Solution for Output Coordination Equations of Several Typical Parallel Six-Dimensional Acceleration Sensing Mechanisms

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Abstract: Aiming at the problem that it is difficult to generate the dynamic decoupling equation of the parallel six-dimensional acceleration sensing mechanism, two typical parallel six-dimensional acceleration sensing mechanisms are taken as examples. By analyzing the scale constraint relationship between the hinge points on the mass block and the hinge points on the base of the sensing mechanism, a new method for establishing the dynamic equation of the sensing mechanism is proposed. Firstly, based on the scale constraint relationship between the hinge points on the mass block and the hinge points on the base of the sensing mechanism, the expression of the branch rod length is obtained. The inherent constraint relationship between the branches is excavated and the branch coordination closed chain of the "12-6" configuration is constructed. The output coordination equation of the sensing mechanism is successfully derived. Secondly, the dynamic equations of "12-4" and "12-6" configurations are constructed by the Newton-Euler method, and the forward decoupling equations of the two configurations are solved by combining the dynamic equations and the output coordination equations. Finally, the virtual prototype experiment is carried out, and the maximum reference errors of the forward decoupling equations of the two configuration sensing mechanisms are 4.23% and 6.53%, respectively. The results show that the proposed method is effective and feasible, and meets the real-time requirements.

Key words: six-dimensional acceleration sensor; parallel mechanism; topological configuration; coordination equation; dynamics

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0 Introduction

With the development of science and technology and the improvement of people's understanding of the objective world, it is more and more important to detect the six-dimensional motion of objects in three-dimensional space. For example, in order to realize the dynamic control of the end effector of the space robot, the complete motion information of the robot body must be obtained in real time. In addition, the space rigid motion of the carrier is involved in inertial navigation, artificial intelligence, space docking and other fields[1-3]. In general, only acceleration measurement cannot obtain signals from the outside world, and there is no need to transmit signals to the outside world. As a result, the concept of "six-dimensional acceleration sensor" is produced, which is an inertial measurement instrument that can simultaneously measure three-dimensional linear acceleration and three-dimensional angular acceleration in space[4].

The measurement performance of the six-axis accelerometer is mainly determined by the operating performance of its sensing mechanism[5-6]. According to the six-dimensional acceleration perceived by the sensing mechanism, the problem of solving the output values of all sensitive components is called the forward dynamic model of the six-dimensional acceleration sensing mechanism. In addition, the so-

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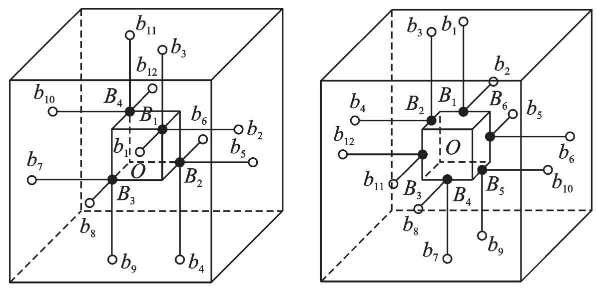
lution for the forward dynamic equation is a prerequisite for the configuration synthesis and fault handling of the sensing mechanism. Therefore, how to solve the forward dynamic equation of the sensing mechanism has attracted wide attention of scholars at home and abroad[7-9].

In general, the research methods of mechanism dynamics include the general equation of dynamics, Hamilton canonical equation, Newton-Euler method, Lagrange equation, Kane method and so on. You et al.[10] studied the dynamics of parallel six-axis accelerometer by using Hamilton canonical equation. But this method is difficult to solve in the configuration space. Lu et al.[11] used Kane method to establish the dynamic model of parallel machine tool, which is difficult to solve. Lebret et al.[12] used Lagrange equation to analyze the dynamics of Stewart mechanism. But this method also has the problems of high coupling degree and difficult solution. There are many studies before the general equation of dynamics, which are not repeated in this paper. Therefore, this paper uses Newton-Euler method to analyze the dynamics of six-dimensional acceleration sensing mechanism[12-15].

When the Newton-Euler method is used to analyze the dynamics of redundant mechanisms, it is difficult to establish supplementary equations. To this end, taking two typical six-axis acceleration sensing mechanisms as the research object, this paper derives the two-configuration complementary equation (that is, the output coordination equation), and solves the forward decoupling equation of the two configurations. The research content of this paper lays a theoretical foundation for fault handling and configuration synthesis of sensing mechanisms.

