CRB for 2-D DOA Estimation in MIMO Radar with UCA

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Abstract: The Cramer-Rao bound (CRB) for two-dimensional (2-D) direction of arrival (DOA) estimation in multiple-input multiple-output (MIMO) radar with uniform circular array (UCA) is studied. Compared with the uniform linear array (ULA), UCA can obtain the similar performance with fewer antennas and can achieve DOA estimation in the range of 360°. This paper investigates the signal model of the MIMO radar with UCA and 2-D DOA estimation with the multiple signal classification (MUSIC) method. The CRB expressions are derived for DOA estimation and the relationship between the CRB and several parameters of the MIMO radar system is discussed. The simulation results show that more antennas and larger radius of the UCA leads to lower CRB and more accurate DOA estimation performance for the monostatic MIMO radar. Also the interference during the 2-D DOA estimation will be well restrained when the number of the transmitting antennas is different from that of the receiving antennas.

Key words: uniform circular array (UCA); Cramer-Rao bound (CRB); direction of arrival (DOA) estimation; multiple-input multiple-output (MIMO) radar

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0 Introduction

The multiple-input multiple-out (MIMO) radar exploits the spatial diversity and the degree of freedom to improve resolution, clutter mitigation and classification performance[1]. Unlike a standard phased-array radar, which transmits scaled versions of a single waveform, MIMO radars emit orthogonal or noncoherent waveforms in each of the transmitting antennas and utilize a bank of matched filters to extract the waveforms at the receiver[2-3]. This paper focuses on array configuration problems of one class of MIMO radars which use closely spaced antennas to achieve coherent processing gain. Many literatures apply the uniform linear array (ULA) for array configuration and there have been various traditional algorithms for its direction of arrival (DOA) estimation. However, compared to ULA, uniform circular array (UCA) has its own advantages of DOA estimation in range of 360° and less antennas required than ULA for the same radar performance[4-5]. Here we will explore the property of UCA to design both the transmitting and receiving arrays and derive the CRB for DOA estimation of the MIMO radar system. The relationship between the parameters of the MIMO radar and the CRB will be investigated in detail.

1 Signal Model for MIMO Radar with UCA

Assume a MIMO radar system that utilizes a UCA with M transmitting antennas and N receiving antennas (Fig. 1). R_1 and R_2 denote the radii of the transmitting and receiving arrays, respec-

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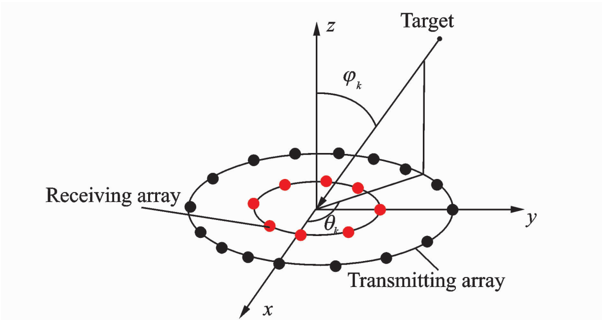


Fig. 1 Signal model for MIMO radar with UCA

tively. Each of the transmitting antennas emits M orthogonal signals $s_m \in C^{L\times 1}$, $m=1,2,\cdots,M$ where L is the number of samples of each pulse. Let $\boldsymbol{S}=[s_1,s_2,\cdots,s_M]$. Assume there are also many (say, K) far field independent scattering point targets with the azimuth angle θ and the elevation angle φ. Then the transmitting and receiving steering vectors can be described by the following expression respectively.

$$\boldsymbol{a}(\theta_k,\varphi_k)=[\xi_{k0},\xi_{k1},\cdots,\xi_{k(M-1)}]^{\mathrm{T}} \tag{1}$$

$$\boldsymbol{b}(\theta_k,\varphi_k)=[\zeta_{k0},\zeta_{k1},\cdots,\zeta_{k(N-1)}]^{\mathrm{T}} \tag{2}$$

where

$$\xi_{ki}=\exp\left\{\frac{\mathrm{j}2\pi R_1}{\lambda}\sin(\varphi_k)\cos\left(\theta_k-\frac{2\pi i}{M}\right)\right\} \tag{3}$$

$$\zeta_{kj}=\exp\left\{\frac{\mathrm{j}2\pi R_2}{\lambda}\sin(\varphi_k)\cos\left(\theta_k-\frac{2\pi j}{N}\right)\right\} \tag{4}$$

and $i=0,1,\cdots,M-1$; $j=0,1,\cdots,N-1$, and $(\cdot)^{\mathrm{T}}$ denotes the transpose of the matrix.

