A New Regularized Minimum Error Thresholding Method

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Abstract：To overcome the shortcoming that the traditional minimum error threshold method can obtain satisfactory image segmentation results only when the object and background of the image strictly obey a certain type of probability distribution，one proposes the regularized minimum error threshold method and treats the traditional minimum error threshold method as its special case. Then one constructs the discrete probability distribution by using the separation between segmentation threshold and the average gray-scale values of the object and background of the image so as to compute the information energy of the probability distribution. The impact of the regularized parameter selection on the optimal segmentation threshold of the regularized minimum error threshold method is investigated. To verify the effectiveness of the proposed regularized minimum error threshold method，one selects typical grey-scale images and performs segmentation tests. The segmentation results obtained by the regularized minimum error threshold method are compared with those obtained with the traditional minimum error threshold method. The segmentation results and their analysis show that the regularized minimum error threshold method is feasible and produces more satisfactory segmentation results than the minimum error threshold method. It does not exert much impact on object acquisition in case of the addition of a certain noise to an image. Therefore，the method can meet the requirements for extracting a real object in the noisy environment.

Key words：image processing；image segmentation；regularized minimum error threshold method；informational divergence；segmentation threshold

0　Introduction

It is difficult for computer vision to analyze，understand and segment an image. Among the many image segmentation methods，the threshold segmentation method is extensively studied and applied due to its simplicity，rapidity，effectiveness，stability and ease of understanding[1]. The minimum error threshold method[2] is applied the most extensively among image segmentation methods because of its firm support by the probability theory. Kittler and Illingworth，the proponents of the method，obtained the threshold segmentation criteria with the Bayesian error theory and some reasonable mathematical treatments on the assumption that the gray scale distributions of ideal objects and backgrounds obey the mixed normal distribution. Many scholars at home and abroad studied the minimum error threshold method[3-10]，for instance，the fast iteration algorithm[3]，the robust minimum error threshold method[4]，the Poisson distribution minimum error threshold method[6]，the Rayleigh distribution minimum error threshold method[7]，the generalized minimum error threshold method[8] and so on. In order to enhance the noise resistance of the minimum error threshold method，they proposed the 2D histogram minimum error threshold method[9] and its fast algorithm[10-12]. To reveal the theoretical foundation of the method more clearly，Refs.[13-14] used informational divergence to give a reasonable explanation to the minimum er-

ror threshold segmentation method, thereby laying a firm theoretical foundation for using the method better. Since the minimum error threshold method is derived from pattern matching, the distribution of the gray scales of the object and background of its hypothetical image strictly obeys the mixed normal distribution, the Poisson distribution, the Rayleigh distribution, etc. However, because of the acquisition environment and transmission interference of an actual image, the distribution of its gray scales cannot strictly obey the mixed normal distribution, the Poisson distribution, the Rayleigh distribution, etc. The thresholds segmented with the minimum error threshold method are not necessarily the ideal ones required by image segmentation, causing severe segmentation errors. To reduce the image segmentation error rate of the minimum error threshold segmentation method, a regularized minimum error threshold method is proposed so as to enhance its segmentation performance, while treating the traditional minimum error threshold method as the former's special case.

1 Minimum Error Threshold Method

Use $f(x,y)$ to denote the gray value of an arbitrary pixel located at (x,y) of a digital image whose size is $M \times N$. The one-dimensional (1D) histogram of the image $h(i)$ denotes the frequency number at which all the gray-scale values of the image appear. Therefore, one can use the 1D histogram to describe the image's probability distribution.

Assume that the ideal distribution of gray scales is a mixed normal distribution $p(g) = \sum_{i=0}^{1} P_i p(g \mid i)$. Here P_i is the prior probability of a sub-distribution. The two sub-distributions $p(g \mid 0)$ and $p(g \mid 1)$ of $p(g)$ obey the normal distribution $p(g \mid i) = \dfrac{1}{\sqrt{2\pi}\sigma_i}\exp\left(-\dfrac{(g-\mu_i)^2}{2\sigma_i^2}\right)$

whose mean value is μ_i and whose standard deviation is σ_i. The estimations of various parameters are given below; if the gray level t is used as the segmentation value, then

$$P_0(t) = \sum_{i=0}^{t} h(i), \quad P_1(t) = \sum_{i=t+1}^{L-1} h(i)$$

$$\mu_0(t) = \frac{\sum_{i=0}^{t} h(i)i}{P_0(t)}, \quad \mu_1(t) = \frac{\sum_{i=t+1}^{L-1} h(i)i}{P_1(t)}$$