1 Working Mechanism of Perception Mechanism

The prototype of the parallel six-axis acceleration sensing mechanism is shown in Fig.1. It mainly includes a base, a cube-shaped mass block and 12 identical branches connecting the two. Here, the mass block is a cube with half side length n; the length of the branched chain is L; the base is a cubic hollow shell with half side length $(n+L)$. Any two branches present parallel or vertical relative geometric relations, and each branch is composed of a cylindrical piezoelectric ceramic (represented by a line segment in Fig.1) and two circular elastic spherical hinges in series. Among them, the solid circle and the hollow circle represent the composite spherical hinge and the general spherical hinge, respectively.



(a) "12-4" configuration (b) "12-6" configuration

Fig.1 Principle prototype of parallel six-dimensional acceleration sensing mechanism

The composite ball hinge in the sensing mechanism is generally located at the midpoint and vertex of the edge line of the mass block. Therefore, two typical sensing mechanisms are selected in this paper, which are the "12-4" configuration of the composite spherical hinge at the vertex of the mass block (Fig.1(a)) and the "12-6" configuration of the composite spherical hinge at the midpoint of the edge line of the mass block (Fig.1(b)). In actual work, the base is fixed with the object to be measured, that is, the acceleration of the base and the acceleration of the object to be measured (including size and direction) are exactly the same. Under the action of inertial force and inertial moment, the mass block compresses or stretches the piezoelectric ceramics on each branch chain. Due to the positive piezoelectric effect, the two polarization surfaces of the piezoelectric ceramic will produce positive and negative charges, respectively. The piezoelectric charge is received by the serial/parallel port of the computer after being processed by signal equipment such as charge amplifier, data acquisition card and so on. Therefore, the input of the system is the six-dimensional acceleration excitation of the base, and the output is the axial force (or positive and negative charge) of the branched chain.

2 Derivation of Coordination Equation

The Newton-Euler method is used to analyze the forward dynamics of redundant mechanisms. The difficulty lies in generating the output coordination equation of the mechanism. The research ideas of this section are as follows: Firstly, the scale constraint relationship between the hinge points on the mass block and the hinge points on the base of the sensing mechanism is analyzed. Secondly, the expression of the rod length of the branch chain is deduced theoretically, and the inherent constraint relationship between the rod lengths is excavated. Finally, Hooke's law is used to derive the output coordination equation.

2.1 The system of coordinates

The fixed coordinate systems $\{Q_1\}$ and $\{Q_2\}$ are established on the mass block and the base, respectively, as shown in Fig.2. In the initial state, their origin coincides with the center of mass.

The quaternion $\boldsymbol{\Lambda}=\lambda_0+\lambda_1\boldsymbol{i}+\lambda_2\boldsymbol{j}+\lambda_3\boldsymbol{k}$ is selected to represent the attitude of the mass block of the sensing mechanism. In Ref.[5], it is found that

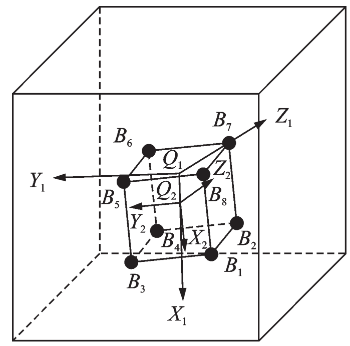


Fig.2 Two coordinate systems in the system

the pose of the mass relative to the base changes very little. Therefore, the rotation matrix $\boldsymbol{R}_{\{Q_1\}}^{\{Q_2\}}$ of $\{Q_1\}$ relative to $\{Q_2\}$ can be approximated as

$$\boldsymbol{R}_{\{Q_1\}}^{\{Q_2\}}\approx\begin{bmatrix}1 & -2\lambda_0\lambda_3 & 2\lambda_0\lambda_2 & 0\\ 2\lambda_0\lambda_3 & 1 & -2\lambda_0\lambda_1 & 0\\ -2\lambda_0\lambda_2 & 2\lambda_0\lambda_1 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \tag{1}$$

2.2 Derivation of output coordination equation

As shown in Fig.1(a), the coordinates of the composite spherical hinges B_h ($h=1, 2, 3, 4$) in $\{Q_2\}$ are expressed as

$$[B_1\ B_2\ B_3\ B_4]=\begin{bmatrix}-n & n & n & -n\\ -n & -n & n & n\\ -n & n & -n & n\end{bmatrix} \tag{2}$$