Then the received signal $\boldsymbol{X}\in C^{N\times L}$ is

$$\boldsymbol{X}=\sum_{k=1}^{K}\boldsymbol{b}(\theta_k,\varphi_k)\eta_k\boldsymbol{a}(\theta_k,\varphi_k)^{\mathrm{T}}\boldsymbol{S}+\boldsymbol{E} \tag{5}$$

where $\boldsymbol{E}\in C^{N\times L}$ is the interference plus Gaussian noise uncorrelated with $\boldsymbol{X}$.

Thus, the output of the matched filters is[6]

$$\boldsymbol{Y}=\mathrm{vec}(\boldsymbol{X}\boldsymbol{S}^{\mathrm{H}}/L) \tag{6}$$

where vec($\cdot$) denotes vectoring of the matrix. The DOA estimation method applied here is the multiple signal classification (MUSIC) algorithm[7-8].

2 CRB for 2-D DOA Estimation

CRB is probably the best known lower bound on the mean square error (MSE) of unbiased estimators[9-10]. In this section we derive the CRB for 2-D DOA estimation of MIMO radar with UCA. Assume that the parameters to be estimated corresponding to the kth target is $\boldsymbol{\Omega}=[\theta,\varphi,\eta,\sigma^2]^{\mathrm{T}}$, where θ is the azimuth angle, φ the elevation angle, η the scattering coefficients, and σ^2 the parameter of the noise. Besides, assume that during the qth pulse, the received signal through the lth sampling is $\boldsymbol{x}(l-1)$, $l=1,\cdots,L$. Then a column vector is given by

$$\boldsymbol{z}=[\boldsymbol{x}(0)^{\mathrm{T}},\boldsymbol{x}(1)^{\mathrm{T}},\cdots,\boldsymbol{x}(L-1)^{\mathrm{T}}]^{\mathrm{T}} \tag{7}$$

The received signal without noise can be expressed as

$$\boldsymbol{u}(l)=\sum_{k=1}^{K}\eta_k\boldsymbol{b}(\theta_k,\varphi_k)\boldsymbol{a}^{\mathrm{T}}(\theta_k,\varphi_k)\boldsymbol{s}(l) \tag{8}$$

Then we have

$$\boldsymbol{u}=[\boldsymbol{u}^{\mathrm{T}}(0),\boldsymbol{u}^{\mathrm{T}}(1),\cdots,\boldsymbol{u}^{\mathrm{T}}(L-1)]^{\mathrm{T}}=E[\boldsymbol{z}] \tag{9}$$

Note that the received signal obeys the Gaussian distribution $\boldsymbol{z}\sim N_C(\boldsymbol{u},\sigma^2\boldsymbol{I}_{NL\times NL})$ and its probability density function is

$$p(\boldsymbol{z},\boldsymbol{\Omega})=\frac{1}{\pi^{NL}\sigma^{2NL}}\exp\left\{\frac{-1}{\sigma^2}(\boldsymbol{z}-\boldsymbol{u})^{\mathrm{H}}(\boldsymbol{z}-\boldsymbol{u})\right\} \tag{10}$$

For each unknown parameter in vector $\boldsymbol{\Omega}=[\theta,\varphi,\eta,\sigma^2]^{\mathrm{T}}$, its estimated variance has a lower bound, which is the CRB. It can be computed by the Fisher information matrix

$$\mathrm{var}(\Omega_i)\geqslant[\boldsymbol{F}^{-1}]_{ii}\quad 1\leqslant i\leqslant 4 \tag{11}$$

where var($\cdot$) denotes the variance, Ω_i the estimation of ith element of $\boldsymbol{\Omega}$, and $[\boldsymbol{F}^{-1}]_{ii}$ the (i,i) element of the inverse matrix of the Fisher information matrix $\boldsymbol{F}$. The element of $\boldsymbol{F}$ can be expressed as

$$F_{i,j}=f(\Omega_i,\Omega_j)=E\left[\frac{\partial\ln p(\boldsymbol{z},\boldsymbol{\Omega})}{\partial\Omega_i}\cdot\frac{\partial\ln p(\boldsymbol{z},\boldsymbol{\Omega})}{\partial\Omega_j}\right] \tag{12}$$

