A Novel Multi-objective Optimization Approach for Gear Hobbing Parameters Considering Hidden Management Constraints

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Abstract: In the multi-variety and small-batch production mode, the effective historical gear hobbing cases of gear hobbing machine tools represent the best practices under the combined constraints of gear hobbing machine tool, gear, and hob characteristics, as well as gear hobbing management requirements. The gear hobbing parameters of samples in historical gear hobbing cases inherently embed hidden management constraints of gear hobbing. This study established the two-stage multi-objective optimization models for gear hobbing parameters considering hidden management constraints, achieving a trade-off among gear hobbing quality, gear hobbing time, and gear hobbing cost under hidden management constraints. A multi-objective hippopotamus optimization (MHO) algorithm is proposed in this study. The diversity, convergence, and coverage of its Pareto optimal solutions are verified via standard test functions, and the running efficiency of MHO is significantly superior to that of the non-dominated sorting genetic algorithm Ⅱ (NSGA-Ⅱ) and the multi-objective particle swarm optimization algorithm (MOPSO). By solving the proposed model with MHO, a set of gear hobbing parameters subjected to hidden management constraints is obtained. By designing different selection strategies, multiple optional gear hobbing parameter combinations are provided for technicians. Experimental results show that the effectiveness of gear hobbing using the proposed method meets the requirements of quality, time, and cost subjected to hidden management constraints. The research results are of significant technical and engineering value for technicians and managers to optimize gear hobbing parameters subjected to hidden management constraints.

Key words: hidden management constraints; gear hobbing parameters; multi-objective hippopotamus optimization algorithm (MHO); non-dominated sorting genetic algorithm Ⅱ (NSGA-Ⅱ); multi-objective particle swarm optimization algorithm (MOPSO)

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0 Introduction

Gear hobbing is a widely used gear machining method. The gear hobbing system consists of the gear hobbing machine tool, hobs, and gears. As shown in Fig.1, the gear is installed on the worktable of the gear hobbing machine tool and rotates together with the worktable. The gear hobbing machine tool provides power for the movement of the hob and gear. The hob and gear rotate at a certain speed ratio. The hob completes gear hobbing through radial and axial motion. The gear hobbing parameters (spindle speed, cutting feed, depth of cut) directly influence the gear hobbing quality, gear hobbing time, and gear hobbing cost[1-2]. Choosing appropriate gear hobbing parameters can achieve satisfactory gear hobbing effectiveness[3-5].

In the multi-variety and small-batch production

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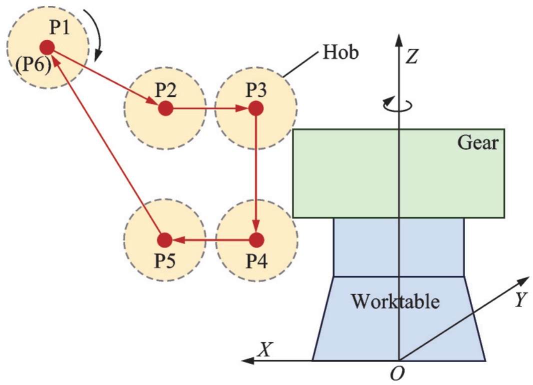

Fig.1 Schematic diagram of gear hobbing system

mode, many categories of gears are machined by hobbing, with small batches (usually between 10 and 1 000). The characteristics of the gears to be hobbed and the hobs used by each gear hobbing machine tool may differ, resulting in differences in the gear hobbing parameters required for different gears. To achieve good effectiveness of gear hobbing, such as good gear hobbing quality, short gear hobbing time, or low gear hobbing cost, technicians need to conduct single or multi-objective optimization research on gear hobbing parameters based on the characteristics of gears and hobs for different gear hobbing machine tools.

At present, studies on the optimization of gear hobbing parameters are mainly classified into two types[6].

Type Ⅰ is the unsupervised optimization of gear hobbing parameters. Through theoretical analysis, the mathematical model for evaluating the effectiveness of gear hobbing is established without referring to historical gear hobbing cases. The model parameters are determined through gear hobbing experiments, and optimization algorithms are used to solve the problem. Finally, the gear hobbing parameters subjected to ideal conditions are obtained[1,7-13].

Type Ⅱ is the supervised optimization of gear hobbing parameters. Based on historical gear hobbing cases, gear hobbing is treated as a "black box" and trained using algorithms or regression fitting to obtain the mapping relationship between optimization objectives and decision variables, yeilding satisfactory gear hobbing parameters[2,6,14-18].

Type Ⅰ methods construct mathematical models solely through theoretical analysis and require extensive experimentation to determine model parameters for each new optimization problem. In the multi-variety and small-batch production mode, this leads to prohibitive experimental costs and poor adaptability. In contrast, Type Ⅱ methods leverage historical cases to capture the intrinsic mapping between parameters and objectives, significantly reducing experimental burden while maintaining practical applicability, making them well-suited for actual production environments. Therefore, this study follows the Type Ⅱ strategy.

In Type Ⅱ, Cao et al.[6, 17-18] effectively solved the optimization problem of gear hobbing parameters supported by a few historical gear hobbing cases. It is found that Cao et al.[17] designed nine sets of experiments and obtained the empirical formula for energy consumption of the optimization problem (i.e., gear to be hobbed, the same later) through regression fitting of the experimental results. In the multi-variety and small-batch production mode, user requirements are constantly changing, and the characteristics of the optimization problem change frequently. If we continue using the method in Ref. [17], it is necessary to design many experiments to obtain the empirical formula for the optimization objective of the problem, which increases the burden on technicians and the gear hobbing cost.

In practical production, the choices of gear hobbing parameters are not only determined by machine and tool capabilities but also by unwritten managerial preferences regarding acceptable quality, time, and cost trade-offs. These preferences are implicitly embedded in the historical successful cases, forming what is termed "hidden management constraints". Capturing and formalizing such constraints allows the optimization to yield results that are both technically feasible and managerially acceptable.

It is recognized that the effective historical gear hobbing cases of a specific gear hobbing machine tool demonstrate the integration of the characteristics of the gear hobbing machine tool, gears, and hobs and the requirements of gear hobbing management. The gear hobbing parameters of each gear are the decision results that the current gear hobbing machine tool, gears, and hobs can support and the managers can accept, containing the hidden management constraints of gear hobbing. Therefore, each gear hobbing machine tool's effective historical gear hobbing cases have important reference values for optimizing the gear hobbing parameters. Plus, technicians can generally provide empirical formulas for evaluating the effectiveness of gear hobbing of samples in historical gear hobbing cases.

Given this, this study calculates the degree of similarity between the optimization problem and samples in historical gear hobbing cases, selects the empirical formula for the optimization objective function of the sample with the highest similarity, used the correction coefficient and correction amount of gear hobbing characteristic differences to establish the empirical formula for the optimization objective function of the optimization problem, and determines the relevant parameters through a small number of experiments. We express hidden management constraints of gear hobbing through a mathematical model, establish two-stage multi-objective optimization models for gear hobbing parameters considering hidden management constraints, and achieve three three-objective optimizations of gear hobbing quality, gear hobbing time, and gear hobbing cost. Considering the advantages of the hippopotamus optimization (HO) algorithm in global optimization and convergence[19], the method for updating the position of the male hippopotamus is improved, and a Pareto optimal mechanism is introduced. Based on this, the multi-objective hippopotamus optimization (MHO) is proposed. After analyzing the influence of gear hobbing parameters on gear hobbing quality, gear hobbing time, and gear hobbing cost, the multi-objective optimization models for gear hobbing parameters are solved by MHO. Multiple combinations of gear hobbing parameters subjected to hidden management constraints are obtained, providing technicians with multiple optional gear hobbing parameter decision-making results. The novelties of this study are listed as follows.

(1) A mathematical model for hidden management constraints in gear hobbing is established and is multi-objective optimization of gear hobbing parameters subjected to hidden management constraints is achieved.

(2) MHO is proposed, and its superiority in comprehensive performance is verified through experiments.

(3) The influence of hidden management constraints on the optimization of gear hobbing parameters is analyzed.

(4) Multiple selection strategies are provided to support the decision-making of gear hobbing parameters considering hidden management constraints.

The rest of the paper is organized as follows: Section 1 provides the standardized description and research methods for the optimization problem. Section 2 describes the evaluation indicators for the effectiveness of gear hobbing and the two-stage multi-objective optimization models for gear hobbing parameters considering hidden management constraints. Section 3 describes the Pareto optimal mechanism and the mathematical modelling of MHO. Section 4 focuses on experimental preparation, MHO validation, feasibility verification of multi-objective optimization, and the analysis of multi-objective optimization experimental results. Section 5 draws conclusions of this study.

1 Problem Description and Research Method

1.1 Problem description

Based on the explanation in Table 1, the optimization problem of gear hobbing parameters can be described as $Q=(G, W)$[17], where $G=\{m_{\mathrm{n}}, \alpha_{\mathrm{n}}, z_1, \beta_{\mathrm{n}}, d_2, B, a_{\mathrm{p}}, d_0, z_0\}$ is the attribute set of the optimization problem; $W=\{w_1, w_2, \cdots, w_i, \cdots, w_M\}$ the sample set of historical gear hobbing cases; $w_i=\{\{m_{\mathrm{n}i}, \alpha_{\mathrm{n}i}, z_{1i}, \beta_{\mathrm{n}i}, d_{2i}, B_i, a_{\mathrm{p}i}, d_{0i}, z_{0i}\}, \{n_i, f_i\}, \{Q_{\mathrm{h}i}, T_{\mathrm{h}i}, C_{\mathrm{h}i}\}\}$ the attribute set of the sample; M an integer representing the sample size and $i=1, 2, \cdots, M$. The gear used for gear hobbing in this study is a small-module involute cylindrical gear (i.e., $m_{\mathrm{n}} \leqslant 3$), hobbed using a single pass[20]. For such gears in small-batch production, the value of a_{p} can be determined according to machining allowance and machining requirements, so a_p is excluded from the decision variables of the optimization model.

In this study, hidden management constraints are operationalized as the requirement that the optimization results should closely approximate the effectiveness of the most similar historical case.

Table 1 Description of attributes

Problem attributes		Optimization objectives (i.e., the evaluation indicators for the effectiveness of gear hobbing)		Decision variables	
Parameter	Description	Parameter	Description	Parameter	Description
m_n	Gear normal modules/mm	Q_h	Gear hobbing quality/mm	n	Spindle speed/(r•min^{-1})
α_n	Gear normal pressure angle/rad	T_h	Gear hobbing time/min	f	Cutting feed/(mm•r^{-1})
z_1	Number of gear teeth	C_h	Gear hobbing cost/CNY		
β_n	Gear helix angle/rad	Q_{h0}	Evaluation index of Q_h with hidden management constraints		
d_2	Gear outside diameter/mm				
B	Face width/mm	T_{h0}	Evaluation index of T_h with hidden management constraints		
a_p	Depth of cut/mm				
d_0	Tip diameter of hob/mm	C_{h0}	Evaluation index of C_h with hidden management constraints		
z_0	Number of hob start				

1.2 Research methods

The research methods adopted in this study determine the evaluation indicators for the effectiveness of gear hobbing and obtain the formula for the evaluation indicators. Then, a two-stage optimization of gear hobbing parameters considering hidden management constraints is carried out to get a set of gear hobbing parameters subjected to hidden management constraints. Finally, appropriate gear hobbing parameters can be selected based on the strategies to provide technicians with multiple optional gear hobbing parameter combinations.

As shown in Fig.2, the evaluation indicators for the effectiveness of gear hobbing include gear hobbing quality, gear hobbing time, and gear hobbing cost. Specifically, the process of obtaining the empirical formula for energy consumption in the gear hobbing cost involves the following steps.

