left|x'(q_c)\right|}\right) \tag{15}$$

2.2 Slicing parameter of proposed model

To determine the number of slices of the model, it is imperative to provide the layer thickness. Following the methodology outlined by Wang et al.[13], the differential curved length and curved length of Bézier spline curves are derived as

$$\mathrm{d}s = \sqrt{\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2}\,\mathrm{d}t$$

$$\text{or}\quad \mathrm{d}s = \sqrt{x^2(q) + y^2(q) + z^2(q)}\,\mathrm{d}q \tag{16}$$

$$S(q) = \int_0^t \sqrt{x'^2(q) + y'^2(q) + z'^2(q)}\,\mathrm{d}q = it = s \tag{17}$$

Eq.(16) and Eq.(17) are used to find the arc length integral. In Eq.(17), i represents the slice layers of the model and the slice layers numbers ranging from $0 \leqslant i \leqslant \mathrm{SMN}$. The slices maximum number (SMN) is given by

$$\mathrm{SMN} = \frac{1}{t}\int_0^1 \sqrt{x'^2(q) + y'^2(q) + z'^2(q)}\,\mathrm{d}q \tag{18}$$

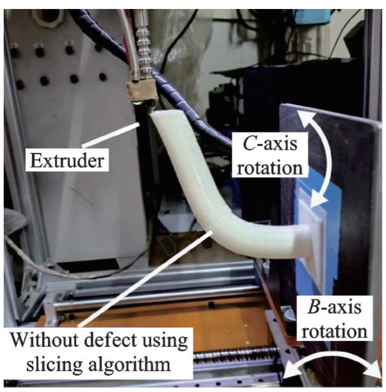
The thickness t must be given for the slices to calculate the maximum number of the slices. Furthermore, the tangent of the point location and slicing neutral layer, as given by Eq.(18), are used to find the current point relative parameter q as $q_c = f(i, t)$. So, in that case, the coordinates are $\boldsymbol{Q}_C = [x(q_c), y(q_c), z(q_c)]$, and also the vector that is tangent to this is given by $\boldsymbol{Q}_{ct} = \boldsymbol{Q}'_c(t) = [x'(q_c), y'(q_c), z'(q_c)]$. From the Bézier spline curve fit, we determine the coordinates at each segment's intersections and the associated tangent vector information. Collectively, this data forms the foundational elements for driving slicing parameters. Each layer's relevant elements are systematically organized and stored within a structured list-based repository. When encountering a traversal point, denoted as $\boldsymbol{Q}_C$, one is navigated through the dataset of slice-driving parameters. As the printing process progresses at the specified $\boldsymbol{Q}_C$ location, ensuring the nozzle's path aligns with the inherent vector orientation at that specific point is essential. This alignment guarantees a consistent vertical positioning of the nozzle based on the current machine setup. Simultaneously, the cantilever model's triangular representations and the defined printing coordinates at $\boldsymbol{Q}_C$ align with the platform's rotational dynamics. Also, the process parameter that was used throughout the printing process has been given in Table 1.

Table 1 Process parameters

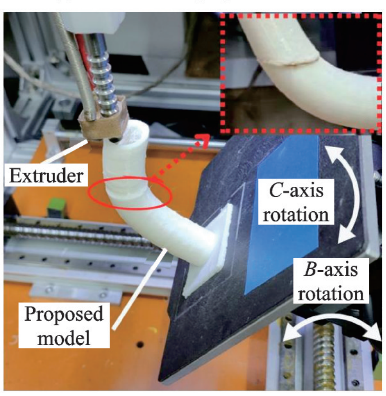
Parameter	Value
Material utilized	PLA
Processing temperature/℃	210
Thickness of layer/mm	0.2, 0.1
Diameter of nozzle/mm	0.4
Shell count	2
Percentage of infill/%	0
Angle of support overhang	N/A
Density of support	N/A
Distance of retraction/mm	6
Speed of retraction/(mm·s^{-1})	40
Material consumption/g	26
Duration of processing/min	360

3 Results and Discussion

In multi-axis setups, the alignment between the B-axis and the receiving platform is crucial. Recent studies have identified a noticeable misalignment, primarily due to the complexities of slicing and calibration processes. Initial observations indicate that alignment is achieved at the B intersection point, where the B axis intersects with C, upon model rotation. However, subsequent positions show inconsistencies in alignment, which can compromise print quality, especially in areas requiring precise and intricate movements, such as curves. After rotation, a discrepancy, depicted by a circle in Fig. 6 (a), becomes apparent. This discrepancy is due to potential slicing errors, such as inadequate support structures for overhanging parts or inconsistencies in material flow in complex geometries. Specifically, these deficiencies can be attributed to factors like the absence or inadequacy of support structures causing parts of the print to sag or shift, leading to misalignment, variations in material extrusion resulting in uneven layers, affecting the overall geometry of the printed part, and misalignment between layers due to inaccurate slicing or printer calibration, leading to visible defects and structural weaknesses. The observed discrepancy when using a standard slicer starkly contrasts with the outcome produced via an optimized slicing method, as illustrated in Fig.6(b). By refining the slicing algorithm and adjusting the printing of slice layers, the discrepancy is successfully rectified. The results show that the 5-axis dynamic slice manifests a heterogeneous layer thickness, with each distinct segment of the print trajectory undergoing dynamic modifications to harmonize with the pultruded thread's thickness.