$$\sigma_0^2(t) = \frac{\sum_{i=0}^{t}(i-\mu_0(t))^2 h(i)}{P_0(t)} \tag{1}$$

$$\sigma_1^2(t) = \frac{\sum_{i=t+1}^{L-1}(i-\mu_1(t))^2 h(i)}{P_1(t)}$$

For the threshold value $t \in \{0,1,\cdots,L-1\}$, Kittler and Illingworth[2] gave the following expression based on the minimum segmentation error

$$J(t) = 1 + 2\left[P_0(t)\ln\left(\frac{\sigma_0(t)}{P_0(t)}\right) + P_1(t)\ln\left(\frac{\sigma_1(t)}{P_1(t)}\right)\right] \tag{2}$$

The criterion for selecting the best segmentation threshold value is $t = t^*$ that minimizes $J(t)$, namely

$$t^* = \arg\min_{0 < t < L-1} J(t) \tag{3}$$

To have a profound understanding of the minimum error threshold method, Refs. [13-14] minimize the informational divergence between the actual distribution $h(l)$ and the hypothetical distribution $p(l)$ and derive the expression again, further clarifying the mathematical mechanism of the method.

The divergence of the parametrical α between the two discrete probability distributions $P = (p_1, p_2, \cdots, p_n)$ and $Q = (q_1, q_2, \cdots, q_n)$ is as follows

$$d_\alpha(P,Q) = \frac{1}{\alpha(1-\alpha)}\sum_{i=1}^{n}\left[\alpha p_i + (1-\alpha)q_i - p_i^\alpha q_i^{1-\alpha}\right] \quad \alpha \in \mathbf{R} \tag{4}$$

The divergence of the parametrical α has the following characteristics:

(1) $d_\alpha(P,Q)$ is concerned with P, Q and is a concave function;

(2) $d_a(P,Q) \geqslant 0$; when and only when $P = Q$, $d_a(P,Q) = 0$;

(3) The divergences of KL and χ^2 are the special cases of α-type divergence, namely

$$\lim_{a \to 1} d_a(P,Q) = \sum_{i=1}^{n} p_i \ln\left(\frac{p_i}{q_i}\right) \tag{5}$$

which is the famous KL divergence. Also

$$\lim_{a \to 0} d_a(P,Q) = \sum_{i=1}^{n} \frac{(p_i - q_i)^2}{p_i} \tag{6}$$

which is the classic χ^2 divergence.

The minimum error threshold method can be explained with the KL divergence as

$$t^* = \arg \min_{0 < t < L-1} \left\{ \sum_{i=0}^{t} h(i) \ln\left(\frac{h(i)}{P_0(t)p(i \mid 0)}\right) + \sum_{i=t+1}^{L-1} h(i) \ln\left(\frac{h(i)}{P_1(t)p(i \mid 1)}\right) \right\} =$$

$$\arg \min_{0 < t < L-1} \left\{ -\sum_{i=0}^{t} h(i) \ln(P_0(t)p(i \mid 0)) - \sum_{i=t+1}^{L-1} h(i) \ln(P_1(t)p(i \mid 1)) \right\} =$$

$$\arg \min_{0 < t < L-1} \left\{ \ln(2e\pi) + 2P_0(t) \ln\left(\frac{\sigma_0(t)}{P_0(t)}\right) + 2P_1(t) \ln\left(\frac{\sigma_1(t)}{P_1(t)}\right) \right\} =$$

$$\arg \min_{0 < t < L-1} \left\{ 1 + 2P_0(t) \ln\left(\frac{\sigma_0(t)}{P_0(t)}\right) + 2P_1(t) \ln\left(\frac{\sigma_1(t)}{P(t)}\right) \right\} = \arg \min_{0 < t < L-1} J(t) \tag{7}$$

This explanation demonstrates that the minimum error threshold method is, in nature, the gray-scale normal distribution template matching method.

2 Information Entropy and Energy

In the finite universe of discourse $X = \{x_1, x_2, \cdots, x_n\}$, the probability distribution that corresponds to an event A is named as $P = (p_1, p_2, \cdots, p_n)$ and satisfies $\sum_{i=1}^{n} p_i = 1$ and $0 \leqslant p_i \leqslant 1$ $(i = 1, 2, \cdots, n)$. The expression of the information entropy that describes the random uncertainty degrees of probability distribution proposed by Shannon in 1948[15] is as follows

$$H(P) = -\sum_{i=1}^{n} p_i \ln p_i \tag{8}$$

The information entropy has the following typical properties:

(1) If $\forall i \in \{1, 2, \cdots, n\}$ and $p_i = 1/n$, the information entropy $H(P)$ reaches the maximum value $\ln(n)$;

(2) If $\exists i \in \{1, 2, \cdots, n\}$ and $p_i = 1$, the information entropy $H(P)$ reaches the minimum value 0.