As shown in Fig.1(a), the coordinates of the spherical hinge b_i in $\{Q_2\}$ are

$$[b_1\ b_2\ b_3\ b_4\ b_5\ b_6\ b_7\ b_8\ b_9\ b_{10}\ b_{11}\ b_{12}]=$$
$$\begin{bmatrix}-n & -n & -n-L & n+L & n & n & n & n & n+L & -n & -n-L & -n\\ -n & -n-L & -n & -n & -n-L & -n & n+L & n & n & n+L & n & n\\ -n-L & -n & -n & n & n & n+L & -n & -n-L & -n & n & n & n+L\end{bmatrix} \tag{3}$$

The branch vector in $\{Q_2\}$ is expressed as

$$l_i=B_h-b_i=O^{\{Q_2\}}+R_{\{Q_1\}}^{\{Q_2\}}B_h-b_i \tag{4}$$

$$h=\begin{cases}1 & i=1,2,3\\ 2 & i=4,5,6\\ 3 & i=7,8,9\\ 4 & i=10,11,12\end{cases} \tag{5}$$

where $O^{\{Q_2\}}$ is the coordinate of the mass center of mass in $\{Q_2\}$.

Substituting Eqs.(1—3) into Eq.(5), using Taylor formula to expand at the original length of the branch chain, and ignoring the infinitesimal above the second order, the analytical formula of the branch chain length can be obtained

$$\begin{cases}l_1=L+z-2n\lambda_0\lambda_1+2n\lambda_0\lambda_2\\ l_2=L+y-2n\lambda_0\lambda_3+2n\lambda_0\lambda_1\\ l_3=L+x-2n\lambda_0\lambda_2+2n\lambda_0\lambda_3\\ l_4=L-x-2n\lambda_0\lambda_3-2n\lambda_0\lambda_2\\ l_5=L+y-2n\lambda_0\lambda_1+2n\lambda_0\lambda_3\\ l_6=L-z+2n\lambda_0\lambda_2+2n\lambda_0\lambda_1\\ l_7=L-y-2n\lambda_0\lambda_3-2n\lambda_0\lambda_1\\ l_8=L+z-2n\lambda_0\lambda_2+2n\lambda_0\lambda_1\\ l_9=L-x+2n\lambda_0\lambda_3+2n\lambda_0\lambda_2\\ l_{10}=L-y+2n\lambda_0\lambda_3+2n\lambda_0\lambda_1\\ l_{11}=L+x-2n\lambda_0\lambda_3+2n\lambda_0\lambda_2\\ l_{12}=L-z-2n\lambda_0\lambda_2-2n\lambda_0\lambda_1\end{cases} \tag{6}$$

According to Eq.(6), it can be obtained that

$$\begin{cases} l_1 + l_6 + l_8 + l_{12} = 4L \\ l_2 + l_5 + l_7 + l_{10} = 4L \\ l_3 + l_4 + l_9 + l_{11} = 4L \\ l_1 + l_6 - l_9 - l_{11} = 0 \\ l_2 + l_{10} - l_6 - l_8 = 0 \\ l_4 - l_2 - l_7 + l_{11} = 0 \end{cases} \tag{7}$$

Using Hooke's law, the output coordination equation is derived as

$$\begin{cases} f_1/k_1 + f_6/k_6 + f_8/k_8 + f_{12}/k_{12} = 0 \\ f_2/k_2 + f_5/k_5 + f_7/k_7 + f_{10}/k_{10} = 0 \\ f_3/k_3 + f_4/k_4 + f_9/k_9 + f_{11}/k_{11} = 0 \\ f_1/k_1 + f_6/k_6 - f_9/k_9 - f_{11}/k_{11} = 0 \\ f_2/k_2 + f_{10}/k_{10} - f_6/k_6 - f_8/k_8 = 0 \\ f_4/k_4 - f_2/k_2 - f_7/k_7 + f_{11}/k_{11} = 0 \end{cases} \tag{8}$$

where k_i is the axial stiffness of the i branch chain, and f_i the axial force of the i branch chain.

Assuming that the stiffness of each branch is consistent, Eq.(8) can be simplified as

$$g_j = 0 \quad j = 1, 2, \cdots, 6 \tag{9}$$

where $g_1 = f_1 + f_6 + f_8 + f_{12}$; $g_2 = f_2 + f_5 + f_7 + f_{10}$; $g_3 = f_3 + f_4 + f_9 + f_{11}$; $g_4 = f_1 + f_6 - f_9 - f_{11}$; $g_5 = f_2 - f_6 - f_8 + f_{10}$; $g_6 = f_4 - f_2 - f_7 + f_{11}$.

