Driving Force Model for Non-pneumatic Elastic Wheel

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Abstract: In order to obtain good driving performance, a driving force model is presented for non-pneumatic elastic wheel. Brush model of pneumatic tyres is introduced and the deformations of elastic supports and tread are also taken into account. The longitudinal slip rate is redefined. The grounding pressure distribution of elastic wheels is analyzed and corrected according to speed, temperature and stiffness. Then rolling resistance equation is developed. Finally, simulation is conducted by software CarSim, and the results show that the estimated values are consistent with simulation values, especially at low longitudinal slip rate. The research can help to optimize design of non-pneumatic elastic wheel.

Key words: driving force; non-pneumatic elastic wheel; brush model; rolling resistance

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0 Introduction

Motions of a vehicle depend on the interaction between tyres and ground, so a reasonable tyre model is a key for vehicle dynamic simulation. Tyre model is the basis for the developments of vehicle dynamics control systems and tyre sensors. Driving force is always an important indicator of tyre mechanical properties, which directly affects vehicle dynamic properties and passing abilities. Early researchers have developed a variety of physical models, theoretical models and semi-empirical models to analyze dynamic performances of pneumatic tyres[1-5]. Gim et al.[2] proposed pure-slip tire model and combined a slip tire model of the brush theory and 3-D spring tire model. Gordon[3] elaborated the tire longitudinal force on slip rate based on brush theory. "Magic Formula", obtained through fair enough experiments by Pacejka[4] was a semi-empirical tire model. However, theoretical models hold complex forms and low computational efficiency. Semi-empirical models require many experimental data and computations with large amounts of parameters. Physical models are easier to understand and requires less parameters, so its computation cost is small.

Many scholars have focused on a non-pneumatic elastic wheels[6-9]. Jagadish[6] proposed a computational investigation of contact pressure for a non-pnemautic wheel with a metamaterial shear band. Bezgam[7] designed and analyzed the alternating spoke pair concepts for a non-pneumatic tire with reduced vibration at high speed rolling. Thyagaraja et al.[8] optimized the shear beam of a non-pneumatic wheel for low rolling resistance. Compared with pneumatic tyres, non-pneumatic elastic wheels utilize longitudinal force model of non-pneumatic elastic wheels, and longitudinal slip rate was redefined by considering the vertical and the longitudinal deformations of treads and elastic supports. The contact pressure distribution function defined by Rhyne and Cron[9] was corrected and the rolling resistance equation was established. To validate the model, a specific vehicle and condition were chosen in

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CarSim. The results showed that the theoretical model agreeed well with simulation data, especially at low longitudinal slip rates.

1 Magic Formula

The "Magic Formula", proposed by Pacejka, has been widely used in tyre simulations[4]. It is a semi-empirical formula from large volumes of measurement data of longitudinal force, lateral force and aligning torque. By entering vertical load, slip rate and roll angle, the formula can be used to calculate force and torque. Longitudinal force calculation is

$$F_x = D\sin(C\arctan(B\varphi)) \tag{1}$$

where

$$\varphi = (1-E)k + \left(\frac{E}{B}\right)\arctan(Bk)$$

$$D = a_1 F_z^2 + a_2 F_z$$

$$C = 1.65$$

$$B = \frac{a_3 F_z^2 + a_4 F_z}{CD\mathrm{e}^{a_5 F_z}}$$

$$E = a_6 F_z^2 + a_7 F_z + a_8$$

k is the slip rate and F_z the vertical load. $\varphi, D, C, B, E, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$ are some specific parameters. The semi-empirical model needs large amounts of data, but the characteristic data of non-pneumatic elastic wheel are incomplete. Therefore, a relatively simple physical model, namely brush model, was selected to solve the problem.

2 Driving Force Model

2.1 Brush model of longitudinal force

We chose Tweel by Michelin (as shown in Fig. 1), a typical non-pneumatic elastic wheel[10], composed of tread, elastic support and rigid wheel hub.