In addition to the logarithmic-type expression of the information entropy, there is a parametric-type Tsallis expression[16] of information entropy given as follows

$$H_a(P) = \frac{\sum_{i=1}^{n} p_i^a - 1}{1 - \alpha} \tag{9}$$

Eq. (9) is related to parametrical α divergence $d_a(P,Q)$ as follows

$$H_a(P) = n^{1-a} \left[\frac{1}{1-\alpha} - \alpha d_a\left(P, \left[\frac{1}{n}\right]\right) \right]$$

where $\left[\frac{1}{n}\right] = \left(\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n}\right)$ is the discrete uniform probability distribution.

The parametric-type expression of information entropy has the following properties:

(1) If the parameter $\alpha \to 1$, then

$$\lim_{a \to 1} H_a(P) = H(P) \tag{10}$$

(2) If the parameter $\alpha = 2$, then

$$H_2(P) = 1 - \sum_{i=1}^{n} p_i^2 \tag{11}$$

(3) If the probability distributions of any two independent events A and B are P_1 and P_2, then

$$H_a(P_1 + P_2) = H_a(P_1) + H_a(P_2) + (1 - \alpha) H_a(P_1) H_a(P_2) \tag{12}$$

In 1966, Onicescu[17] proposed his information energy that describes the probability distribution certainty degree, whose expression is $E(P) = \sum_{i=1}^{n} p_i^2$. The information energy has the following typical properties:

(1) If $\forall i \in \{1, 2, \cdots, n\}$ and $p_i = 1/n$, the information energy $E(P)$ reaches the minimum value $1/n$;

(2) If $\exists i \in \{1, 2, \cdots, n\}$ and $p_i = 1$, the information energy $E(P)$ reaches the maximum value 1.

From the above definitions, one can see that the information entropy and the information energy are in pairs. In particular, $H_2(P)$ and $E(P)$ satisfy the validity of $H_2(P) = 1 - E(P)$, so that information energy is related to divergence $d_a(P, Q)$ as follows

$$E(P) = 1 + \frac{1}{n}\left[1 + 2d_2\left(P, \left[\frac{1}{n}\right]\right)\right]$$

This expression provides the possibility of combining information energy with the minimum error threshold criteria.

3 Regularized Minimum Error Threshold Method Based on Informational Divergence

Since the threshold segmentation method is a special type of clustering segmentation method which, however, has uncertainty, the regularization theory has been applied to the study of clustering algorithm[18-19] so as to enhance its performance. The informational divergence theory has also been extensively studied in the construction of the clustering algorithm[20], whose generalized clustering model has been established. The regularization theory has been applied to the improvement of the Otsu threshold method, thus obtaining the improved global threshold segmentation method for fused image gray-scale contrast[21] and the regularized parameter selection method. Saha et al. [22] proposed the regularization variance minimum and maximum local threshold segmentation method and worked out the method for adaptively selecting regularization parameters. To segment an image, one develops the regularized minimum error threshold method based on informational divergence by combining the information energy of object and background of the image with the informational divergence that corresponds to the template matching method.

3. 1 Construction of regular items with information energy

It is assumed that an image has the L gray level $G = \{0, 1, 2, \cdots, L-1\}$ and its histogram is

$h(i)$ $(i = 0, 1, \cdots, L-1)$. Given that its segmentation threshold is t $(0 < t < L-1)$, one segments the image into object and background, each of which corresponds to the following average gray scales

$$m_0(t) = \frac{\sum\limits_{i=0}^{t} h(i)i}{P_0(t)}, \quad m_1(t) = \frac{\sum\limits_{i=t+1}^{L-1} h(i)i}{P_1(t)} \quad (13)$$

where $P_0(t)$ and $P_1(t)$ are a posteriori probability values that correspond to object and background respectively.