Hence, the Fisher information matrix has the following expression

$$\boldsymbol{F}=\begin{bmatrix} f(\theta,\theta) & f(\theta,\varphi) & f(\theta,\eta) & f(\theta,\sigma^2) \\ f(\varphi,\theta) & f(\varphi,\varphi) & f(\varphi,\eta) & f(\varphi,\sigma^2) \\ f(\eta,\theta) & f(\eta,\varphi) & f(\eta,\eta) & f(\eta,\sigma^2) \\ f(\sigma^2,\theta) & f(\sigma^2,\varphi) & f(\sigma^2,\eta) & f(\sigma^2,\sigma^2) \end{bmatrix} \tag{13}$$

Since $\boldsymbol{F}$ is a symmetric matrix, only one part of its elements needs to be calculated. Define a matrix $\boldsymbol{C}=\sigma^2\boldsymbol{I}_{NL\times NL}$, the following expression can be obtained from Eq. (12)

$$f(\Omega_i,\Omega_j)=\operatorname{tr}\left[\boldsymbol{C}^{-1}\frac{\partial\boldsymbol{C}}{\partial\Omega_i}\boldsymbol{C}^{-1}\frac{\partial\boldsymbol{C}}{\partial\Omega_j}\right]+2\operatorname{Re}\left[\frac{\partial\boldsymbol{u}^{\mathrm{H}}}{\partial\Omega_i}\boldsymbol{C}^{-1}\frac{\partial\boldsymbol{u}}{\partial\Omega_j}\right]=\frac{NL}{\sigma^4}\frac{\partial\sigma^2}{\partial\Omega_i}\frac{\partial\sigma^2}{\partial\Omega_j}+\frac{2}{\sigma^2}\operatorname{Re}\left[\frac{\partial\boldsymbol{u}^{\mathrm{H}}}{\partial\Omega_i}\frac{\partial\boldsymbol{u}}{\partial\Omega_j}\right]\tag{14}$$

The transmitting array of the MIMO radar with UCA emits orthogonal signal, therefore, we can obtain $\boldsymbol{R}=\mu^2\boldsymbol{I}_{M\times M}$, where μ^2 denotes the power of the signal. Let $\boldsymbol{d}(\theta,\varphi)=\boldsymbol{b}(\theta,\varphi)\boldsymbol{a}^{\mathrm{T}}(\theta,\varphi)$ and $\boldsymbol{R}=\frac{1}{L}\sum_{l=0}^{L-1}\boldsymbol{s}(l)\boldsymbol{s}^{\mathrm{H}}(l)$, the element of $\boldsymbol{F}$ can be calculated

$$\begin{aligned}f(\theta,\theta)&=\frac{2}{\sigma^2}\sum_{l=0}^{L-1}\operatorname{Re}\left[\frac{\partial\boldsymbol{u}^{\mathrm{H}}(l)}{\partial\theta}\frac{\partial\boldsymbol{u}(l)}{\partial\theta}\right]=\\&\frac{2}{\sigma^2}\sum_{l=0}^{L-1}\operatorname{Re}\left[\operatorname{tr}\left(\frac{\partial\boldsymbol{u}(l)}{\partial\theta}\frac{\partial\boldsymbol{u}^{\mathrm{H}}(l)}{\partial\theta}\right)\right]=\\&\frac{2L\eta^2}{\sigma^2}\operatorname{Re}\left[\operatorname{tr}\left(\frac{\partial\boldsymbol{d}(\theta,\varphi)}{\partial\theta}\boldsymbol{R}\frac{\partial\boldsymbol{d}^{\mathrm{H}}(\theta,\varphi)}{\partial\theta}\right)\right]=\\&\frac{2L\mu^2\eta^2}{\sigma^2}\operatorname{Re}\left[\operatorname{tr}\left(\frac{\partial\boldsymbol{d}(\theta,\varphi)}{\partial\theta}\frac{\partial\boldsymbol{d}^{\mathrm{H}}(\theta,\varphi)}{\partial\theta}\right)\right]\end{aligned}\tag{15}$$