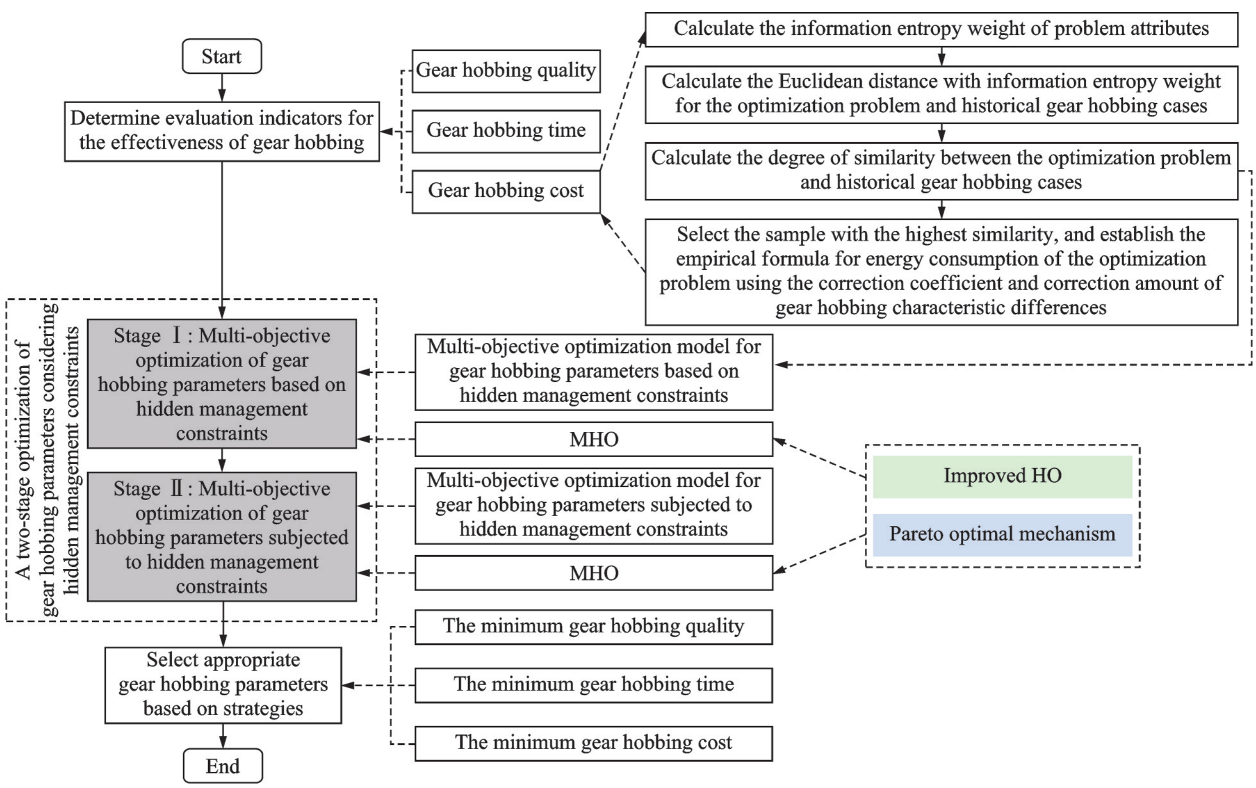

Fig.2 Research method of this study

(1) Calculate the information entropy weight of problem attributes.

(2) Calculate the Euclidean distance with information entropy weight for the optimization problem and the samples in historical gear hobbing cases.

(3) Calculate the degree of similarity between the optimization problem and the samples in historical gear hobbing cases.

(4) Select the sample with the highest similarity, and establish an empirical formula for energy consumption of the optimization problem using the correction coefficient and correction amount of gear hobbing characteristic differences.

The optimization of gear hobbing parameters considering hidden management constraints includes two stages. In Stage Ⅰ, MHO is used to solve the multi-objective optimization model for gear hobbing parameters based on hidden management constraints and obtain the range of gear hobbing parameters. In Stage Ⅱ, based on the range of gear hobbing parameters from Stage Ⅰ, MHO is used to solve the multi-objective optimization model for gear hobbing parameters subjected to hidden management constraints, and a set of gear hobbing parameters that meet hidden management constraints is obtained.

2 Mathematical Model for Optimization of Gear Hobbing Parameters

2.1 Optimization objectives

2.1.1 Gear hobbing quality

Gear hobbing quality can be evaluated through tooth curve error, tooth profile error, and surface roughness[17], expressed as

$$Q_{\mathrm{h}}=\omega_{\mathrm{q}1}\times Q_{\mathrm{hx}}+\omega_{\mathrm{q}2}\times Q_{\mathrm{hs}}+\omega_{\mathrm{q}3}\times Q_{\mathrm{hr}} \tag{1}$$

$$Q_{\mathrm{hx}}=\left(\frac{f}{\cos\beta_{\mathrm{n}}}\right)^2\times\frac{\sin\alpha_{\mathrm{n}}}{4\pi d_0} \tag{2}$$

$$Q_{\mathrm{hs}}=\frac{\pi^2 m_{\mathrm{n}} z_0^2\times\sin\alpha_{\mathrm{n}}}{4z_2\zeta^2} \tag{3}$$

$$Q_{\mathrm{hr}}=\frac{0.032\,1\times f^2}{r_{\mathrm{a}}} \tag{4}$$

$$\omega_{\mathrm{q}1}+\omega_{\mathrm{q}2}+\omega_{\mathrm{q}3}=1 \tag{5}$$

where Q_{hx} is the tooth curve error; Q_{hs} the tooth profile error; Q_{hr} the surface roughness; $\omega_{\mathrm{q}1}$, $\omega_{\mathrm{q}2}$ and $\omega_{\mathrm{q}3}$ represent the weight coefficients of Q_{hx}, Q_{hs}, and Q_{hr}, respectively; ζ represents the slot number of the hob; and r_{a} the radius of the rounded corner of the hob tip. According to expert review opinions, $\omega_{\mathrm{q}1}=0.25$, $\omega_{\mathrm{q}2}=0.25$, $\omega_{\mathrm{q}3}=0.5$[17].

Eq.(1) shows that Q_{h} is related to the attributes of gears and hobs and the decision variable f. The formula for gear hobbing quality can be directly obtained based on the optimization problem.

2.1.2 Gear hobbing time

The entire gear hobbing process includes machine tool acceleration operation, gear hobbing cutting operation, and gear hobbing auxiliary operation[9, 11]. Due to the short time, the time can be ignored for the acceleration operation of the gear hobbing machine tool. As shown in Fig.1, gear hobbing cutting operations include fast feed (P1 → P2), slow feed (P2 → P3), axial gear hobbing (P3 → P4), slow tool retraction (P4 → P5), fast reset (P5 → P6 (P1)). The operation time of this part is directly related to the gear hobbing parameters. Gear hobbing auxiliary operation includes auxiliary work such as installing gears, disassembling gears, replacing hobs, cooling, etc. The duration of this part is related to the parameters of the gear hobbing machine tool, gears, and hobs themselves. For the specific gear hobbing machine tool, gear, and hob, the time for gear hobbing auxiliary operations is constant[9]. The gear hobbing time is expressed as

$$T_{\mathrm{h}}=T_{\mathrm{hs}}+T_{\mathrm{hc}}+T_{\mathrm{ha}} \tag{6}$$

where T_{hs} represents the machine tool acceleration operation time; T_{hc} the gear hobbing cutting operation time; and T_{ha} the gear hobbing auxiliary operation time. Among them, we have

$$T_{\mathrm{hc}}=\frac{(L_{\mathrm{in}}+B+L_{\mathrm{out}})\times z_1}{n\times f\times z_0}+\frac{L_{\mathrm{fa}}}{v_{\mathrm{fa}}}+\frac{L_{\mathrm{sl}}}{v_{\mathrm{sl}}} \tag{7}$$

$$L_{\mathrm{in}}=\sqrt{d_0\times h}+A \tag{8}$$

$$L_{\mathrm{in}}=\sqrt{[(d_0+d_2)\times\tan^2\delta+d_0]\times h}+A \tag{9}$$

$$L_{\mathrm{out}}=\frac{1.25\times m_{\mathrm{n}}\times\sin\delta}{\tan\alpha_{\mathrm{n}}}+A \tag{10}$$

where L_{in} represents the axial cut-in stroke of gear hobbing calculated by Eq.(8) for cylindrical spur gears and Eq.(9) for cylindrical helical gears[20]; L_{out} the axial cut-out stroke of gear hobbing calculated by Eq.(10)[20]; L_{fa} the fast idle cutting stroke (refer to P1 → P2 and P5 → P6 in Fig.1); v_{fa} the fast cutting speed; L_{sl} the slow idle cutting stroke (refer to P2 → P3 and P4 → P5 in Fig.1); and v_{sl} the slow cutting speed. In Eqs.(8—10), h is set as $2.25\times m_{\text{n}}$ when machining small-module gears, A is a constant value ranging between $[2, 5]$, and δ is the installation angle of the hob and a constant.

Based on practical experience, in Eq.(6), $T_{\text{hs}}=0$. T_{hc} can be directly calculated by Eq.(7) based on the attributes of the optimization problem. T_{ha} is a constant, which can be obtained through practical experience of technicians.

2.1.3 Gear hobbing cost

Gear hobbing is a complex process in which workers, materials (i.e., gears), gear hobbing machine tools (including computer numerical control (CNC) systems, cutting fluids, lighting, etc.), hobs, and other gear hobbing system elements operate in close collaboration. From the perspective of optimization of gear hobbing parameters, gear hobbing cost C_{h} includes the gear hobbing machine tool depreciation cost C_{mt}, personnel cost C_{pt}, hob cost C_{hb}, cutting fluid cost C_{cf}, and electrical energy consumption cost C_{he}[9-11,17]. Specifically, it can be expressed as

$$C_{\text{h}}=C_{\text{mt}}+C_{\text{pt}}+C_{\text{hb}}+C_{\text{cf}}+C_{\text{he}} \tag{11}$$

$$C_{\text{mt}}=c_{\text{mt}}\times T_{\text{h}} \tag{12}$$

$$C_{\text{pt}}=c_{\text{pt}}\times T_{\text{h}} \tag{13}$$

$$C_{\text{hb}}=c_{\text{hb}}\times\frac{T_{\text{h}}}{T_{\text{hb}}} \tag{14}$$

$$C_{\text{cf}}=c_{\text{cf}}\times\frac{T_{\text{h}}}{T_{\text{cf}}} \tag{15}$$

$$C_{\text{he}}=c_{\text{he}}\times E_{\text{h}} \tag{16}$$

where c_{mt} represents the unit time depreciation cost of the gear hobbing machine tool; c_{pt} the salary per unit time of the personnel; c_{hb} the price of the hob; T_{hb} the estimated service life of the hob; c_{cf} the price of cutting fluid; T_{cf} the replacement cycle of cutting fluid; c_{he} the unit cost of electricity determined according to the local industrial electricity billing standards; and E_{h} the energy consumption during the gear hobbing.

The description in Refs.[1, 10] shows that the theoretical formula of E_{h} involves multiple parameters. For different gear hobbing machine tools and gears, many experiments need to be carried out to determine the parameters in the theoretical formula. In the multi-variety and small-batch production mode, gears to be hobbed are frequently replaced, and the cost of determining the parameters in the theoretical formula through many experiments is relatively high. Since the empirical formula for energy consumption of the samples in the historical gear hobbing cases of a specific gear hobbing machine tool is determined, this study introduces the correction coefficient and correction amount of gear hobbing characteristic differences to establish an empirical formula.

$$E_{\text{h}}=\rho\times E_{\text{h}}^{\text{bs}}+\Delta \tag{17}$$

where ρ is the correction coefficient of gear hobbing characteristic differences; E_{h}^{bs} the empirical formula for energy consumption of the sample with the highest similarity to the optimization problem in the historical gear hobbing cases; and Δ the correction amount of gear hobbing characteristic differences and constant. ρ and Δ can be determined through experiments.