With the weighted Otsu threshold coefficient constructed in Ref. [23], one uses the segmentation threshold t, the average gray-scale value of the object $m_0(t)$ and the average gray-scale value of the background $m_1(t)$ to construct the discrete probability distribution as follows:

The discrete probability distribution P': $(p'_1 \quad p'_2)$, where

$$\begin{aligned} p'_1 &= \frac{(t - m_0(t))^2}{(t - m_0(t))^2 + (t - m_1(t))^2} \\ p'_2 &= \frac{(t - m_1(t))^2}{(t - m_0(t))^2 + (t - m_1(t))^2} \end{aligned} \quad (14)$$

The information energy possessed by the discrete probability distribution P' is

$$E(P') = \sum_{i=1}^{2} (p'_i)^2 = \frac{(t - m_0(t))^4 + (t - m_1(t))^4}{((t - m_0(t))^2 + (t - m_1(t))^2)^2} \quad (15)$$

The value of information energy increases with increasing deviation between the segmentation threshold and the average gray-scale values of the object and background respectively. On the contrary, the value of information energy decreases with decreasing deviation between the segmentation threshold and the average gray-scale values of the object and background respectively.

3. 2 Regularized informational divergence threshold method based on information energy

Under the condition that the segmentation threshold t $(0 < t < L-1)$ is given, the probability that corresponds to any gray scale i $(i = 0, 1, \cdots, L-1)$ is $h(i)$ $(i = 0, 1, \cdots, L-1)$.

The minimization of the deviation between

the probability and the joint normal probability distribution of the object and background with the assumed ideal normal distribution constitutes the minimum error threshold criteria. One uses the separate information energy between object and background to construct the regularized informational divergence threshold method，namely

$$t^* = \arg \min_{0<t<L-1} \{J(t) + \lambda E(P')\} \quad (16)$$

In Eq. (16)，the selection of the regularized factor λ is of great importance for obtaining the ideal segmentation threshold. The value of the parameter λ may be positive or negative，and its symbol can be determined with the method to be presented in Section 4.

The informational divergence for the regularized threshold segmentation should be selected as the logarithmic-type expression as proposed in this paper. The parameters are λ and γ ，where $\lambda=4\gamma$ and $|\gamma| \leqslant 3$；the typical values of the parameter γ should be selected as $0,1,2,3$. If the parameter γ is 0，the regularized minimum error threshold method degenerates the traditional minimum error threshold method，which，therefore，is the special case of the regularized minimum error threshold method proposed by this paper.

4　Method for Selecting Symbol of Regularized Parameter

If the object deviation and the background deviation that correspond to a given segmentation value t_2^* are $V_0(t_2^*)$ and $V_1(t_2^*)$ ；and if $V_0(t_2^*) - V_2(t_2^*)$ is larger than 0，the parameter λ is selected as a positive symbol；if $V_0(t_2^*) - V_2(t_2^*)$ is smaller than 0，the parameter λ is selected as a negative symbol；if $V_0(t_2^*) - V_2(t_2^*)$ is equal to 0，the parameter λ is selected as 0.

$$V_0(t) = \sum_{i=0}^{t} h(i)(i - m_0(t))^2$$
$$V_1(t) = \sum_{i=t+1}^{L-1} h(i)(i - m_1(t))^2 \quad (17)$$

where the criterion for selecting the parameter t_2^* is

$$t_2^* = \arg \min_{0<t<L-1} \{(P_0(t))^2 + (P_1(t))^2\} \quad (18)$$

5　Explanation on Rationality of Regularized Minimum Error Threshold Method

Assuming that the probability of the image's histogram is a continuous function $h(x)$ ，one constructs the objective function that corresponds to the regularized minimum error threshold criteria as follows

$$J(t) = 1 - 2(P_0(t)\ln P_0(t) + P_1(t)\ln P_1(t)) + P_0(t)\ln\sigma_0^2(t) + P_1(t)\ln\sigma_1^2(t) + \lambda E(P') \quad (19)$$

where $P_0(t) = \int_0^t h(x)\mathrm{d}x$, $P_1(t) = \int_t^{+\infty} h(x)\mathrm{d}x$

$$\sigma_0^2(t) = \frac{\int_0^t h(x)(x - m_0(t))^2 \mathrm{d}x}{P_0(t)}$$

$$\sigma_1^2(t) = \frac{\int_t^{+\infty} h(x)(x - m_1(t))^2 \mathrm{d}x}{P_1(t)}$$

$$m_0(t) = \frac{\int_0^t h(x)x\mathrm{d}x}{P_0(t)}, \quad m_1(t) = \frac{\int_t^{+\infty} h(x)x\mathrm{d}x}{P_1(t)}$$
$$(20)$$