Considering that the transmitting array and receiving array of the MIMO radar satisfy the following expression

$$\begin{cases}\boldsymbol{a}^{\mathrm{T}}(\theta,\varphi)\boldsymbol{a}^{*}(\theta,\varphi)=M,\ \boldsymbol{b}^{\mathrm{H}}(\theta,\varphi)\boldsymbol{b}(\theta,\varphi)=N\\ \dfrac{\partial\boldsymbol{a}^{\mathrm{T}}(\theta,\varphi)}{\partial\theta}\boldsymbol{a}^{*}(\theta,\varphi)=\boldsymbol{a}^{\mathrm{T}}(\theta,\varphi)\dfrac{\partial\boldsymbol{a}^{*}(\theta,\varphi)}{\partial\theta}=0\\ \dfrac{\partial\boldsymbol{b}^{\mathrm{H}}(\theta,\varphi)}{\partial\theta}\boldsymbol{b}(\theta,\varphi)=\boldsymbol{b}^{\mathrm{H}}(\theta,\varphi)\dfrac{\partial\boldsymbol{b}(\theta,\varphi)}{\partial\theta}=0\\ \dfrac{\partial\boldsymbol{a}^{\mathrm{T}}(\theta,\varphi)}{\partial\theta}\dfrac{\partial\boldsymbol{a}^{*}(\theta,\varphi)}{\partial\theta}=\dfrac{2\pi^2R_1^2M(\sin\varphi)^2}{\lambda^2}\\ \dfrac{\partial\boldsymbol{b}^{\mathrm{H}}(\theta,\varphi)}{\partial\theta}\dfrac{\partial\boldsymbol{b}(\theta,\varphi)}{\partial\theta}=\dfrac{2\pi^2R_2^2N(\cos\varphi)^2}{\lambda^2}\end{cases}\tag{16}$$

Eq. (15) can be simplified as

$$f(\theta,\theta)=4\pi^2\eta^2LMN\nu(\sin\varphi)^2\left[\left(\frac{R_1}{\lambda}\right)^2+\left(\frac{R_2}{\lambda}\right)^2\right]\tag{17}$$

where $\nu=\frac{\mu^2}{\sigma^2}$ is the signal-to-noise ratio.

Similarly, the other elements of $\boldsymbol{F}$ can be calculated by

$$f(\varphi,\varphi)=4\pi^2\eta^2LMN\nu(\cos\varphi)^2\left[\left(\frac{R_1}{\lambda}\right)^2+\left(\frac{R_2}{\lambda}\right)^2\right]\tag{18}$$

$$f(\eta,\eta)=2LMN\nu\tag{19}$$

$$f(\sigma^2,\sigma^2)=\frac{NL}{\sigma^4}\tag{20}$$

$$f(\theta,\varphi)=f(\varphi,\theta)=f(\theta,\eta)=f(\eta,\theta)=0\tag{21}$$

$$f(\theta,\sigma^2)=f(\sigma^2,\theta)=f(\varphi,\sigma^2)=f(\sigma^2,\varphi)=0\tag{22}$$

$$f(\eta,\sigma^2)=f(\sigma^2,\eta)=f(\varphi,\eta)=f(\eta,\varphi)=0\tag{23}$$

Thus, the Fisher information matrix can be simplified as

$$\boldsymbol{F}=\begin{bmatrix}f(\theta,\theta)&0&0&0\\0&f(\varphi,\varphi)&0&0\\0&0&f(\eta,\eta)&0\\0&0&0&f(\sigma^2,\sigma^2)\end{bmatrix}\tag{24}$$

and its inverse matrix is

$$\boldsymbol{F}^{-1}=\begin{bmatrix}f^{-1}(\theta,\theta)&0&0&0\\0&f^{-1}(\varphi,\varphi)&0&0\\0&0&f^{-1}(\eta,\eta)&0\\0&0&0&f^{-1}(\sigma^2,\sigma^2)\end{bmatrix}\tag{25}$$

Finally, the CRB of azimuth angle can be obtained.