The calculation method for the degree of similarity between samples in historical gear hobbing cases and the optimization problem is as follows.

(1) Standardize data

The set consisting of attributes of the optimization problem G and samples W in historical gear hobbing cases is defined as $X_1, X_2, \cdots, X_j, \cdots, X_9$, corresponding to the value range of m_{n}, α_{n}, z_1, β_{n}, d_2, B, a_{p}, d_0, z_0, $X_j=\{x_{0j}, x_j, \cdots, x_{ij}, \cdots, x_{Mj}\}$, $i=0, 1, 2, \cdots, M$, $j=1, 2, \cdots, 9$, where $i=0$ represents the optimization problem. If the set that standardizes $X_1, X_2, \cdots, X_j, \cdots, X_9$ is $Y_1, Y_2, \cdots, Y_j, \cdots, Y_9$, then

$$y_{ij}=\frac{x_{ij}-\min(X_j)}{\max(X_j)-\min(X_j)} \tag{18}$$

(2) Determine the information entropy of the attribute

The information entropy of a set of data Y_j can be expressed as[21]

$$e_j = -\ln\left(\frac{1}{M+1}\right) \times \sum_{i=0}^{M}(\eta_{ij} \times \ln \eta_{ij}) \tag{19}$$

where $\eta_{ij} = \dfrac{y_{ij}}{\sum_{i=0}^{M} y_{ij}}$. If $\eta_{ij} = 0$, $\lim_{\eta_{ij} \to 0}(\eta_{ij} \times \ln \eta_{ij}) = 0$ is defined.

(3) Determine the weights of the attribute

According to the calculated information entropy $e_1, e_2, \cdots, e_j, \cdots, e_9$, the weights for different attributes are

$$w_j = \frac{1 - e_i}{9 - \sum e_i} \tag{20}$$

(4) Calculate the degree of similarity between samples in historical gear hobbing cases and the optimization problem

According to the weights $w_1, w_2, \cdots, w_j, \cdots, w_9$, the Euclidean distance between the samples and the optimization problem are calculated as

$$\tau_i = \sqrt{\sum_{j=1}^{9}\left(w_j \times (y_{ij} - y_{0j})^2\right)} \tag{21}$$

where $i = 1, 2, \cdots, M$.

The degree of similarity between the samples and the optimization problem is

$$\varphi_i = \frac{1}{1 + \tau_i} \tag{22}$$

2.2 Two-stage multi-objective optimization models

Based on the optimization objectives, the mathematical models for multi-objective optimization of gear hobbing parameters considering hidden management constraints are established as follows.

Stage Ⅰ Multi-objective optimization model for gear hobbing parameters based on hidden management constraints is defined as

$$\text{Minimize} \quad F(n, f) = (Q_{h0}, T_{h0}, C_{h0}) \tag{23}$$

s.t.

$$Q_{h0} = \left\| \frac{Q_h}{Q_0} - \varphi_0 \right| - \left| \frac{Q_h}{Q_0} - \frac{1}{\varphi_0} \right\| \tag{24}$$

$$T_{h0} = \left\| \frac{T_h}{T_0} - \varphi_0 \right| - \left| \frac{T_h}{T_0} - \frac{1}{\varphi_0} \right\| \tag{25}$$

$$C_{h0} = \left\| \frac{C_h}{C_0} - \varphi_0 \right| - \left| \frac{C_h}{C_0} - \frac{1}{\varphi_0} \right\| \tag{26}$$

$$n_{lb} \leqslant n \leqslant n_{ub} \tag{27}$$

$$f_{lb} \leqslant f \leqslant f_{ub} \tag{28}$$

where Eq.(23) indicates that the optimization objective in Stage Ⅰ is to achieve the optimal effectiveness of gear hobbing of the optimization problem as closely as possible to the effectiveness of gear hobbing of the sample with the highest similarity in historical gear hobbing cases, which is the mathematical expression of hidden management constraints. Eq.(24) represents the degree of closeness between the gear hobbing quality of the optimization problem and the gear hobbing quality of the sample with the highest similarity. Eq.(25) represents the degree of closeness between the gear hobbing time of the optimization problem and the gear hobbing time of the sample with the highest similarity. Eq.(26) represents the degree of closeness between the gear hobbing cost of the optimization problem and the gear hobbing cost of the sample with the highest similarity. In Eqs.(24—26), φ_0 represents the maximum similarity between the sample from historical gear hobbing cases and the optimization problem. In Eq.(27), n_{lb} and n_{ub} represent the lower and the upper bounds of n, respectively. In Eq.(28), f_{lb} and f_{ub} represent the lower and the upper bounds of f, respectively.

Stage Ⅱ Multi-objective optimization model for gear hobbing parameters subjected to hidden management constraints is defined as

$$\text{Minimize} \quad F(n, f) = (Q_h, T_h, C_h) \tag{29}$$

s.t.

$$n_{lb}^{near} \leqslant n \leqslant n_{ub}^{near} \tag{30}$$

$$f_{lb}^{near} \leqslant f \leqslant f_{ub}^{near} \tag{31}$$

where Eq.(29) indicates that the optimization objective in Stage Ⅱ is to minimize gear hobbing quality, gear hobbing time, and gear hobbing cost subjected to hidden management constraints. In Eq.(30), n_{lb}^{near} and n_{ub}^{near} represent the minimum and the maximum values of n subjected to hidden management constraints, respectively. In Eq.(31), f_{lb}^{near} and f_{ub}^{near} represent the minimum and the maximum values of f subjected to hidden management constraints, respectively.

3 Optimization Solution via MHO

Eqs.(23, 29) are typical multi-objective optimization problems that can be solved using two methods: Type Ⅰ involves integrating multiple objectives into a single optimization objective, which is solved using a single-objective optimization algorithm; Type Ⅱ is based on the Pareto optimal mechanism, which compares and balances the optimal solutions to obtain more satisfactory solutions. This study adopts Type Ⅱ.

The HO algorithm is a metaheuristic algorithm[19]. Its advantages in global optimization and convergence have been fully elaborated by Amiri et al.[19], laying a good foundation for its application in the optimization of machining process parameters. Therefore, on the basis of HO, this study improves the method for updating the position of the male hippopotamus and introduces the Pareto optimal mechanism. Thus, MHO is proposed to solve multi-objective optimization problems of gear hobbing parameters. The mathematical modeling of MHO is described as follows.

The hippopotamus is a semi-aquatic social mammal with a population size of 10 to 30, consisting of several female hippopotamuses, immature hippopotamuses, multiple mature male hippopotamuses, and a dominant male hippopotamus[19]. HO is a population-based optimization algorithm and inspired by three important behavioral patterns in hippopotamus daily activities[19]. In HO, search agents are hippopotamuses, and hippopotamuses are candidate solutions for the optimization problem, meaning that the position update of each hippopotamus in the search space represents values for the decision variables. Thus, each hippopotamus is represented as a vector, and the population of hippopotamuses is mathematically characterized by a matrix

$$\boldsymbol{X}=\begin{pmatrix}\boldsymbol{X}_1\\ \boldsymbol{X}_2\\ \vdots\\ \boldsymbol{X}_i\\ \vdots\\ \boldsymbol{X}_N\end{pmatrix}=\begin{pmatrix}x_{11} & \cdots & x_{1j} & \cdots & x_{1m}\\ \vdots & & \vdots & & \vdots\\ x_{i1} & \cdots & x_{ij} & \cdots & x_{im}\\ \vdots & & \vdots & & \vdots\\ x_{N1} & \cdots & x_{Nj} & \cdots & x_{Nm}\end{pmatrix} \tag{32}$$

where N represents the population size; vector $\boldsymbol{X}_i=(x_{i1}\ \ x_{i2}\ \cdots\ x_{ij}\ \cdots\ x_{im})$ the position of individual i; x_{ij} the value of the decision variable, $i=1, 2, \cdots, N$; and m the number of decision variables (for the multi-objective optimization problem in this study, $m=2$, i.e., n and f), $j=1, 2, \cdots, m$.

To solve multi-objective optimization problems of gear hobbing parameters, this study introduces the Pareto optimal mechanism[22-25] on the basis of HO and proposes MHO. The specific implementation strategies are as follows.

Strategy Ⅰ Construct a matrix $\boldsymbol{X}^{\text{Pareto}}$ (i.e., repository) to store the current Pareto optimal solutions to solve the problem of having only one solution for HO. The size of repository is $\lambda\times m$, and λ is an integer constant. During optimization, each individual is compared against all the repository residents using the Pareto optimal mechanism. If an individual dominates a solution in the repository, they have to be swapped. If an individual dominates a set of solutions in the repository, they all should be removed from the repository and the individual should be added in the repository. If at least one of the repository residents dominates an individual in the new population, it should be discarded straight away. If an individual is non-dominated in comparison with all repository residents, it has to be added to the repository.

Strategy Ⅱ To ensure that the optimal solution in repository can be uniformly distributed, (1) divide the solution space into λ segments, with each segment having a spatial range of $\boldsymbol{d}=\dfrac{\boldsymbol{V}_{\max}-\boldsymbol{V}_{\min}}{\lambda}$, where $\boldsymbol{V}_{\max}$ and $\boldsymbol{V}_{\min}$ are used to store vectors for the maximum and minimum values of different objective functions[25]. (2) Determine the number of elimination priority levels based on the number of adjacent solutions in each segment. The more neighboring solutions there are, the greater the number of elimination priority levels. (3) When repository is full, the elimination object is selected using the roulette wheel algorithm based on the number of elimination priority levels.

Strategy Ⅲ The number of Pareto optimal solutions may be multiple. When selecting one solution as the dominant male hippopotamus, it is also based on $\boldsymbol{d}$ in Strategy Ⅱ. On the contrary, the lower the elimination priority level, the greater the possibility of selection, and the roulette wheel algorithm is used for implementation.

3.1 Population initialization

Similar to conventional optimization algorithms, the initialization stage of the MHO involves the generation of randomized initial solutions. The vector is defined as $\boldsymbol{R}_e=(r_{e1}, r_{e2}, \cdots, r_{ej}, \cdots, r_{em})$, where $e=0, 1, 2, 3, 4, 9, 11$, and r_{ej} is a random number within the range of $[0, 1]$. x_{ij} is the same as Eq.(32) and initialized

$$x_{ij}=\mathrm{lb}_j+r_{0j}\times(\mathrm{ub}_j-\mathrm{lb}_j) \tag{33}$$

where lb_j and ub_j represent the lower and the upper bounds of the jth decision variable, respectively.

3.2 Stage 1: The hippopotamuses position update in the river or pond (exploration)

Hippopotamuses adopt a social approach to resist external threats. The dominant male hippopotamus protects the population and their territory in the hippopotamus population. Multiple female hippopotamuses are generally located around the dominant male hippopotamus. When the male hippopotamus matures, they will be expelled from the herd by the dominant male hippopotamus, forced to absorb female hippopotamus or compete with other male hippopotamus to establish their own dominance[19]. The modelling approach for this behavior is as follows.

According to $\boldsymbol{X}$, the objective function matrix of the hippopotamus population can be determined as

$$F=\begin{pmatrix}\boldsymbol{F}_1\\ \vdots\\ \boldsymbol{F}_i\\ \vdots\\ \boldsymbol{F}_N\end{pmatrix} \tag{34}$$

where $\boldsymbol{F}_i$ is the vector of objective function values for the individual i, $\boldsymbol{F}_i=(F_{i1}, \cdots, F_{ik}, \cdots, F_{iK})$, K is the number of objective functions; and F_{ik} represents the kth objective function value of the individual i.