Accordingly，one obtains the derivative of $J(t)$ with the segmentation threshold t as follows

$$J'(t) = -2h(t)\ln\left(\frac{P_0(t)}{P_1(t)}\right) + h(t)\ln(\sigma_0^2(t)) + \frac{P_0(t)}{\sigma_0^2(t)}\frac{\mathrm{d}\sigma_0^2(t)}{\mathrm{d}t} - h(t)\ln(\sigma_1^2(t)) + \frac{P_1(t)}{\sigma_1^2(t)}\frac{\mathrm{d}\sigma_1^2(t)}{\mathrm{d}t} + \lambda\frac{\mathrm{d}E(P')}{\mathrm{d}t} \quad (21)$$

where

$$\frac{\mathrm{d}\sigma_0^2(t)}{\mathrm{d}t} = \frac{h(t)(t - m_0(t))^2}{P_0(t)} - \frac{h(t)}{P_0(t)}\sigma_0^2(t)$$

$$\frac{\mathrm{d}\sigma_1^2(t)}{\mathrm{d}t} = \frac{-h(t)(t - m_0(t))^2}{P_1(t)} + \frac{h(t)}{P_1(t)}\sigma_1^2(t)$$

$$E(P') = \frac{(t - m_0(t))^4 + (t - m_1(t))^4}{((t - m_0(t))^2 + (t - m_1(t))^2)^2}$$

$$\frac{\mathrm{d}E(P')}{\mathrm{d}t} = $$

$$\frac{4((t - m_0(t))^3(1 - m'_0(t)) + (t - m_1(t))^3(1 - m'_1(t)))}{((t - m_0(t))^2 + (t - m_1(t))^2)^2} -$$

$$\frac{4((t - m_0(t))^4 + (t - m_1(t))^4)(t - m_0(t))(1 - m'_0(t))}{((t - m_0(t))^2 + (t - m_1(t))^2)^3} -$$

$$\frac{4((t - m_0(t))^4 + (t - m_1(t))^4)(t - m_1(t))(1 - m'_1(t))}{((t - m_0(t))^2 + (t - m_1(t))^2)^3}$$

$$m'_0(t) = \frac{h(t)t}{P_0(t)} - \frac{m_0(t)h(t)}{P_0(t)}$$

$$m'_1(t) = \frac{-h(t)t}{P_1(t)} + \frac{m_1(t)h(t)}{P_1(t)} \quad (22)$$

If its derivative is $J'(t) = 0$, one can obtain the iteration expression for acquiring the segmentation threshold

$$t = \frac{g_1(t)}{g_2(t)} + \lambda \frac{E'(P')}{g_2(t)} \qquad (23)$$

where

$$g_1(t) = 2h(t)\ln\left(\frac{P_0(t)}{P_1(t)}\right) + h(t)\ln\left(\frac{\sigma_1^2(t)}{\sigma_0^2(t)}\right) +$$

$$h(t)\left(\frac{t^2 + m_1^2(t)}{\sigma_1^2(t)} - \frac{t^2 + m_0^2(t)}{\sigma_0^2(t)}\right)$$

$$g_2(t) = 2h(t)\left(\frac{m_1(t)}{\sigma_1^2(t)} - \frac{m_0(t)}{\sigma_0^2(t)}\right)$$

$$E'(P') = \frac{\mathrm{d}E(P')}{\mathrm{d}t} \qquad (24)$$

where $E'(P')$ is the derivative of $E(P')$ to the parameter t.

Since the probability distribution of the histogram of an actual image is discrete and its integral can be discretely and approximately summed rather than calculated, the discrete iteration formula that corresponds to the regularized minimum error threshold method can be obtained, whereas the discrete iteration formula that corresponds to the traditional minimum error threshold method is

$$t^{(k+1)} = \frac{g_1(t^{(k)})}{g_2(t^{(k)})} \quad k = 0, 1, \cdots \qquad (25)$$

Therefore, the regularized minimum error threshold method, in nature, is to add the threshold obtained with the traditional minimum error threshold method to the allowance $E'(P')/g_2(t^{(k)})$ controlled by the parameter λ so that the obtained segmentation threshold will be more approximate to the ideal segmentation threshold of the image itself, thus reducing the image's segmentation error rate.

The above analysis shows that the regularized minimum error threshold method, in nature, corrects the segmentation threshold obtained with the traditional minimum error threshold method and treats the latter as a special case of the former.

6 Experiments and Result Analysis

To verify the effectiveness of our regularized minimum error threshold method based on infor-mational divergence, one selects some typical grey-scale images and carries out the test analysis of the images obtained with our regularized minimum error threshold method and the traditional minimum error threshold method respectively.