$$\boldsymbol{C}(\theta)=[\boldsymbol{F}^{-1}]_{11}=f^{-1}(\theta,\theta)\tag{26}$$

that is

$$\boldsymbol{C}(\theta)=\left\{4\pi^2\eta^2LMN\nu(\sin\varphi)^2\left[\left(\frac{R_1}{\lambda}\right)^2+\left(\frac{R_2}{\lambda}\right)^2\right]\right\}^{-1}\tag{27}$$

And the CRB of the elevation angle is

$$\boldsymbol{C}(\varphi)=[\boldsymbol{F}^{-1}]_{22}=f^{-1}(\varphi,\varphi)\tag{28}$$

Namely,

$$\boldsymbol{C}(\varphi)=\left\{4\pi^2\eta^2LMN\nu(\cos\varphi)^2\left[\left(\frac{R_1}{\lambda}\right)^2+\left(\frac{R_2}{\lambda}\right)^2\right]\right\}^{-1}\tag{29}$$

It can be inferred from Eqs. (27,29) that for the MIMO radar system with UCA, the more the number of antennas and samples is, the lower the CRB of the azimuth and elevation angle will become and the higher the estimation accuracy will be. Meanwhile, the CRB of the azimuth and elevation angle will be lower when the radius of the transmitting or receiving circular arrays and the scattering coefficient become larger. When the elevation angle is close to 0° (90°) and $\sin\varphi\rightarrow 0$ ($\cos\varphi\rightarrow 0$), the CRB of azimuth angle (elevation angle) estimation tends to infinity and the estimation accuracy of azimuth angle (elevation angle) will become poor.

3 Simulation Results

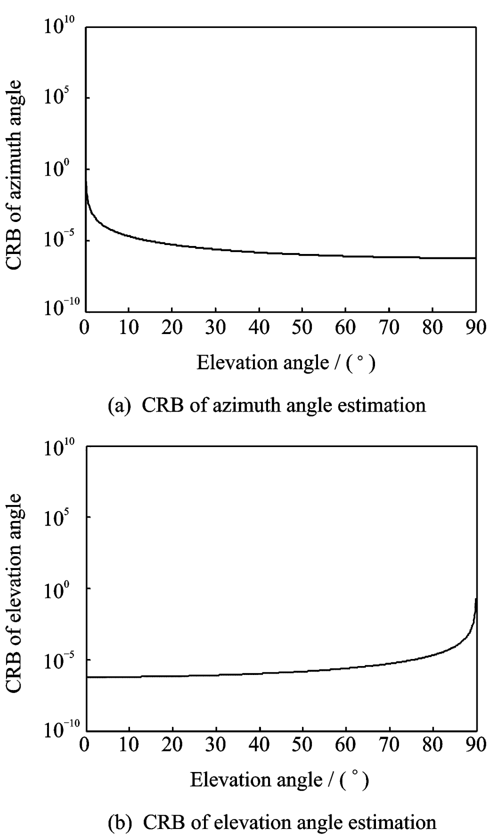
The relationship between CRBs of the 2-D

DOA estimation and the several parameters of a monostatic MIMO radar system is analyzed according to the simulation results. Assume that the signal-to-noise ratio (SNR) is $\nu=5$ dB, the number of pulses is $Q=512$, and the number of samples during one pulse is $L=256$. The scattering coefficient of the target obeys the Rayleigh distribution with the parameter to be 1. All the simulations are conducted by the Matlab software.

First, the number of the transmitting (receiving) antennas is set to be eight and the radius of the circular array be twice of the wavelength. The CRB of the 2-D DOA estimation is shown in Fig. 2. The simulation results show that the CRB of the angle estimation relates to elevation angle. Fig. 2(a) illustrates that the CRB for the azimuth angle estimation becomes lower and lower when the elevation angle increases from 0° to 90°. Contrarily, Fig. 2(b) demonstrates that the CRB of the elevation angle increases as the elevation angle changes from 0° to 90°. This is in consistent with the theoretical analysis in the previous section.

Then we test the relations between the CRB of the angle estimation and the number of the transmitting and receiving antennas. All the simulation parameters are the same as those in the above example except for the number of antennas. Set the number of antennas to be 5, 8, 10 and 15, respectively. The simulation results are shown in Fig. 3. Both Fig. 3(a) and Fig. 3(b) indicate that the CRB of the angle estimation decrease when the number of antennas increase. It means that estimation accuracy will be improved when the number of the antennas increases. However, as the number of the antennas climbs, the complexity of the system and the computational load highly increases, which should be thought about seriously by radar designers.