Fig. 1 Tweel structure

Elastic support is for damping, similar to inner tube of pneumatic wheel. Tweel is simplified according to brush model[11], as shown in Figs. 2, 3. At the loaded state, the tread and elastic support are deformed longitudinally and laterally under ground forces (See Fig. 2). The involved parametres include rigid hub angular velocity w, wheel longitudinal sliding speed v_{lx}, longitudinal force f_x, lateral force f_y, wheel hub diameter r, effective thickness of elastic support and tread with the vertical load h.

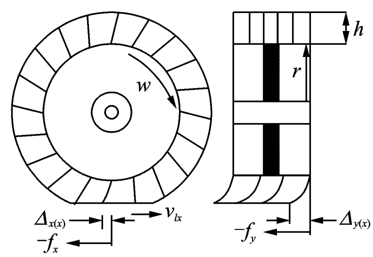

Fig. 2 Deformation of wheel

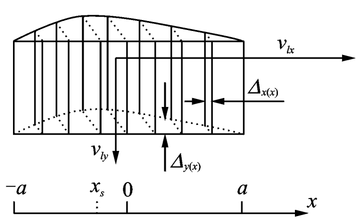

Fig. 3 Deformation situation of wheel grounding based on brush model

The wheel deformation in contact with ground is shown in Fig. 3. The longitudinal length of contact area is $2a$, and the breakpoint coordinate of slip area and adhesion area is x_s. As the role of longitudinal and lateral forces, in longitudinal direction, bristles deformations are $\Delta_{x(x)}$ and $\Delta_{y(x)}$. Then longitudinal force f_x and lateral force f_y are solved.

Bristles deformation analysis in adhesion area is

$$\begin{cases} \Delta_{x(x),st} = \dfrac{a-x}{v_{lx}}\left(w - \dfrac{v_{lx}}{r+h}\right)h \\ \Delta_{y(x),st} = \dfrac{a-x}{v_{lx}} v_{ly} \end{cases} \tag{2}$$

where $\Delta_{x(x),st}$ is the bristles longitudinal deformation in adhesion area, $\Delta_{y(x),st}$ the bristles lateral deformation in adhesion area, $\frac{a-x}{v_{lx}}=\Delta t$ the time from bristles entering contact area to the point x, $\left(w-\frac{v_{lx}}{r+h}\right)h = \Delta v_{lx}$ the relatively longitudinal speed of birstles top and bottom. Assume wheel longitudinal slip rate is

$$s_x = \frac{\left(w-\frac{v_{lx}}{r+h}\right)h}{v_{lx}} = \frac{\Delta v_{lx}}{v_{lx}} \tag{3}$$

and $\tan\alpha = \frac{v_{ly}}{\Delta v_{lx}}$, therefore

$$\begin{cases}\Delta_{x(x),st} = (a-x)s_x \\ \Delta_{y(x),st} = (a-x)s_x\tan\alpha\end{cases} \tag{4}$$

Longitudinal stress and lateral stress are

$$\begin{cases}f_{x(x),st} = k_x(a-x)s_x \\ f_{y(x),st} = k_y(a-x)s_x\tan\alpha\end{cases} \tag{5}$$

where $f_{x(x),st}$ is the longitudinal stress in adhesion area, $f_{y(x),st}$ the lateral stress in adhesion area, k_x the longitudinal stiffness, and k_y the lateral stiffness. Similarly, the deformation in slip area is

$$\begin{cases}\Delta_{x(x),sl} = \frac{\mu q(x)\cos\theta}{k_x} \\ \Delta_{y(x),sl} = \frac{\mu q(x)\sin\theta}{k_y}\end{cases} \tag{6}$$