6.1 Parameter selection and test

The regularized parameter λ is important for the regularized minimum error threshold method. The selection of inappropriate parameters may cause immeasurable loss to the subsequent image processing. Therefore, one investigates the impact of the regularized parameter selection on the optimal segmentation threshold of the regularized minimum error threshold method.

One uses the four standard gray-scale images, i. e., Lena, cameraman, mug and the moon (Fig. 1) to investigate the impact of the regularized parameter λ of the regularized minimum error threshold method on its segmentation threshold. The image segmentation results are presented in Table 1. The regularized parameter is $\lambda = 4\gamma$, whose symbol uses the object and background a posteriori probability information energy minimum determination method.
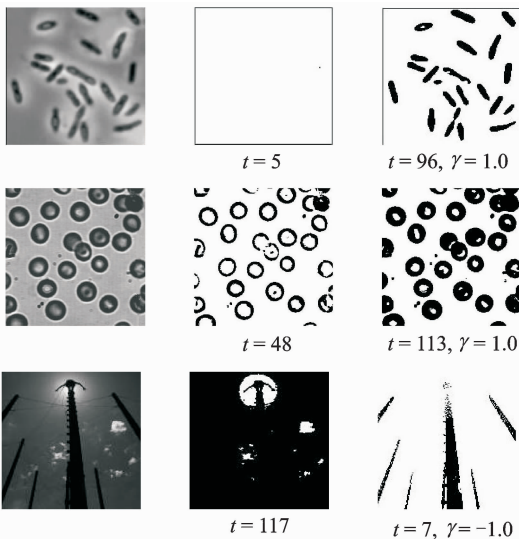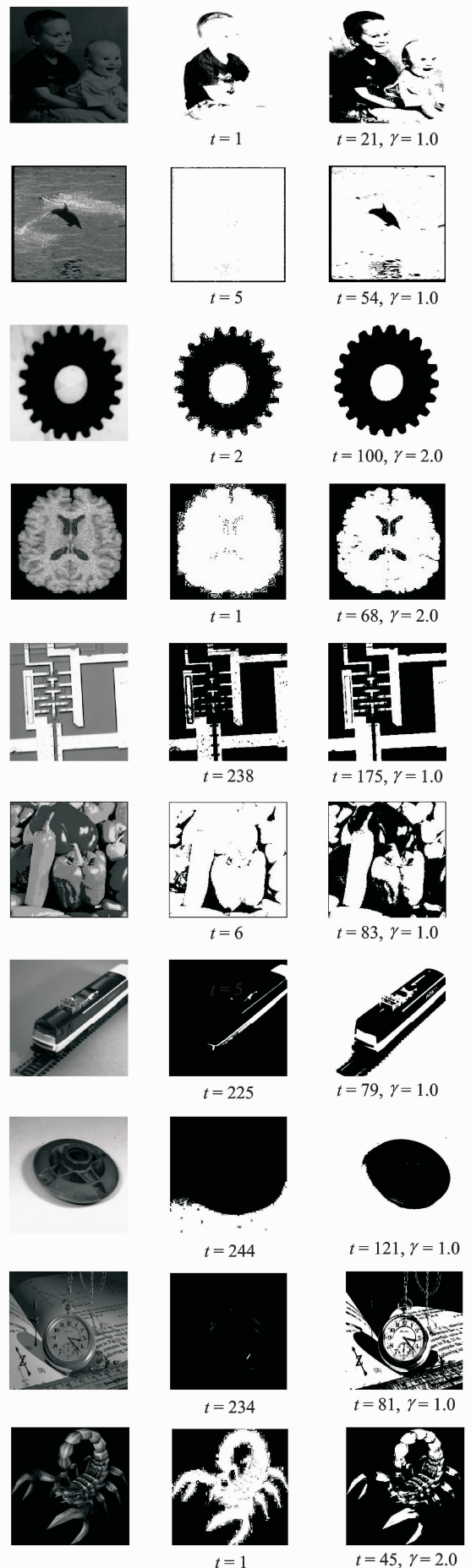