Using the method of the previous section, the branch chain expression of the "12-6" configuration is derived as

$$\begin{cases} l_1 = L + x + 2n\lambda_0\lambda_2 \\ l_2 = L - z - 2n\lambda_0\lambda_2 \\ l_3 = L + x - 2n\lambda_0\lambda_3 \\ l_4 = L - y + 2n\lambda_0\lambda_3 \\ l_5 = L - z + 2n\lambda_0\lambda_1 \\ l_6 = L + y - 2n\lambda_0\lambda_1 \\ l_7 = L - x + 2n\lambda_0\lambda_2 \\ l_8 = L + z - 2n\lambda_0\lambda_2 \\ l_9 = L - x - 2n\lambda_0\lambda_3 \\ l_{10} = L + y + 2n\lambda_0\lambda_3 \\ l_{11} = L + z + 2n\lambda_0\lambda_1 \\ l_{12} = L - y - 2n\lambda_0\lambda_1 \end{cases} \tag{10}$$

The analytical expression of the branch chain length shown in the Eq.(10) shows that there is a fixed constraint relationship between the branch chain lengths of the "12-6" configuration, as shown below

$$\begin{cases} l_1 - l_3 - l_7 + l_9 = 0 \\ l_4 + l_6 - l_{10} - l_{12} = 0 \\ l_2 - l_5 - l_8 + l_{11} = 0 \\ l_5 + l_6 + l_{11} + l_{12} = 4L \\ l_1 + l_2 + l_7 + l_8 = 4L \\ l_3 + l_4 + l_9 + l_{10} = 4L \end{cases} \tag{11}$$

Further, the output coordination equation is derived according to Eq.(11), shown as

$$g_p = 0 \quad p = 1, 2, \cdots, 6 \tag{12}$$

where $g_1 = f_1 - f_3 - f_7 + f_9$; $g_2 = f_4 + f_6 - f_{10} - f_{12}$; $g_3 = f_2 - f_5 - f_8 + f_{11}$; $g_4 = f_5 + f_6 + f_{11} + f_{12}$; $g_5 = f_1 + f_2 + f_7 + f_8$; $g_6 = f_3 + f_4 + f_9 + f_{10}$.

Differently, after studying the output coordination equation of "12-6" type, it is found that Eq.(12) satisfies the regular hexagon coordinated closed chain, as shown in Fig.3. In the figure, two branch forces are marked on each side of the regular hexagon, and two adjacent sides form an output coordination equation. The six vertices of the regular hexagon correspond to the six output coordination equations in Eq.(12).

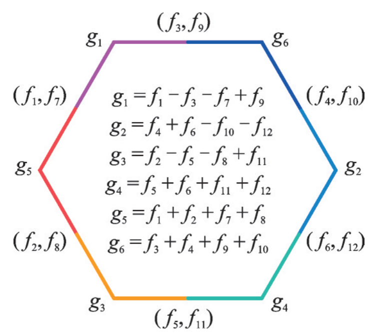


Fig.3 Regular hexagon coordinated closed chain of "12-6" configuration

3 Direct Dynamics

The decoupling of the forward dynamics of the parallel six-dimensional acceleration sensing mechanism belongs to the second-order statically indeterminate problem, so it is necessary to find a supplementary equation to transform the statically indeterminate problem into a statically indeterminate problem. The output coordination equation of the two configurations has been derived above, that is, the supplementary equation. The research ideas of this section are as follows: Firstly, based on the Newton-Euler method, the dynamic equations of the two configurations are constructed; then, the dynamic solution of the two configurations is completed by combining the dynamic equation and the coordination equation.