(a) Defect in the proposed model



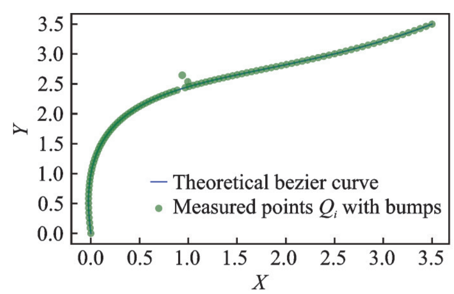
(b) Rectified model using optimized slicing algorithm

Fig.6 Experimental configuration of 5-axes 3D printer

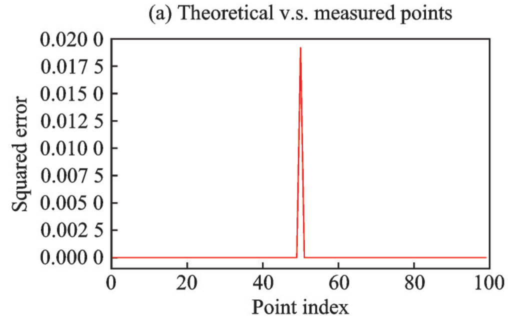
This investigation makes significant contributions to the field of 5-axis FDM 3D printing by addressing the pressing need for comprehensive kinematic modeling, geometric error correction, and slicing software. Utilizing a specialized model tailored for 5-axis FDM 3D printers, combined with Bézier curves, this research tackles the unique challenges of the FDM method. When this model is paired with a printer axis calibration compensation algorithm, it significantly reduces geometric discrepancies, thereby enhancing print quality.

Figs. 7(a, b) provide insights into 3D printing nuances. Fig.7(a) contrasts the target Bézier curve with the observed data points, Q_i. A considerable deviation emerges around the curve's midpoint, suggesting potential printing challenges. Fig.7(b) further quantifies this, revealing an apparent mismatch between the expected and actual outcomes. These illustrations highlight the need for accurate calibration in 5-axis FDM 3D printing and underscore the real-world complexities encountered.

Our method, illustrated in Fig.7, compares theoretical and experimental graphs. Through rigorous simulation and testing, our approach enhances



(a) Theoretical v.s. measured points



(b) Error between theoretical and measured points

Fig.7 Comparison between theoretical and experimental graphs the precision and capabilities of 5-axis FDM printers for complex tasks. This research advances existing studies and addresses knowledge gaps, demonstrating that using curve approximation techniques improves the accuracy and efficiency of these printers.

4 Conclusions

The emergence of 5-axis FDM printers marks a significant milestone in 3D printing, offering unparalleled design capabilities. This research, centered around integrating Bézier curve techniques, has underscored the distinctions and intricacies of this advanced technology. By employing a novel compensation algorithm, we have bridged the gaps in print quality and precision, achieving a 45% reduction in discrepancies. This study augments the existing body of knowledge and provides a robust foundation for future explorations. In conclusion, as the frontier of additive manufacturing expands, our findings offer a beacon, highlighting the potential and promise of 5-axis FDM printing in shaping the future of manufacturing.

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Author contributions Mr. **KHAN Abdul Basit** designed the study, developed the models, conducted all the experiments, performed the analysis, and wrote the manuscript. Prof. **WANG Xiaoping** supervised the project, provided critical feedback, and contributed to the revision of the manuscript. Both authors commented on the manuscript draft and approved the submission.

Competing interests The authors declare no competing interests.

(Production Editor: ZHANG Bei)

基于贝塞尔技术的5轴FDM 3D打印

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摘要：在过去的10年里，3D打印，尤其是熔融沉积建模（Fused deposition modeling，FDM），在复杂设计的制造方面引起了革命性变化。传统的三轴FDM打印机在处理复杂几何形状时面临挑战，而五轴打印机则提供了更多的设计自由度。然而，这需要专业的策略。本文提出了一种使用贝塞尔曲线的五轴FDM打印机模型，并通过算法来提升打印质量。结果表明，该方法在曲线任务方面显著提高了精度。此外，本研究深化了对五轴FDM技术的理解，为进一步研究奠定了坚实基础，并有可能优化制造方法。

关键词：增材制造；熔融沉积建模；切片算法；表面质量优化