Fig. 1   Raw images

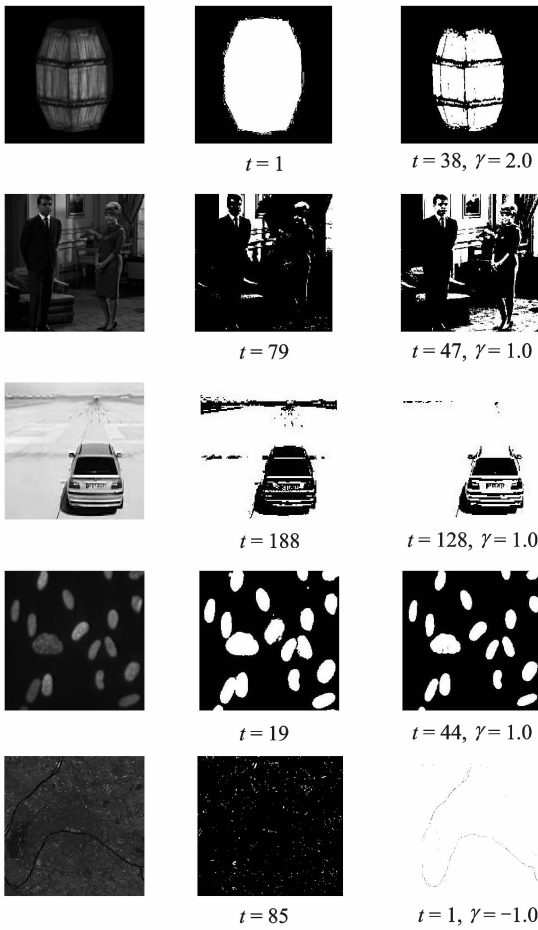Table 1   Image segmentation results on four images

$|\gamma|$	Segmentation threshold			
	Lena	Cameraman	Mug	The moon
0.00	63	24	58	10
0.10	71	25	78	10
0.25	11	81	88	11
0.50	113	84	91	11
0.75	114	86	93	11
1.00	114	87	94	76
1.25	115	87	94	79
1.50	115	87	94	82
1.75	115	87	118	82
2.00	115	87	118	84
2.25	115	88	118	85
2.50	115	88	118	85
2.75	115	88	118	85
3.00	115	88	118	86

The segmentation results on the four images (Table 1) show that，if the absolute value of the parametric γ in the regularized parameter λ takes 0，the regularized minimum error thresholding method proposed in this paper may function the same as the classic minimum error thresholding method，whose optimal threshold for segmenting a gray image is the smallest. As the absolute value of the parametric γ in the regularized parameter λ increases gradually，the optimal threshold of the gray image segmented by the regularized minimum error threshold method changes monotonously. When the parameter $|\gamma|$ is greater than 2，the optimal threshold of gray image does not change obviously. In general，if the absolute value of the parametric γ in the regularized parameter λ in the majority of grey-scale images is less than or equals to 1，satisfactory segmentation results are obtained. But if the ratio of mean value to variation of gray image is greater than the constant of 0.05，the parametric absolute value γ in the regularized parameter λ must be 2 in order to obtain better segmentation quality.

6.2　Image segmentation result comparison

The gray-level image segmentation results，given in Fig. 2，and their analysis show that the regularized minimum error threshold method is feasible and produces more satisfactory segmenta-

$t=1$　　　　$t=21, \gamma=1.0$

$t=5$　　　　$t=54, \gamma=1.0$

$t=2$　　　　$t=100, \gamma=2.0$

$t=1$　　　　$t=68, \gamma=2.0$

$t=238$　　　　$t=175, \gamma=1.0$

$t=6$　　　　$t=83, \gamma=1.0$

$t=225$　　　　$t=79, \gamma=1.0$

$t=244$　　　　$t=121, \gamma=1.0$

$t=234$　　　　$t=81, \gamma=1.0$

$t=1$　　　　$t=45, \gamma=2.0$

$t=5$　　　　$t=96, \gamma=1.0$

$t=48$　　　　$t=113, \gamma=1.0$

$t=117$　　　　$t=7, \gamma=-1.0$

(a) Raw images   (b) Minimum error threshold method   (c) Regularized minimum error threshold method

Fig. 2   Result comparison between the regularized minimum error threshold method and the minimum error threshold method

tion results than the minimum error threshold method.

6.3   Robutness test of the regularized minimum error threshold method

To test robustness of the regularized minimum error threshold method, one compares the results of segmenting the standard images of Lena and cameraman respectively. The segmentation results are given in Figs. 3, 4. The mean value of the image with the Gaussian noise is 0, and its standard deviations are 0, 8.0, 16.0, and 25.5, respectively.