(1) Determine the position of the dominant male hippopotamus

Based on the objective function matrix $\boldsymbol{F}$ of the hippopotamus population, determine the Pareto optimal solution according to the Pareto optimal mechanism and update the Pareto optimal solution to $\boldsymbol{X}^{\text{Pareto}}$ one by one according to Strategy Ⅰ. If $\boldsymbol{X}^{\text{Pareto}}$ is full, eliminate the corresponding individuals according to Strategy Ⅱ and add Pareto optimal solutions with fewer neighboring solutions to $\boldsymbol{X}^{\text{Pareto}}$. Finally, based on Strategy Ⅲ, the position of the dominant male hippopotamus in $\boldsymbol{X}^{\text{Pareto}}$ is determined as $\boldsymbol{X}^d=(x_j^d)_{1\times m}$.

(2) Update the position of individuals based on the position of male hippopotamuses

Among the $\left[\dfrac{N}{3}\right]$ individuals in the hippopotamus population, individuals can update its position based on the position of the male hippopotamus. At this point $i=1, 2, \cdots, \left[\dfrac{N}{3}\right]$, the position of the male hippopotamus $\boldsymbol{X}_i^{\text{Ma}}$ can be represented as

$$\boldsymbol{X}_i^{\text{Ma}}=\boldsymbol{X}_i+y_1\times\boldsymbol{E}_i \tag{35}$$

where y_1 is a random number within the range of $[-1, 1]$, $\boldsymbol{E}_i=(E_{ij})_{1\times m}$, $E_{ij}=\max\{E_{ij}^{2d}, E_{ij}^{2l}, E_{ij}^{2u}\}$, $E_{ij}^{2d}=|x_{ij}-x_j^d|$, indicating the distance from the dominant male hippopotamus, $E_{ij}^{2l}=|x_{ij}-\mathrm{lb}_j|$, indicating the distance from the lower bound; $E_{ij}^{2u}=|x_{ij}-\mathrm{ub}_j|$, indicating the distance from the upper bound. The objective function corresponding to $\boldsymbol{X}_i^{\text{Ma}}$ is represented as $\boldsymbol{F}_i^{\text{Ma}}$. Then the position of individual i can be updated based on the position of the male hippopotamus $\boldsymbol{X}_i^{\text{Ma}}$

$$\boldsymbol{X}_i=\begin{cases}\boldsymbol{X}_i & \boldsymbol{X}_i<\boldsymbol{X}_i^{\text{Ma}}\\ \boldsymbol{X}_i^{\text{Ma}} & \text{Others}\end{cases} \tag{36}$$

Eq.(36) indicates that if $\boldsymbol{X}_i$ dominates $\boldsymbol{X}_i^{\text{Ma}}$, $\boldsymbol{X}_i$ remains unchanged; If $\boldsymbol{X}_i^{\text{Ma}}$ dominates $\boldsymbol{X}_i$ or there is no dominance relationship between them, $\boldsymbol{X}_i=\boldsymbol{X}_i^{\text{Ma}}$ is updated.

(3) Update the position of individuals based on the position of female hippopotamuses

Among the $\left[\frac{N}{3}\right]$ individuals in the hippopotamus population, individuals can update their position based on the position of the female hippopotamus. At this point, $i=\left[\frac{N}{3}\right]+1,\left[\frac{N}{3}\right]+2,\cdots,\left[\frac{2N}{3}\right]$.

Matrix $\boldsymbol{H}=\begin{pmatrix}\boldsymbol{H}_1\\ \boldsymbol{H}_2\\ \boldsymbol{H}_3\\ \boldsymbol{H}_4\\ \boldsymbol{H}_5\end{pmatrix}=\begin{pmatrix}I_2\times\boldsymbol{R}_1+\boldsymbol{Q}_1\\ 2\times\boldsymbol{R}_2-(1)_{1\times m}\\ \boldsymbol{R}_3\\ I_1\times\boldsymbol{R}_4+\boldsymbol{Q}_2\\ (r_5)_{1\times m}\end{pmatrix}$ is defined, where I_1 and I_2 are random integers within the range of [1, 2]; $\boldsymbol{Q}_1=\begin{cases}(0)_{1\times m} & q_1=1\\ (1)_{1\times m} & q_1=0\end{cases}$; $\boldsymbol{Q}_2=\begin{cases}(0)_{1\times m} & q_2=1\\ (1)_{1\times m} & q_2=0\end{cases}$, q_1 and q_2 are random integers within the range of [0, 1]; and r_5 is a random number within the range of [0, 1]. The position of the female hippopotamus $\boldsymbol{X}_i^{\mathrm{FM}}$ can be represented as[19]

$$\boldsymbol{X}_i^{\mathrm{FM}}=\begin{cases}\boldsymbol{X}_i+\boldsymbol{h}_1\odot(\boldsymbol{X}^d-I_2\times\boldsymbol{X}_i^{\mathrm{Mean}}) & T>0.6\\ \boldsymbol{Y} & T\leqslant 0.6\end{cases} \tag{37}$$

$$\boldsymbol{Y}=\begin{cases}\boldsymbol{X}_i+\boldsymbol{h}_2\odot(\boldsymbol{X}_i^{\mathrm{Mean}}-\boldsymbol{X}^d) & r_6>0.5\\ \boldsymbol{Z} & r_6\leqslant 0.5\end{cases} \tag{38}$$

$$\boldsymbol{Z}=(z_1,z_2,\cdots,z_j,\cdots,z_m) \tag{39}$$

where $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ are randomly selected vectors in $\boldsymbol{H}$; $\boldsymbol{X}_i^{\mathrm{Mean}}$ is the average vector of positions of the randomly-selected population; $T=\mathrm{e}^{-\frac{l}{L}}$ where l is the current iteration number and L the total number of iterations; r_6 a random number within the range of [0, 1]; $z_j=\mathrm{lb}_j+r_7\times(\mathrm{ub}_j-\mathrm{lb}_j)$; and r_7 a random number within the range of [0, 1]. The objective function corresponding to $\boldsymbol{X}_i^{\mathrm{FM}}$ represents $\boldsymbol{F}_i^{\mathrm{FM}}$. Then, the position of individual i can be updated based on the position of the female hippopotamus $\boldsymbol{X}_i^{\mathrm{FM}}$

$$\boldsymbol{X}_i=\begin{cases}\boldsymbol{X}_i & \boldsymbol{X}_i<\boldsymbol{X}_i^{\mathrm{FM}}\\ \boldsymbol{X}_i^{\mathrm{FM}} & \text{Others}\end{cases} \tag{40}$$

Eq.(40) indicates that if $\boldsymbol{X}_i$ dominates $\boldsymbol{X}_i^{\mathrm{FM}}$, $\boldsymbol{X}_i$ remains unchanged; If $\boldsymbol{X}_i^{\mathrm{FM}}$ dominates $\boldsymbol{X}_i$ or there is no dominance relationship between them, update $\boldsymbol{X}_i=\boldsymbol{X}_i^{\mathrm{FM}}$ is defined.

3.3 Stage 2: Hippopotamus defence against predators (exploration)

In the hippopotamus population, immature hippopotamuses, driven by inherent curiosity, often leave the population to search for food or others and are considered the best potential targets by external predators such as lions, crocodiles, and spotted dogs. Similarly, sickly hippopotamuses can also become potential targets for predators. When a hippopotamus is threatened, it immediately turns its body towards the predator, opens its mouth wide and reveals its teeth, and makes a sound to prevent the predator from approaching[19]. The modelling approach for this behavior is as follows.

For individual i, $i=\left[\frac{2N}{3}\right]+1,\left[\frac{2N}{3}\right]+2,\cdots,N$, the predator's position $\boldsymbol{P}$ is defined as

$$\boldsymbol{P}=(p_1,p_2,\cdots,p_j,\cdots,p_m) \tag{41}$$

where $p_j=\mathrm{lb}_j+r_8\times(\mathrm{ub}_j-\mathrm{lb}_j)$, r_8 is a random number within the range of [0, 1]. The objective function corresponding to $\boldsymbol{P}$ represents $\boldsymbol{F}_i^P$.

In the vector $\boldsymbol{D}=(d_1,d_2,\cdots,d_j,\cdots,d_m)$, we have $d_j=|p_j-x_{ij}|$. The position of individual i facing the predator can be expressed as[19]

$$\boldsymbol{X}_i^P=\begin{cases}\boldsymbol{R}_{\mathrm{L}}\oplus\boldsymbol{P}+\left[\dfrac{\varepsilon}{\alpha-\beta\times\cos 2\pi\theta}\right]\times\boldsymbol{D}' & \boldsymbol{P}<\boldsymbol{X}_i\\ \boldsymbol{R}_{\mathrm{L}}\oplus\boldsymbol{P}+\left[\dfrac{\varepsilon}{\alpha-\beta\times\cos 2\pi\theta}\right]\times\boldsymbol{D}'' & \text{Others}\end{cases} \tag{42}$$

$$\boldsymbol{D}'=\left(\frac{1}{d_j}\right)_{1\times m} \tag{43}$$

$$\boldsymbol{D}''=\left(\frac{1}{2\times d_j+r_{9j}}\right)_{1\times m} \tag{44}$$

$$\mathrm{Levy}(\vartheta)=0.05\times\frac{\omega\times\sigma_\omega}{|\upsilon|^{\frac{1}{\vartheta}}} \tag{45}$$

$$\sigma_\omega=\left[\frac{\Gamma(1+\vartheta)\times\sin\left(\frac{\pi\vartheta}{2}\right)}{\Gamma\left(\frac{1+\vartheta}{2}\right)\times\vartheta\times 2^{\frac{\vartheta-1}{2}}}\right]^{\frac{1}{\vartheta}} \tag{46}$$

where $\boldsymbol{R}_{\mathrm{L}}$ is a random vector with a Levy distribution, utilized for sudden changes in the predator's position during an attack on the hippopotamus; ε a uniform random number within the range of [2, 4]; α a uniform random number within the range of [1, 1.5]; β a uniform random number within the range of [2, 3]; and θ a uniform random number within the range of [−1, 1]. The mathematical model for the random movement of Levy movement[26] is calculated by Eq.(45), where ω and υ are the random numbers in [0, 1], respectively. σ_ω can be obtained by Eq.(46), where ϑ is a constant (ϑ =1.5), and Γ is an abbreviation for Gamma function. Then the position of individual i is updated based on $\boldsymbol{X}_i^P$

$$\boldsymbol{X}_i=\begin{cases}\boldsymbol{X}_i & \boldsymbol{X}_i<\boldsymbol{X}_i^P\\ \boldsymbol{X}_i^P & \text{Others}\end{cases} \tag{47}$$

Eq.(47) indicates that if $\boldsymbol{X}_i$ dominates $\boldsymbol{X}_i^P$, $\boldsymbol{X}_i$ remains unchanged; If $\boldsymbol{X}_i^P$ dominates $\boldsymbol{X}_i$ or there is no dominance relationship between them, $\boldsymbol{X}_i=\boldsymbol{X}_i^P$ is updated.