3.1 Derivation of dynamic equation

Since the quality of the mass block of the sensing mechanism is far greater than that of the branch chain, the 12 branches of the sensing mechanism can be regarded as two-force rods. In this section, we also use the quaternion $\boldsymbol{\Phi}=\varphi_0+\varphi_1\boldsymbol{i}+\varphi_2\boldsymbol{j}+\varphi_3\boldsymbol{k}$ to represent the rotation matrix of the base relative to the inertial frame, and its expression is

$$\boldsymbol{R}=\left(\boldsymbol{\varphi}^-\right)^{\mathrm{T}}\boldsymbol{\varphi}^+ \tag{13}$$

where $\boldsymbol{\varphi}^+$, $\boldsymbol{\varphi}^-$ denote the pre-matrix and the post-matrix, respectively.

$$\boldsymbol{\varphi}^+=\begin{bmatrix}\varphi_0\boldsymbol{E}_3+\hat{\boldsymbol{\psi}} & \boldsymbol{\psi}\\ -\boldsymbol{\psi}^{\mathrm{T}} & \varphi_0\end{bmatrix},\boldsymbol{\varphi}^-=\begin{bmatrix}\varphi_0\boldsymbol{E}_3-\hat{\boldsymbol{\psi}} & \boldsymbol{\psi}\\ -\boldsymbol{\psi}^{\mathrm{T}} & \varphi_0\end{bmatrix}$$

where $\boldsymbol{E}_3$ is a third-order unit matrix.

The dynamic equations of the "12-4" configuration are constructed by Newton-Euler method, we have

$$\boldsymbol{\alpha}=\frac{1}{m}\boldsymbol{R}\boldsymbol{F}_1-\boldsymbol{G}_1 \tag{14}$$

$$\boldsymbol{\varepsilon}=\frac{3}{2mn}\boldsymbol{R}\boldsymbol{F}_2 \tag{15}$$

where $\boldsymbol{\alpha},\boldsymbol{\varepsilon}$ are the three-dimensional linear acceleration vector and the three-dimensional angular acceleration vector in the basic excitation are represented, respectively; $\boldsymbol{G}_1=\begin{bmatrix}g & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$; m represents the mass of the mass block and g the acceleration of gravity; $\boldsymbol{F}_1=\begin{bmatrix}f_3-f_4-f_9+f_{11}\\ f_2+f_5-f_7-f_{10}\\ f_1-f_6+f_8-f_{12}\\ 0\end{bmatrix}$; $\boldsymbol{F}_2=\begin{bmatrix}f_2-f_1-f_5+f_6-f_7+f_8+f_{10}-f_{12}\\ f_1-f_3-f_4+f_6-f_8+f_9+f_{11}-f_{12}\\ f_3-f_2-f_4+f_5-f_7+f_9+f_{10}-f_{11}\\ 0\end{bmatrix}$.

The configurational kinetic equations of "12-6" are constructed as follows

$$\boldsymbol{\alpha}=\frac{1}{m}\boldsymbol{R}\boldsymbol{F}_3-\boldsymbol{G}_1 \tag{16}$$

$$\boldsymbol{\varepsilon}=\frac{3}{2mn}\boldsymbol{R}\boldsymbol{F}_4 \tag{17}$$

where $\boldsymbol{F}_3=\begin{bmatrix}f_1+f_3-f_7-f_9\\ -f_4+f_6+f_{10}-f_{12}\\ -f_2-f_5+f_8+f_{11}\\ 0\end{bmatrix}$, $\boldsymbol{F}_4=\begin{bmatrix}f_5-f_6+f_{11}-f_{12}\\ f_1-f_2+f_7-f_8\\ -f_3+f_4-f_9+f_{10}\\ 0\end{bmatrix}$.

3.2 Forward dynamics decoupling

Combining the coordination Eq.(9) and the kinetic Eqs.(14, 15), the forward decoupling equation of the "12-4" configuration is derived as follows

$$\boldsymbol{C}\boldsymbol{F}=\boldsymbol{D} \tag{18}$$

where $\boldsymbol{D}=\begin{bmatrix}\left(m\boldsymbol{R}^{\mathrm{T}}(\boldsymbol{a}-\boldsymbol{G}_1)\right)_{1,2,3}\\ \left(\dfrac{2mn}{3}\boldsymbol{R}^{\mathrm{T}}\boldsymbol{\varepsilon}\right)_{4,5,6}\\ \boldsymbol{0}\end{bmatrix}$, $\boldsymbol{F}=\begin{bmatrix}f_1 & f_2 & f_3 & f_4 & f_5 & f_6 & f_7 & f_8 & f_9 & f_{10} & f_{11} & f_{12}\end{bmatrix}^{\mathrm{T}}$, $(\bullet)_k$ represents the kth element of the vector, $\boldsymbol{0}$ the sixth-order zero column vector, and $\boldsymbol{C}$ a 12×12 matrix with -1, 0, and 1 as elements.