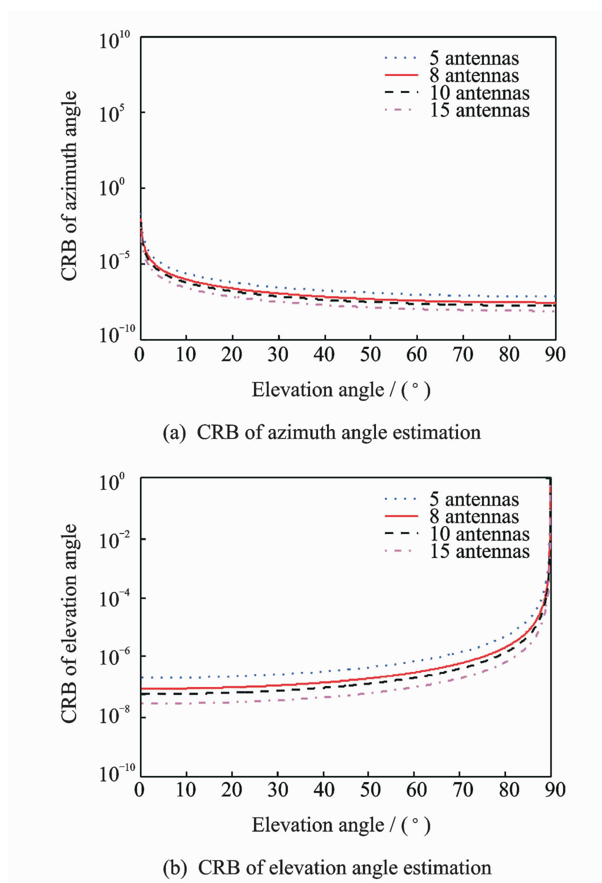
Finally, the relations between the CRB of the angle estimation and the radius of the circular array are studied by tests. All the simulation parameters are the same as in the first example ex-



(a) CRB of azimuth angle estimation

(b) CRB of elevation angle estimation

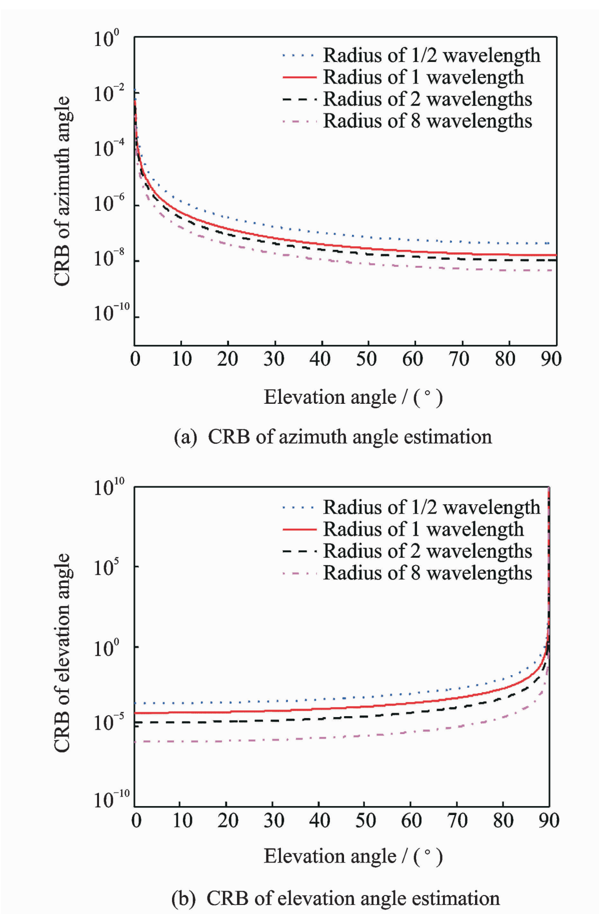
Fig. 2 Relationship between CRB of 2-D DOA estimation and elevation angle in MIMO radar with UCA