3.4 Stage 3: Hippopotamus escaping from the predator (exploitation)

Due to predators such as lions, crocodiles, and spotted dogs not liking to wade through water, when individuals face groups of predators or cannot react to scare off predators, they quickly run towards the nearest lake or pond to protect themselves. Modelling this behavior results in an enhanced ability of MHO for exploitation in local search[19]. The modelling approach for this behavior is defined as

$$\boldsymbol{B}_{\mathrm{L}}^l=(\mathrm{lb}_j^l)_{n\times 1} \tag{48}$$

$$\boldsymbol{B}_{\mathrm{U}}^l=(\mathrm{ub}_j^l)_{n\times 1} \tag{49}$$

where $\mathrm{lb}_j^l=\dfrac{\mathrm{lb}_j}{l}$, $\mathrm{ub}_j^l=\dfrac{\mathrm{ub}_j}{l}$, $l=1,2,\cdots,L$. The neighboring position of individual i ($i=1,2,\cdots,N$) can be represented as[19]

$$\boldsymbol{X}_i^{\mathrm{Near}}=\boldsymbol{X}_i+r_{10}\times(\boldsymbol{B}_{\mathrm{L}}^l+\boldsymbol{s}_1\odot(\boldsymbol{B}_{\mathrm{U}}^l-\boldsymbol{B}_{\mathrm{L}}^l)) \tag{50}$$

$$\boldsymbol{S}=\begin{bmatrix}\boldsymbol{S}_1\\ \boldsymbol{S}_2\\ \boldsymbol{S}_3\end{bmatrix}=\begin{pmatrix}2\times\boldsymbol{R}_{11}-(1)_{1\times m}\\ (r_{12})_{1\times m}\\ (r_{13})_{1\times m}\end{pmatrix} \tag{51}$$

where r_{10}, r_{12}, r_{13} are random numbers within the range of [0, 1]; $\boldsymbol{s}_1$ is a randomly selected vector in $\boldsymbol{S}$. Then the position of individual i is updated based on $\boldsymbol{X}_i^{\mathrm{Near}}$

$$\boldsymbol{X}_i=\begin{cases}\boldsymbol{X}_i & \boldsymbol{X}_i<\boldsymbol{X}_i^{\mathrm{Near}}\\ \boldsymbol{X}_i^{\mathrm{Near}} & \text{Others}\end{cases} \tag{52}$$

Eq.(52) indicates that if $\boldsymbol{X}_i$ dominates $\boldsymbol{X}_i^{\mathrm{Near}}$, $\boldsymbol{X}_i$ remains unchanged; If $\boldsymbol{X}_i^{\mathrm{Near}}$ dominates $\boldsymbol{X}_i$ or there is no dominance relationship between them, $\boldsymbol{X}_i=\boldsymbol{X}_i^{\mathrm{Near}}$ is updated.

It should be noted that the treatment methods of Eqs.(36, 40, 47, 52) can improve the diversity of hippopotamus populations and avoid early local optimization.

According to the mathematical modelling of MHO, the pseudocode for MHO is shown as follows.

```
Initialize the hippopotamus population
while (end criterion is not met)
  Calculate the fitness of each hippopotamus
  Determine the non-dominated hippopotamuses
  Update the repository considering the obtained non-dominated hippopotamuses by Strategy I
  if the repository becomes full
    Remove the repository resident by Strategy Ⅱ
    Add the non-dominated hippopotamus to the repository
  end
  Choose a dominat hippopotamus from repository by Strategy Ⅲ
  /*Stage 1: The hippopotamuses position update in the river or pond (Exploration)*/
  for i=1:[N/3]
    Calculate the new position of ith hippopotamus using Eq.(35)
    Amend the new position of ith hippopotamus based on the ub_j and lb_j
    Update the position of ith hippopotamus using Eq.(36)
  end
  for i=[N/3]+1:[2×N/3]
    Calculate the new position of ith hippopotamus using Eq.(37)
    Amend the new position of ith hippopota-
```

mus based on the ub_j and lb_j

Update the position of ith hippopotamus using Eq.(40)

end

/*Stage 2: Hippopotamus defence against predators (Exploration)*/

for i=[2×N/3]+1:N

Generate rand position for predator using Eq.(41)

Calculate the new position of ith hippopotamus using Eq.(42)

Amend the new position of ith hippopotamus based on the ub_j and lb_j

Update the position of ith hippopotamus using Eq.(47)

end

/*Stage 3: Hippopotamus Escaping from the Predator (Exploitation)*/

for i=1:N

Calculate the new bounds of decision variables using Eqs.(48, 49)

Calculate the new position of ith hippopotamus using Eq.(50)

Amend the new position of ith hippopotamus based on the ub_j and lb_j

Update the position of ith hippopotamus using Eq.(52)

end

end

return repository

4 Case Study

4.1 Experimental preparation

The MHO validation experiment requires a test function library, MATLAB and computers. To verify the comprehensive performance of MHO, this study selected the non-dominated sorting genetic algorithm Ⅱ (NSGA-Ⅱ)[22-24], the MOPSO[27-28] and the multi-objective salp swarm algorithm (MSSA)[25] for comparative experiments. The parameter settings for the algorithms are shown in Table 2.

Table 2 Parameter settings for comparative experiments

Item	Parameter setting
Test functions	ZDT1, ZDT2, ZDT3, ZDT6[29] ZDT1 with the linear front, ZDT2 with three objectives[30]
Algorithm run times	30
MHO	N=30, m=5, L=200, λ=101
NSGA-Ⅱ	N=30, m=5, L=200, λ=101, p_C=0.7, p_M=0.02
MOPSO	N=30, m=5, L=200, λ=101, c_1=2, c_2=2, w=0.9[28]
MSSA	N=30, m=5, L=200, λ=101
Operating environment configuration	CPU: Intel(R) Core(TM) i7-7700HQ CPU @ 2.80 GHz RAM: 16 GB MATLAB 2014a

For the optimization problem, to determine the exact parameters in the evaluation indicators for the effectiveness of gear hobbing and verify the feasibility of multi-objective optimization between optimization objectives, historical gear hobbing cases[6, 16], gear hobbing machine tool, energy consumption monitoring systems (LDMS)[31], computer, and Minitab software are required. The experimental setup information is shown in Table 3. The parameters of the optimization problem are converted into CNC programs through technicians' programming. During the gear hobbing experiment, the gear hobbing time is recorded and monitored through the CNC system of the gear hobbing machine tool, and energy consumption data is collected through LDMS. The data acquisition process of evaluation indicators for the effectiveness of gear hobbing is shown in Fig.3.

Table 3 Parameter settings for optimization problem, gear hobbing machine tool and hob

Object	Item	Parameter setting
Optimization problem	G	{2, 0.349, 41, 0.456, 94.011, 13, 4.005, 71, 3}
	Material	Low carbon alloy steel
Gear hobbing machine tool	Model	YS3120CNC6-s
	Maximum modules of gear to be hobbed/mm	6
	Maximum diameter of gear to be hobbed/mm	200
	Maximum of n/(r•min^{-1})	1 200
	CNC system	SINUMERIK 840Dsl CNC system
Hob	Material	Hard alloy
	ζ	14
	r_a/mm	0.6
	δ/rad	0.124
	A/mm	2.939
	L_{fa}/mm	183.084
	L_{sl}/mm	10.01
	L_{in}/mm	21.135
	L_{out}/mm	3.792
	v_{fa}/(mm•min^{-1})	800
	v_{sl}/(mm•min^{-1})	90

4.2 MHO validation

The ZDT test functions are selected to verify the effectiveness of the MHO algorithm. Three indicators, namely hypervolume (HV), generative distance (GD), and spacing (SP), are used to evaluate the diversity, convergence, and coverage of Pareto optimality[32]. The algorithm's performance is evaluated using runtime (RT). Qualitative performance results are shown in Figs. 4—9, where PF means Pareto front, and quantitative analysis results are listed in Table 4.

For ZDT1, Fig.4 and Table 4 show that the HV value of MHO is comparable to those of NSGA-Ⅱ and MOPSO, and significantly outperforms that of MSSA. MHO achieves balanced in GD and SP metrics, with a running time of 13.185 s, far lower than those of NSGA-Ⅱ and MOPSO, showing a stable comprehensive performance.

For ZDT2, Fig.5 and Table 4 show that MHO obtains a similar HV value to those of NSGA-Ⅱ and MOPSO and surpasses that of MSSA. It exhibits stable GD convergence and SP coverage, with a running time of 13.305 s, remarkably more efficient than NSGA-Ⅱ and MOPSO.

For ZDT3, Fig.6 and Table 4 show that the HV

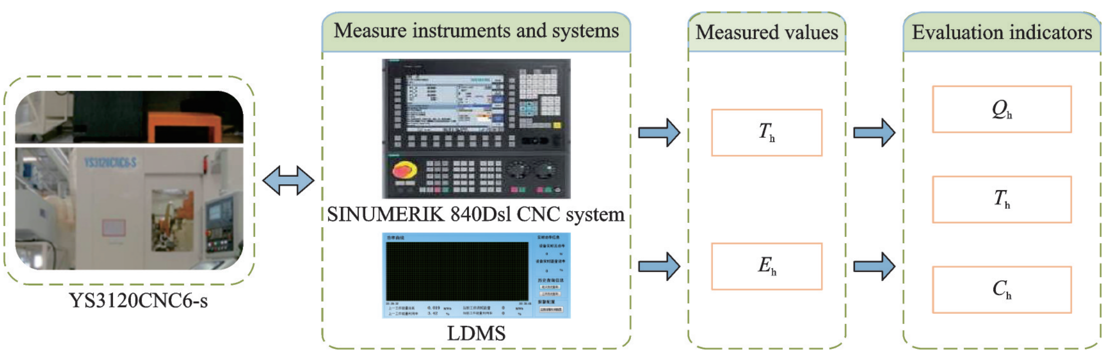

Fig.3 Data acquisition process of evaluation indicators for the effectiveness of gear hobbing