According to the theory of the basic solution system of $\det[\boldsymbol{C}]=4\,096$ and linear equations, $\boldsymbol{F}$ must have a definite and unique solution, that is

$$\boldsymbol{F}=\boldsymbol{C}^{-1}\boldsymbol{D} \tag{19}$$

The solution of the forward decoupling equation of the "12-6" configuration is exactly the same as the above process, and the calculation results also show that the branched chain force of the configuration also has a definite and unique solution.

3.3 Example verification

The virtual prototype of the "12-4" sensing mechanism is established in the software package ADAMS, as shown in Fig.4. Table 1 gives the input motion parameters, which are used for dynamic simulation verification of "12-4" configuration. Similarly, the "12-6" configuration can be verified by the same method.

Table 2 shows the maximum reference error between the theoretical derivation value and the virtual experimental value of the axial force of the two configurations. The maximum reference errors of

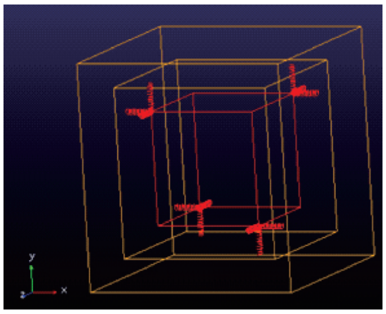


Fig.4 Regular hexagon coordinated closed chain of "12-4" configuration

Table 1 Simulation parameter

Item	Simulation parameter
X direction translation /mm	$1\times\cos(10\pi t)-1$
Y direction translation /mm	$0.3\times\cos(10\pi t)-0.3$
Z direction translation /mm	$3\times\cos(10\pi t)-3$
Rotate around X /rad	$0.3\times\cos(10\pi t)-0.3$
Rotate around Y/rad	$0.3\times\cos(10\pi t)-0.3$
Rotate around Z/rad	$3\times\cos(10\pi t)-3$
n/mm	50
m/kg	62.408
L/mm	50
$k/(\mathrm{N\cdot mm^{-1}})$	5.02×10^8
Simulation time /s	5
Simulation step size /s	0.002

Table 2 Forward decoupling equation verification

Configuration	12-4	12-6
Maximum reference error/%	4.23	6.53

the branched chain force of the "12-4" configuration and the "12-6" configuration are 4.23% and 6.53%, respectively. In addition, the results also show that both configurations meet the real-time requirements.

4 Conclusions

Taking two typical six-dimensional acceleration sensing mechanisms as examples, by analyzing the scale constraint relationship between the hinge points on the mass block and the hinge points on the base of the sensing mechanism, the analytical formula of the branch length is derived theoretically. Based on this, the inherent constraint relationship between all branches of the sensing mechanism is explored, and the output coordination equation of the sensing mechanism is established. The forward dynamic equation of the mechanism is established by Newton-Euler method, and the forward decoupling equation of the sensing mechanism is solved by combining the output coordination equations. This method lays a theoretical foundation for improving the dynamic decoupling accuracy of multi-dimensional sensing system.

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Author contributions Dr. **YOU Jingjing** proposed the model, designed the research program for analysis. Mr. **ZHANG Xianzhu** explained the results and wrote the manuscript. Ms. **ZHANG Yuanwei** contributed to the discussion and background of the study. All authors commented on the manuscript draft and approved the submission.

Competing interests The authors declare no competing interests.

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几种典型并联式六维加速度感知机构的输出协调方程推导

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摘要: 针对并联式六维加速度感知机构动力学解耦方程难以建立的问题,以两种典型的并联式六维加速度感知机构为例,通过剖析感知机构质量块上铰链点和基座上铰链点之间的尺度约束关系,提出了一种适用于一类的并联式六维加速度感知机构动力学方程解耦方法。首先,基于感知机构质量块上铰链点和基座上铰链点之间的尺度约束关系得到支链杆长表达式,挖掘出支链之间固有约束关系并构造了“12-6”式构型的支链协调闭链,成功推导出感知机构的输出协调方程;其次,采用Newton-Euler法构建了“12-4”和“12-6”两构型的动力学方程,结合动力学方程以及输出协调方程求解了两构型的正向解耦方程;最后,开展了虚拟样机实验,两种构型感知机构正向解耦方程的最大引用误差分别为4.23%和6.53%。结果表明:本文所提方法的有效性、可行性,且均满足实时性要求。

关键词: 六维加速度传感器;并联机构;拓扑构型;协调方程;动力学