The image segmentation results, given in Figs. 3, 4, and their analysis show that, compared with the traditional minimum error threshold method, the regularized minimum error threshold
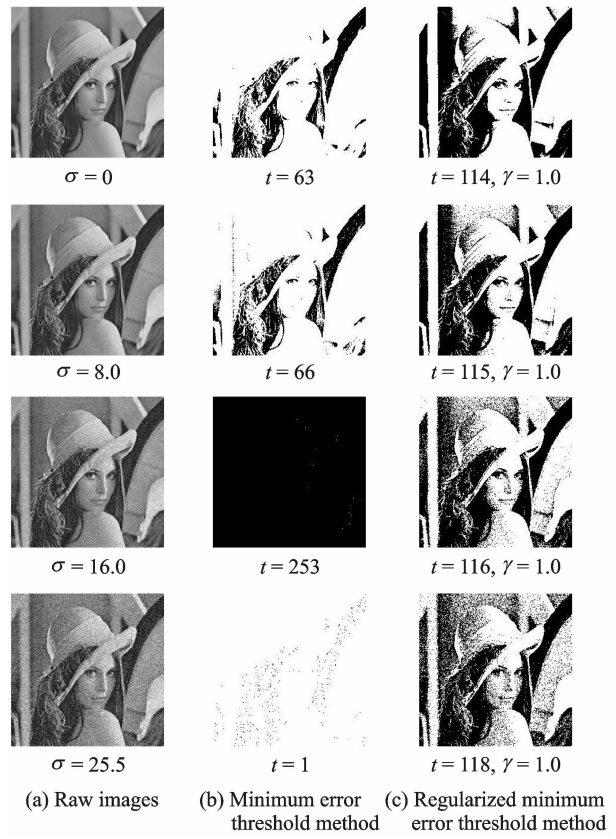

(a) Raw images   (b) Minimum error threshold method   (c) Regularized minimum error threshold method

Fig. 3   Segmentation results on image of Lena with the Gaussian noise mean value as 0

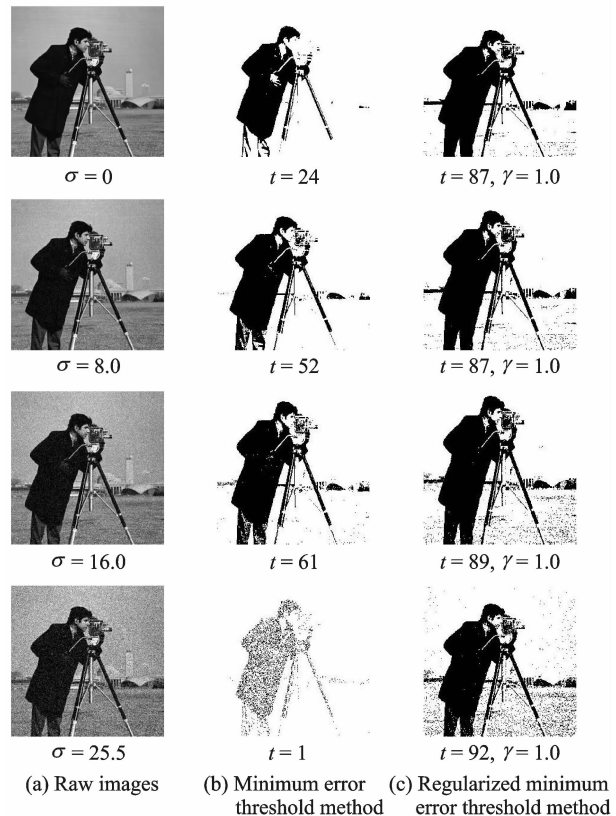(a) Raw images   (b) Minimum error threshold method   (c) Regularized minimum error threshold method

Fig. 4   Segmentation results on image of a cameraman with the Gaussian noise mean value as 0

method does not exert much impact on object acquisition in case of the addition of a certain noise to an image. However，the traditional minimum error threshold method is quite sensitive to noise and cannot acquire the precise information on its object when the noise is large. Therefore，the proposed method can meet the requirements for extracting a real object in the noisy environment.

7 Conclusions

The uncertainty-reducing regularization theory is introduced into the study of the image segmentation with the minimum error threshold method and a new regularized minimum error threshold method is proposed to solve the degree of separation between the segmented object and the segmented background，where the traditional minimum error threshold method is not taken into account. The new method treats the traditional minimum error threshold method as its special case and can enhance the segmentation performance of the latter. Besides，the regularized minimum error threshold method can be extended to the scenarios with multiple threshold values and other various minimum error threshold methods so long as the object and background in the image obey the Poisson distribution or the Rayleigh distribution.

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