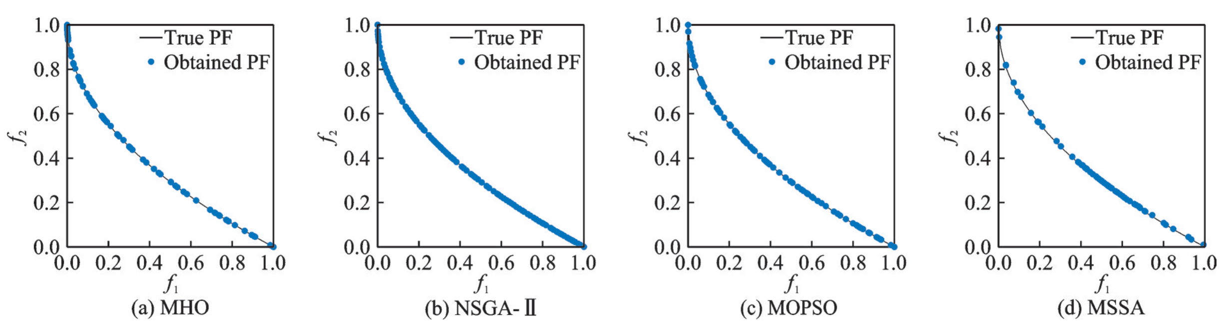

Fig.4 Pareto front of ZDT1

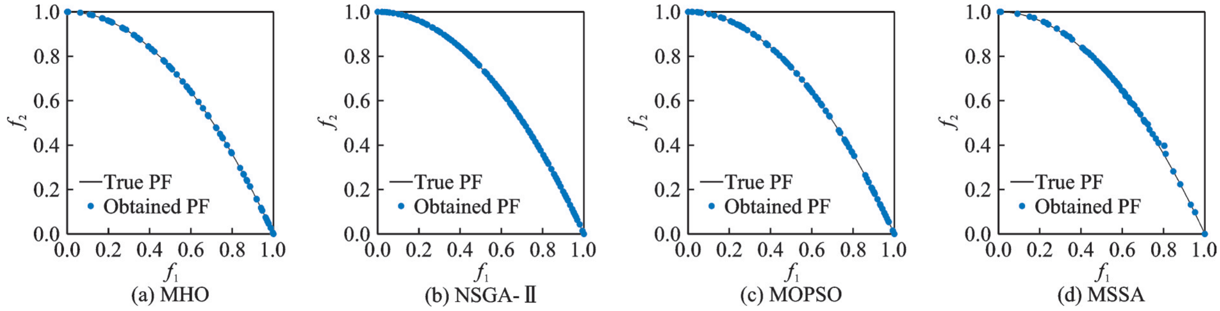

Fig.5 Pareto front of ZDT2

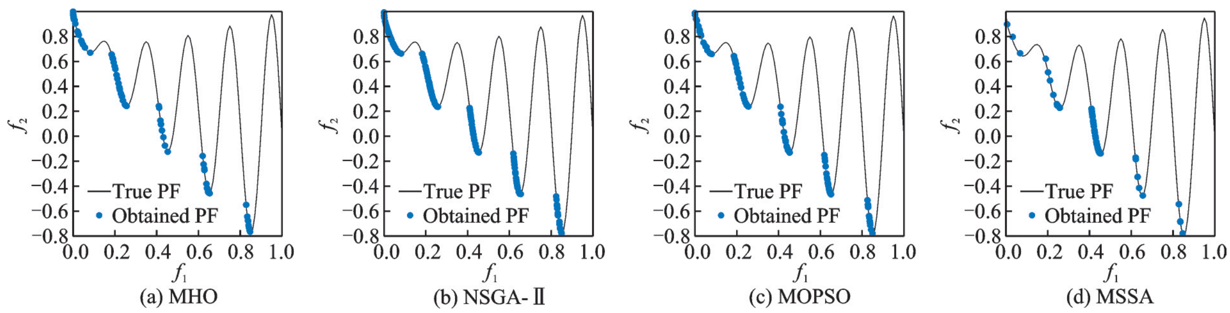

Fig.6 Pareto front of ZDT3

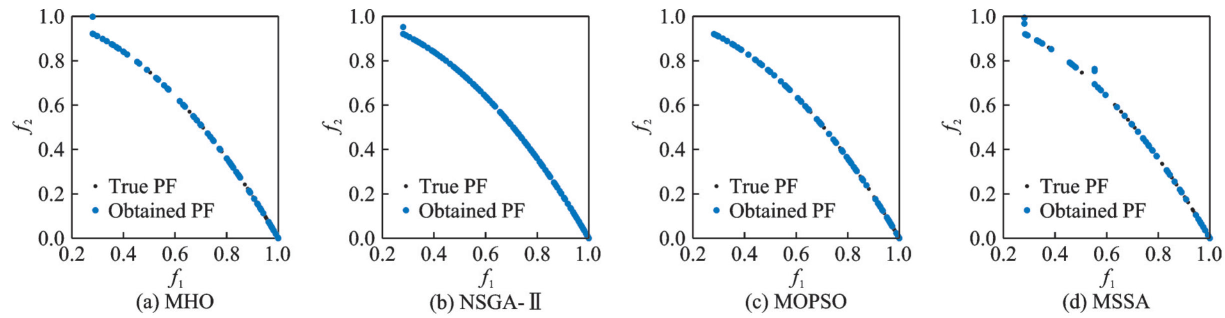

Fig.7 Pareto front of ZDT6

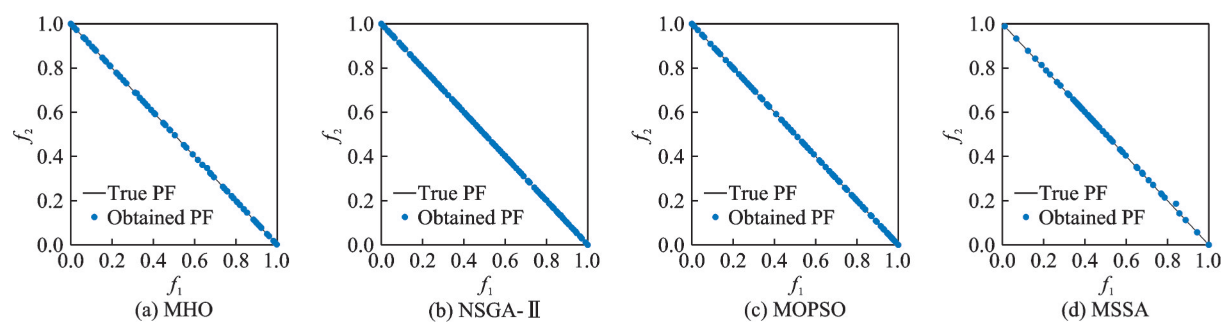

Fig.8 Pareto front of ZDT1 with the linear front

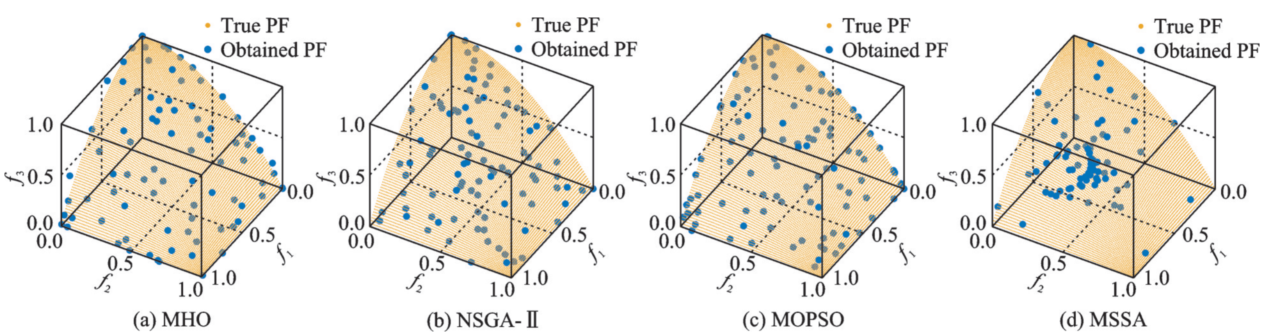

Fig.9 Pareto front of ZDT2 with three objectives

Table 4 Metric indicators for ZDT test functions

Function	Statistics	HV-diversity metric				GD-convergence metric				SP-coverage metric				RT/s			
		MHO	NSGA-Ⅱ	MOPSO	MSSA	MHO	NSGA-Ⅱ	MOPSO	MSSA	MHO	NSGA-Ⅱ	MOPSO	MSSA	MHO	NSGA-Ⅱ	MOPSO	MSSA
ZDT1	Ave	0.712 723	0.717 277	0.715 010	0.697 397	0.001 543	0.000 810	0.000 630	0.000 753	0.013 690	0.007 360	0.011 560	0.012 513				
	Std	0.000 900	0.007 245	0.001 405	0.008 764	0.000 117	0.000 080	0.000 182	0.000 287	0.001 263	0.001 700	0.001 185	0.005 609				
	Median	0.712 9	0.719 25	0.715 3	0.699 7	0.001 5	0.000 8	0.000 6	0.000 75	0.013 8	0.006 95	0.011 65	0.011 3	13.185	76.098	24.641	5.686
	Best	0.714 3	0.719 9	0.717 1	0.708 7	0.001 3	0.000 5	0.000 4	0.000 4	0.011 6	0.004 7	0.009 0	0.003 6				
	Worst	0.711 1	0.681 0	0.710 5	0.666 1	0.001 8	0.000 9	0.001 3	0.001 5	0.016 5	0.012 3	0.014 7	0.029 8				
ZDT2	Ave	0.439 973	0.441 000	0.439 830	0.432 063	0.000 427	0.000 473	0.000 473	0.000 530	0.011 877	0.008 727	0.010 993	0.013 250				
	Std	0.000 690	0.007 896	0.000 944	0.002 703	0.000 045	0.000 045	0.000 074	0.000 121	0.001 383	0.005 464	0.001 500	0.002 871				
	Median	0.439 95	0.443 75	0.440 1	0.432 45	0.000 4	0.000 5	0.000 5	0.000 5	0.011 75	0.007 1	0.010 75	0.013 3	13.305	80.246	27.022	5.760
	Best	0.441 1	0.444 3	0.441 5	0.437 9	0.000 4	0.000 4	0.000 4	0.000 3	0.009 4	0.005 0	0.008 4	0.007 7				
	Worst	0.438 9	0.412 3	0.437 6	0.425 2	0.000 5	0.000 5	0.000 7	0.000 9	0.014 6	0.032 9	0.014 0	0.018 0				
ZDT3	Ave	0.646 573	0.688 593	0.643 470	0.652 913	0.002 327	0.002 387	0.002 303	0.002 690	0.018 027	0.011 867	0.013 467	0.013 053				
	Std	0.009 026	0.040 560	0.001 516	0.029 099	0.000 141	0.000 136	0.000 197	0.000 501	0.003 059	0.010 697	0.002 871	0.005 332				
	Median	0.642 9	0.676 8	0.643 45	0.643 15	0.002 3	0.002 4	0.002 4	0.002 55	0.017 9	0.007 75	0.012 9	0.012 85	13.393	81.382	21.902	4.601
	Best	0.684 5	0.753 8	0.647 7	0.748 8	0.002 1	0.002 1	0.001 6	0.002 0	0.012 6	0.050 1	0.009 1	0.004 2				
	Worst	0.639 8	0.642 7	0.640 1	0.622 1	0.002 6	0.002 7	0.002 5	0.003 6	0.023 3	0.005 6	0.023 6	0.024 1				
ZDT6	Ave	0.378 631	0.385 290	0.384 980	0.371 313	0.001 917	0.002 537	0.002 313	0.003 323	0.016 303	0.008 393	0.008 420	0.009 977				
	Std	0.002 868	0.010 532	0.000 685	0.013 364	0.000 336	0.000 244	0.000 333	0.001 751	0.003 563	0.003 004	0.001 263	0.001 885				
	Median	0.378 8	0.387 8	0.385 15	0.373 7	0.002 0	0.002 6	0.002 2	0.002 95	0.015 8	0.007 7	0.008 25	0.009 55	10.708	78.937	30.254	3.998
	Best	0.384 0	0.388 0	0.385 9	0.380 5	0.001 3	0.001 9	0.001 9	0.001 8	0.009 8	0.005 8	0.005 9	0.006 7				
	Worst	0.370 8	0.330 4	0.382 7	0.310 6	0.002 4	0.003 2	0.003 1	0.011 9	0.024 2	0.021 1	0.011 1	0.013 8				
ZDT1 with the linear front	Ave	0.576 117	0.579 273	0.572 747	0.565 830	0.000 393	0.000 403	0.000 660	0.000 567	0.012 007	0.007 830	0.011 830	0.013 320				
	Std	0.000 929	0.005 072	0.005 805	0.002 827	0.000 037	0.000 018	0.000 631	0.000 215	0.001 127	0.001 929	0.001 548	0.003 696				
	Median	0.576 15	0.581 0	0.574 3	0.565 75	0.000 4	0.000 4	0.000 4	0.000 5	0.011 9	0.007 55	0.011 9	0.012 5	12.974	79.779	25.321	5.667
	Best	0.577 6	0.581 4	0.579 8	0.571 2	0.000 3	0.000 4	0.000 3	0.000 4	0.010 1	0.005 5	0.008 7	0.008 0				
	Worst	0.573 8	0.558 7	0.549 5	0.560 6	0.000 5	0.000 5	0.003 6	0.001 4	0.014 6	0.013 6	0.015 5	0.021 6				
ZDT2 with three objectives	Ave	0.602 790	0.558 680	0.613 380	0.491 343	0.001 317	0.001 360	0.000 970	0.001 617	0.075 487	0.050 400	0.053 400	0.081 400				
	Std	0.005 387	0.019 933	0.003 927	0.037 571	0.000 753	0.001 289	0.000 350	0.000 549	0.006 024	0.005 110	0.004 941	0.016 394				
	Median	0.604 15	0.557 25	0.613 3	0.495 75	0.001 0	0.001 05	0.000 9	0.001 65	0.074 75	0.049 75	0.052 7	0.080 05	13.695	75.581	32.893	7.506
	Best	0.611 0	0.596 6	0.621 6	0.547 1	0.000 7	0.000 6	0.000 5	0.000 8	0.066 2	0.041 1	0.041 2	0.047 3				
	Worst	0.590 6	0.500 2	0.601 9	0.426 9	0.003 9	0.007 6	0.001 7	0.002 7	0.095 4	0.064 3	0.065 6	0.117 3				

of MHO is slightly lower than that of NSGA-Ⅱ, close to that of MOPSO, and better than that of MSSA. MHO maintains balanced in GD and SP indicators, with a running time of 13.393 s, much shorter than those of NSGA-Ⅱ and MOPSO.