(a) CRB of azimuth angle estimation

(b) CRB of elevation angle estimation

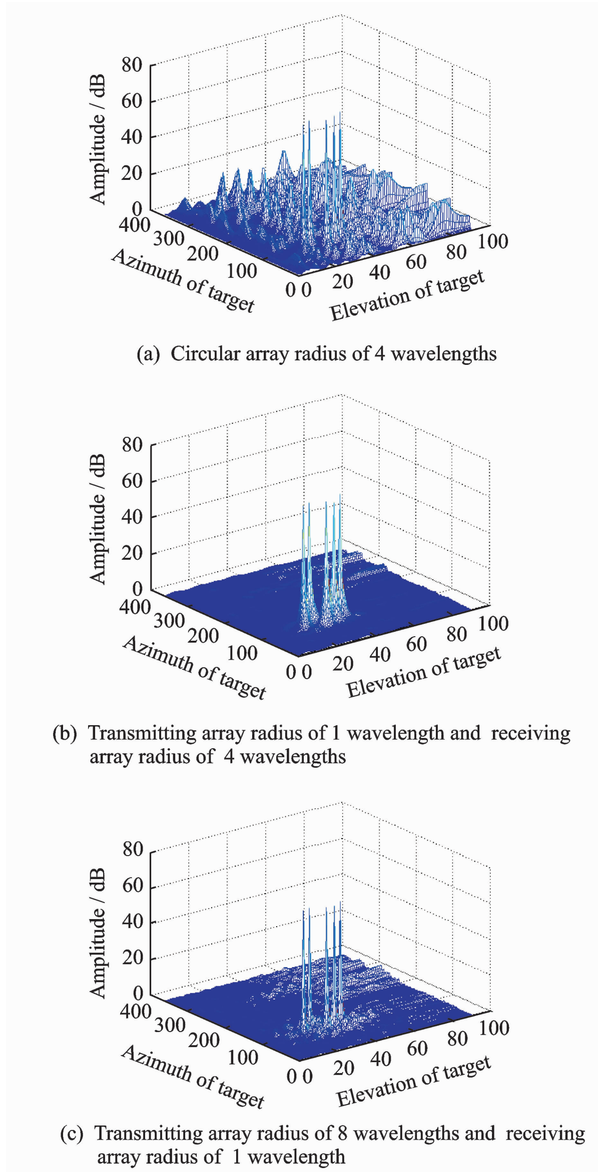
Fig. 3 Relationship between CRB of 2-D DOA estimation and UCA antenna number cept for the radius of circular array. Set the radius of the circular array to be half of wavelength, one wavelength, two wavelengths and eight wavelengths, respectively. The simulation results are shown in Fig. 4. It is obvious that both for the azimuth angle and the elevation angle, the CRB of angle estimation becomes lower with the larger radius of UCA. However, during the simulation the increasing of the radius brings in strong interference, which seriously affects the estimation accuracy. Moreover, when the radii of the transmitting array and the receiving array differ from each other, the interference problem will be restrained. The simulation results are shown in Fig. 5. The 2-D DOA estimation performance of the MIMO radar shown in Figs. 5(b, c) is much better than that of Fig. 5(a).



(a) CRB of azimuth angle estimation

(b) CRB of elevation angle estimation

Fig. 4 Relationship between CRB of 2-D DOA estimation and radius of UCA in MIMO radar



(a) Circular array radius of 4 wavelengths

(b) Transmitting array radius of 1 wavelength and receiving array radius of 4 wavelengths

(c) Transmitting array radius of 8 wavelengths and receiving array radius of 1 wavelength

Fig. 5 2-D DOA estimation results of MIMO radar with different radii of UCA

4 Conclusions

A signal model is developed for 2-D DOA estimation in MIMO radar with UCA. The MUSIC method is applied to angle estimation. The CRB for DOA estimation with UCA is derived. The simulation results show that all the changes of the elevation angle, the number of the antennas, and the radius of the circular array have impacts on the CRB of the 2-D DOA estimation. Generally, the CRB of angle estimation is lower when the MIMO radar system has more antennas in a circular array and larger radius for the circular array. Besides, the simulation results also reveal that when the number of the transmitting and receiving antennas are different from each other, the interference problem during the DOA estimation will be effectively restrained. The work has implications for MIMO radar design.

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