For ZDT6, Fig.7 and Table 4 show that MHO's HV is inferior only to those of NSGA-Ⅱ and MOPSO and superior to that of MSSA. It delivers favorable GD and SP performance, with a running time of 10.708 s, balancing solution quality and efficiency.

For ZDT1 with the linear front, Fig.8 and Table 4 show that MHO's HV approximates to that of NSGA-Ⅱ and outperforms those of MOPSO and MSSA. It presents stable GD and SP metrics, with a running time of 12.974 s, far lower than NSGA-Ⅱ and MOPSO.

For ZDT2 with three objectives, Fig.9 and Table 4 show that MHO achieves a significantly higher HV than those of NSGA-Ⅱ and MSSA, with excellent GD convergence and reasonable SP coverage. Its running time of 13.695 s is far shorter than those of NSGA-Ⅱ and MOPSO.

The comprehensive comparison across six ZDT test functions confirms that MHO achieves the best trade-off and reliable performance among the four algorithms. Although other algorithms may excel in specific metrics on individual functions, MHO consistently delivers strong results in convergence, diversity and coverage while maintaining exceptional computational efficiency. Most importantly, MHO shows remarkable scalability in higher-dimensional objective spaces, making it the most efficient and robust algorithm for solving complex three-objective optimization problems.

4.3 Feasibility verification of multi-objective optimization

4.3.1 Parameter determination of optimization objectives

(1) Gear hobbing quality

Based on the parameters of the optimization problem and the hob in Table 3, the parameters are input into Eq.(1), and the gear hobbing quality is calculated as

$$Q_h = 0.026\,869 \times f^2 + 0.006\,615 \tag{53}$$

(2) Gear hobbing time

Based on the practical experience of technicians, $T_{\mathrm{ha}} = 0.328$. According to the parameters in Table 3, the relevant parameters are input into Eq.(6), and the gear hobbing time is calculated as

$$T_{\mathrm{h}} = \frac{518.336}{n \times f} + 0.668 \tag{54}$$

(3) Gear hobbing cost

① Select the empirical formula for energy consumption of the sample with the highest similarity

The historical gear hobbing cases used in this study are from Cao et al.'s research[6, 16], as shown in Table 5. According to Eqs.(18—20), combined with the description of the optimization problem, the weights of m_{n}, α_{n}, z_1, β_{n}, d_2, B, a_{p}, d_0, z_0 are obtained, which are 0.094, 0.099, 0.104, 0.162, 0.090, 0.079, 0.168, 0.085, and 0.119, respectively. According to Eqs.(21—22), the similarity φ_i between each sample in the historical gear hoobing cases and the optimization problem is shown in Table 5. Due to the highest similarity of Case No. 10 in Ref.[6], the empirical formula for energy consumption of this sample is selected.

From Table 5, Refs.[6, 16] does not list the empirical formula for energy consumption of each sample. Therefore, to verify the effectiveness of the proposed method, this study takes the gear of Case No.10 as the machining object, conducts gear hobbing experiments using the YS3120CNC6-s gear hobbing machine tool, and collects energy consumption data through LDMS, as shown in Table 6.

It should be noted that in the multi-variety and small-batch production mode, the empirical formula for energy consumption of each gear is clear. The setting of gear hobbing experiments here is to illustrate the formation process of the empirical formula and provids a reference for implementing the proposed method in this study. During practical production, there is no need to conduct nine experiments for Case No.10. After verifying the correctness of Eq.(17), only a small number of experiments are needed to clarify the empirical formula for energy consumption of the optimization problem.

Table 5 Historical gear hobbing cases

Source	Case No.	m_n/mm	α_n/rad	z_1	β_n/rad	d_2/mm	B/mm	a_p/mm	d_0/mm	z_0	n/(r•min^{-1})	f/(mm•r^{-1})	φ_i
	1	2.45	0.401	46	0.436	128.55	13.50	5.815	80	3	360	1.5	0.849
	2	1.80	0.297	33	0.576	76.30	14.50	5.782	80	2	360	2.0	0.825
	3	1.89	0.288	39	0.576	93.55	13.70	6.321	80	2	360	1.8	0.815
	4	3.65	0.349	23	0.339	101.74	17.50	0.800	90	2	360	1.5	0.717
	6	2.77	0.349	34	0.339	109.81	18.50	6.453	80	2	360	1.5	0.794
	7	1.96	0.305	71	0.559	170.75	24.50	6.013	80	4	360	1.5	0.754
	8	1.94	0.297	73	0.559	172.35	25.50	5.732	80	4	360	1.5	0.751
	9	2.00	0.307	45	0.550	109.68	13.80	6.000	70	3	410	1.5	0.865
Ref.[6]	10	2.00	0.349	53	0.463	132.20	13.20	5.400	70	3	410	1.5	0.880
	12	1.73	0.305	31	0.576	67.80	14.50	5.295	80	2	360	2.0	0.832
	13	1.80	0.297	32	0.576	74.15	14.00	5.778	80	2	360	2.0	0.824
	14	1.89	0.288	36	0.576	86.80	13.00	6.085	80	2	360	2.0	0.821
	16	2.50	0.401	26	0	72.00	14.15	4.700	80	2	360	1.8	0.740
	17	2.65	0.428	26	0	74.75	15.00	5.448	80	2	360	1.8	0.728
	18	2.00	0.340	21	0.532	54.30	16.90	5.957	80	2	360	1.8	0.808
	19	2.40	0.349	17	0.524	54.10	29.10	6.745	80	1	360	1.8	0.725
	20	2.02	0.253	31	0.583	79.10	18.00	5.524	80	2	360	2.0	0.813
	1	2.50	0.401	46	0.436	128.55	13.5	5.815	80	3	380	1.5	0.846
	2	2.00	0.305	44	0.611	110.00	12.6	5.742	80	3	360	1.5	0.851
	3	1.86	0.279	30	0.628	73.90	13.8	5.784	80	2	360	1.8	0.809
	6	1.75	0.305	31	0.576	67.80	14.5	5.297	80	2	400	2.0	0.832
	7	2.50	0.333	17	0.578	59.80	30.4	7.150	80	1	380	1.6	0.712
	8	1.81	0.286	47	0.579	106.22	18.0	4.000	70	3	450	1.5	0.886
Ref.[16]	9	2.75	0.436	18	0.486	62.45	38.0	6.200	80	1	360	1.6	0.700
	11	3.00	0.524	17	0	56.30	40.4	4.020	110	1	230	2.5	0.618
	14	1.63	0.288	35	0.607	74.28	13.7	5.300	70	2	395	1.8	0.832
	16	2.40	0.349	69	0.524	194.10	25.5	6.734	80	4	350	1.2	0.732
	17	2.54	0.333	17	0.578	59.80	30.4	7.150	80	1	380	1.6	0.711
	20	1.86	0.279	40	0.628	96.70	11.9	5.812	76	3	375	1.8	0.844
	28	2.50	0.332	71	0.497	207.70	29.0	6.997	80	4	350	1.2	0.711

Table 6 Case No.10's energy consumption data

n/(r•min^{-1})	f/(mm•r^{-1})	Energy consumption/ (kW•h)	G
370	1.5	0.228 810	0.221 236
370	1.6	0.221 238	0.215 178
370	1.7	0.215 180	0.210 634
410	1.5	0.222 683	0.216 499
410	1.6	0.216 502	0.211 863
410	1.7	0.210 320	0.208 773
450	1.5	0.218 086	0.214 925
450	1.6	0.211 781	0.210 197
450	1.7	0.207 052	0.207 044

Using Minitab software for regression analysis of the energy consumption data (as shown in Fig.10), it is found that in Eq.(17), $\rho = 0.644\,7$, $\Delta = 0.073\,13$, and the standard deviation is 0.000 151 7, which proves the correctness of the assumption. Using Minitab software to regress the energy consumption data of Case No.10, the empirical formula for energy consumption, i.e., Eq.(55) is obtained with a standard deviation of 4.56E−05.

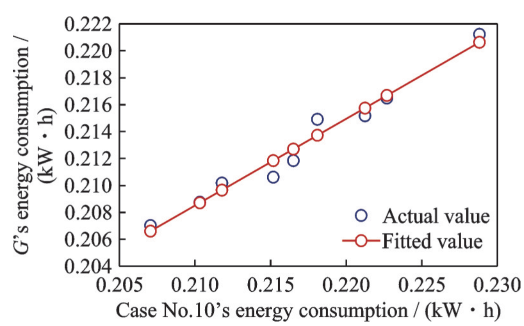

Fig.10 Energy consumption relationship

$$E_h^{bs}=21.19486158-0.101573015\times n-26.04932784\times f+0.00012353\times n^2+8.105398444\times f^2+0.126730886\times n\times f-0.000154503\times n^2\times f-0.039577523\times n\times f^2+4.83125E-05\times n^2\times f^2 \tag{55}$$

②Calculate gear hobbing cost based on the empirical formula for energy consumption

According to the actual gear hobbing situation, the parameter settings for the gear hobbing cost are shown in Table 7. The gear hobbing cost of the optimization problem is defined as

$$C_h=2.037\times T_h+1.025\times E_h \tag{56}$$

$$E_h=0.6447\times E_h^{bs}+0.07313 \tag{57}$$

Table 7 Parameter settings for C_h

Item	Parameter setting
c_{mt}/(CNY·min^{-1})	0.549 8[17]
c_{pt}/(CNY·min^{-1})	0.52
c_{hb}/CNY	2 600
T_{hb}/min	2 800
c_{cf}/CNY	10 000
T_{cf}/min	259 200[17]
c_{he}/(CNY·(kW·h)$^{-1}$)	1.025

4.3.2 Feasibility verification of multi-objective optimization

Effective historical gear hobbing cases are comprehensive results of technicians' process optimization and management feasibility. To better meet the needs of the production site, this study selects the gear hobbing parameters of samples with the similarity greater than 0.8 in historical gear hobbing cases as the constraints for the optimization problem. From Table 5, it can be seen that $n_{lb}=360$, $n_{ub}=450$, $f_{lb}=1.5$ and $f_{ub}=2$.

Without considering hidden management constraints, i. e., $360\leqslant n\leqslant 450$ and $1.5\leqslant f\leqslant 2$ in Eq.(29), the relationship between gear hobbing quality Q_h, gear hobbing time T_h, and gear hobbing cost C_h of the optimization problem is shown in Fig.11. From Fig.11(b), it can be seen that there is a negative correlation between Q_h and T_h: That is, as Q_h decreases, T_h increases. From Fig.11(c), it can be seen that there is a negative correlation between C_h and Q_h: That is, as Q_h decreases, C_h increases. From Fig.11(d), it can be seen that there is a positive correlation between C_h and T_h: That is, as T_h increases, C_h also increases synchronously. In summary, the sufficient and necessary conditions for Pareto domination between Q_h, T_h, and C_h prove the feasibility of multi-objective optimization with Q_h, T_h, and C_h as the objectives.

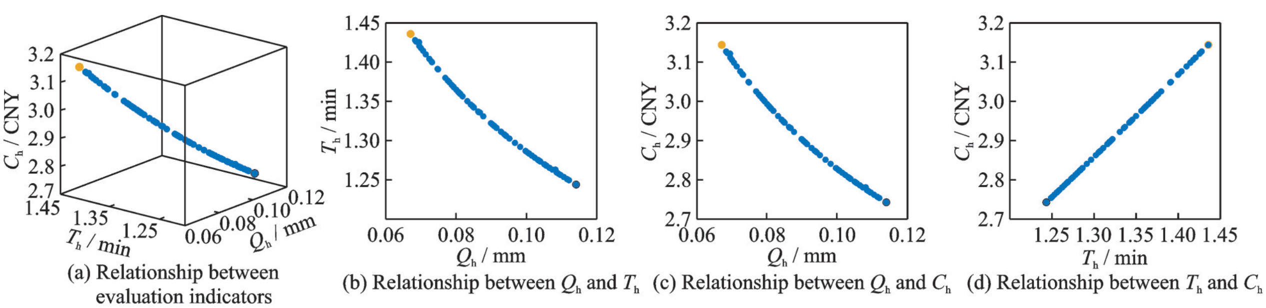

Fig.11 Relationship between evaluation indicators for the effectiveness of gear hobbing without considering hidden management constraints

4.4 Multi-objective optimization of gear hobbing parameters

4.4.1 Influence of gear hobbing parameters on the effectiveness of gear hobbing

Further analysis of Eqs. (53, 54) shows that when n and f are greater than 0, Q_h and T_h decrease as f increases; As n increases, T_h decreases.

To illustrate the influence of n and f on C_h (obtained through Eq.(56)), Table 8 shows that for a specific n, C_h decreases with the increase of f; for a specific f, C_h also decreases as n increases.

The above results indicate that n and f can influence Q_h, T_h, and C_h. Technicians can choose appropriate n and f to achieve satisfactory gear hobbing results.

4.4.2 Analysis of comparative experiments

Based on the problem attributes in Table 5, the $Q_0=0.060826$, $T_0=1.566667$, and $C_0=3.41955$

Table 8 Statistics of gear hobbing cost

n/(r•min^{-1})	f/(mm•r^{-1})	C_h/CNY
370	1.5	3.485
370	1.6	3.345
370	1.7	3.239
410	1.5	3.278
410	1.6	3.172
410	1.7	3.100
450	1.5	3.139
450	1.6	3.033
450	1.7	2.962

for Case No.10 are obtained. These values are then applied to the multi-objective optimization models for gear hobbing parameters and solved by MHO. The Pareto front is shown in Fig.12, and the distribution of Pareto optimal solutions is shown in Fig.13(a). The detailed Pareto optimal solution and front data are shown in Table 9. In Fig.13(a), the yellow dot represents the solution with the minimum gear hobbing quality (such as No.26 in Table 9), the red dot represents the solution with the minimum gear hobbing time and cost(such as No.101 in Table 9).

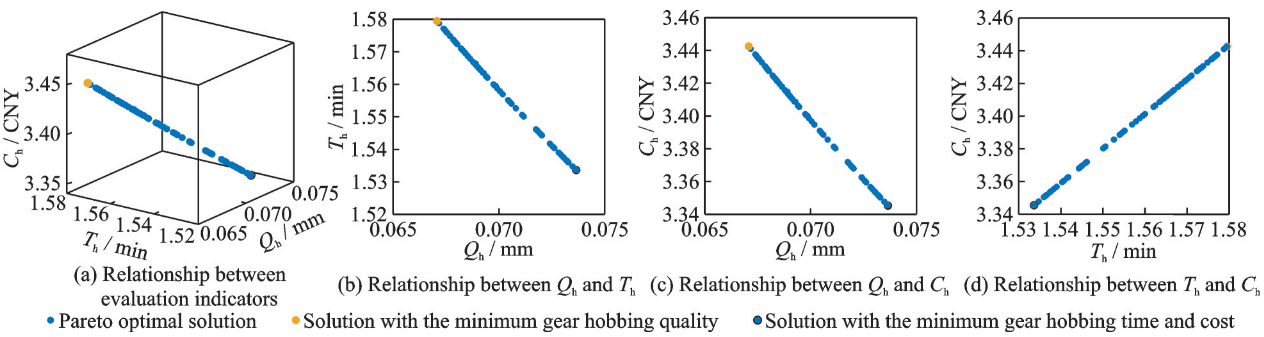

Fig.12 Relationship between evaluation indicators for the effectiveness of gear hobbing considering hidden management constraints

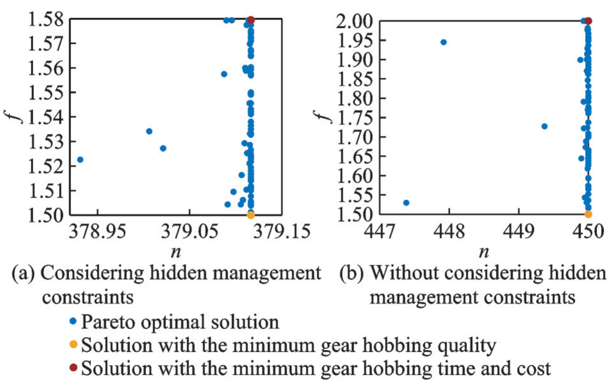

Fig.13 Distribution of Pareto optimal solutions

Table 9 Pareto optimal solution and front

No.	n/(r•min^{-1})	f/(mm•r^{-1})	Q_h/mm	T_h/min	C_h/CNY
1	378.931	1.523	0.068 904	1.566 399	3.414 780
2	379.006	1.534	0.069 852	1.559 466	3.400 092
3	379.021	1.527	0.069 282	1.563 477	3.408 595
4	379.087	1.557	0.071 791	1.545 916	3.371 376
5	379.091	1.504	0.067 419	1.576 926	3.437 108
6	379.096	1.579	0.073 639	1.533 713	3.345 510
7	379.098	1.509	0.067 838	1.573 792	3.430 466
8	379.106	1.504	0.067 419	1.576 886	3.437 024
9	379.106	1.516	0.068 391	1.569 706	3.421 807
⋮	⋮	⋮	⋮	⋮	⋮
26	379.116	1.500	0.067 070	1.579 481	3.442 524
⋮	⋮	⋮	⋮	⋮	⋮
101	379.116	1.579	0.073 646	1.533 618	3.345 311

Two sets of comparative experiments are set up to analyze the practicality of the proposed method in this study, as shown in Table 10. The gear hobbing experiment of Comparison Ⅰ does not consider hidden management constraints; that is, Eq.(29) is solved directly without considering them, and the Pareto optimal front (as shown in Fig.11) and optimal solution (as shown in Fig.13(b)) are obtained.

Table 10 shows that compared with Comparison Ⅰ, the proposed method achieves better gear hobbing quality while pursuing the minimum gear hobbing time and cost. In pursuing the minimum gear hobbing quality, the proposed method has higher gear hobbing time and cost, which indicates that hidden management constraints of gear hobbing limit the optimization of time and cost and further indicates that when optimizing gear hobbing parameters, it is necessary to make a reasonable trade-off between gear hobbing management requirements and actual production constraints, to obtain suitable gear hobbing parameters.

Compared with Comparison Ⅱ, under the same gear hobbing quality, the proposed method has higher gear hobbing time and cost, which indicates that

Table 10 Comparative analysis

Method	Selection strategy	$\{n, f\}$/{r•min^{-1}, mm•r^{-1}}	$\{Q_h, T_h, C_h\}$/{mm, min, CNY}
The proposed method	Minimum gear hobbing quality	{379.116, 1.5}	{0.067 070, 1.579 481, 3.442 524}
	Minimum gear hobbing time	{379.116, 1.579}	{0.073 646, 1.533 618, 3.345 311}
	Minimum gear hobbing cost	{379.116, 1.579}	{0.073 646, 1.533 618, 3.345 311}
Comparison Ⅰ: Without considering hidden management constraints	Minimum gear hobbing quality	{450, 1.5}	{0.067 070, 1.435 905, 3.143 975}
	Minimum gear hobbing time	{450, 2}	{0.114 091, 1.243 929, 2.742 487}
	Minimum gear hobbing cost	{450, 2}	{0.114 091, 1.243 929, 2.742 487}
Comparison Ⅱ: Experience selection	Gear hobbing parameters of the sample with highest similarity	{410, 1.5}	{0.067 070, 1.510 823, 3.299 626}

selecting the gear hobbing parameters of samples in historical gear hobbing cases does not result in the optimal effectiveness of gear hobbing subjected to the currently hidden management constraints.

Therefore, in response to the optimization problem, three selection strategies are proposed according to the proposed method in this study: (1) When pursuing the minimum gear hobbing quality, gear hobbing parameters {379.116, 1.5} could be selected. (2) When pursuing the minimum gear hobbing time, the gear hobbing parameters {379.116, 1.579} could be selected. (3) When pursuing the minimum gear hobbing cost, the gear hobbing parameters {379.116, 1.579} could be selected. Through the above selection strategies, technicians could choose appropriate combinations of gear hobbing parameters as needed, achieving Pareto optimization of gear hobbing quality, gear hobbing time, and gear hobbing cost subjected to hidden management constraints.

5 Conclusions

In the multi-variety and small-batch production mode, the gears to be hobbed are frequently replaced, and it is of great significance to research the optimization of gear hobbing parameters. This study conducts the above research and draws the following conclusions.

(1) Effective historical gear hobbing cases are the best practices for integrating the characteristics of gear hobbing machine tools, gears, hobs, and hidden management constraints of gear hobbing. The two-stage multi-objective optimization models for gear hobbing parameters considering hidden management constraints are established by characterizing the similarity between the optimization problem and the samples in historical gear hobbing cases. The trade-off between gear hobbing quality, processing time, and processing cost is optimized.

(2) MHO is proposed by improving the method for updating the position of the male hippopotamus and introducing the Pareto optimal mechanism. Experimental results on the ZDT test functions show that the obtained Pareto optimal solutions exhibit satisfactory diversity, convergence, and coverage. Moreover, the computational runtime of MHO is significantly superior to that of NSGA-Ⅱ and MOPSO. These findings verify the superiority of MHO in solving multi-objective optimization problems and provide a novel approach for addressing such problems.

(3) By solving the optimization model with MHO, a set of gear hobbing parameters considering hidden management constraints is obtained. Multiple optional gear hobbing parameter combinations are obtained through different selection strategies, by which the effectiveness of gear hobbing meets the requirements of hidden management constraints and can better guide practical gear hobbing, providing a theoretical and scientific basis for technicians and managers to optimize gear hobbing parameters subjected to hidden management constraints.

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考虑隐性管理约束的滚齿工艺参数多目标优化方法

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摘要:在多品种小批量生产模式下,滚齿机的有效历史加工案例是滚齿机、齿轮、滚刀特性及滚齿加工管理要求约束下的最佳实践,案例中样本的滚齿工艺参数蕴含着滚齿加工的隐性管理约束。本文通过构建考虑隐性管理约束的两阶段滚齿工艺参数多目标优化模型,实现隐性管理约束条件下的加工质量、加工时间和加工成本的权衡优化。提出多目标河马优化(Multi-objective hippopotamus optimization, MHO)算法,并通过ZDT系列测试函数验证该算法得到的Pareto最优解的多样性、收敛性和分布性,其运行效率明显优于非支配排序遗传算法Ⅱ(Non-dominated sorting genetic algorithm Ⅱ, NSGA-Ⅱ)、多目标粒子群优化(Multi-objective particle swarm optimization, MOPSO)算法。利用MHO算法求解滚齿工艺参数多目标优化模型,得到隐性管理约束下的滚齿工艺参数决策结果集合,并通过设计不同的选择策略,为技术人员提供多种可选的滚齿工艺参数组合。实验结果表明提出的方法的加工效果更加符合隐性管理约束下的质量、时间和成本的要求。本文的研究成果能为技术人员和管理人员开展滚齿工艺参数决策提供理论和科学依据。

关键词:隐性管理约束;滚齿工艺参数;多目标河马优化算法;非支配排序遗传算法Ⅱ;多目标粒子群优化算法

研究亮点:

1.建立了滚齿加工过程中隐性管理约束的数学模型,实现了隐性管理约束下滚齿工艺参数的多目标优化。

2.提出了MHO,并通过实验验证了其综合表现的优越性。

3.分析了隐性管理约束对滚齿工艺参数优化的影响规律。

4.提供了多种选择策略,为考虑隐性管理约束的滚齿工艺参数决策提供支持。