Longitudinal stress and lateral stress are

$$\begin{cases}f_{x(x),sl} = \mu q(x)\cos\theta \\ f_{y(x),sl} = \mu q(x)\sin\theta\end{cases} \tag{7}$$

where $\Delta_{x(x),sl}$ is the bristles longitudinal deformation in slip area, $\Delta_{x(x),sl}$ the bristles lateral deformation in slip area, $f_{x(x),sl}$ the longitudinal stress in adhesion area, $f_{y(x),sl}$ the lateral stress in adhesion area, θ the angle between wheel resultant force in contact with ground and the axis x, and $\tan\theta = \frac{k_y\tan\alpha}{k_x}$, μ the ground friction coefficient, and $q(x)$ the pressure distribution function in contact area.

Suppose that in the cut-off point x_s of slip area and adhesion area, absolute value of the stress vector in Eqs. (5), (7) is equal.

$$\sqrt{(f_{x(x),st})^2 + (f_{y(x),st})^2} = \sqrt{(f_{x(x),sl})^2 + (f_{y(x),sl})^2} \tag{8}$$

Therefore

$$(a-x)\sqrt{k_x^2 s_x^2 + k_y^2 s_x^2\tan^2\alpha} = \mu\frac{3F_z}{4a}\left[1-\left(\frac{x}{a}\right)^2\right] \tag{9}$$

Therefore, the longitudinal force F_x can be solved as follows by integration within the entire contact area with ground.

$$F_x = b\int_{-a}^{x_s} f_{x(x),sl}\,dx + b\int_{x_s}^{a} f_{x(x),st}\,dx = \frac{b\mu F_z\cos\theta}{4a}\left(3x_s - \frac{x_s^3}{a^2} + 2a\right) + bs_xk_x\left(\frac{a^2}{2} - ax_s + \frac{x_s^2}{2}\right) \tag{10}$$

where b is the wheel width and F_z the wheel vertical load.

2.2 Rolling resistance analysis

2.2.1 Grounding pressure distribution of non-pneumatic elastic wheel

According to the elasticity theory, in Fig. 4, the shear deformation occurs under static load. The shear strain λ_{xz} can be given as

$$\lambda_{xz} = \arctan\left(\frac{(r+h)\theta - r\theta}{h}\right) = \arctan\left(\frac{h\theta}{h}\right) = \arctan\theta \tag{11}$$

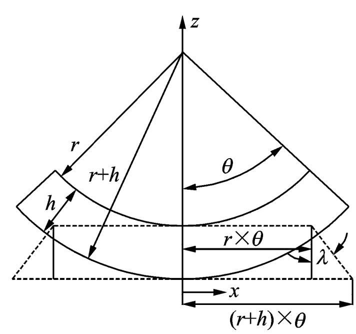

Fig. 4 Length of a shear beam deformed to a flat surface

Using $x = R\theta$ and $R = r + h$ in Fig. 4, the shear strain becomes

$$\lambda_{xz} = \arctan\left(\frac{x}{R}\right) \tag{12}$$

Assuming the composite shear modulus is G, the shear stress can be shown as

$$\sigma_{xz} = G\lambda_{xz} = G\arctan\left(\frac{x}{R}\right) \tag{13}$$

Consider the vertical stress distribution of an infinitesimal length of the wheel outer ring, as shown in Fig. 5, and assume that the contact pressure is $p(x)$ and the stress distribution coordinate system is the same as that in Fig. 4. Then summing the forces in the z direction yields

$$\left(\sigma_{xz}+\frac{\partial\sigma_{xz}}{\partial x}\mathrm{d}x\right)h-\sigma_{xz}h-p(x)\mathrm{d}x=0 \quad (14)$$

Solving Eq. (14)

$$p(x)=\frac{\partial\sigma_{xz}}{\partial x}h \quad (15)$$

As the shear stress from Eq. (13), Eq. (15) becomes

$$p(x)=G\frac{1}{R\left(1+\dfrac{x^2}{R^2}\right)}h=\frac{GhR}{R^2+x^2} \quad (16)$$

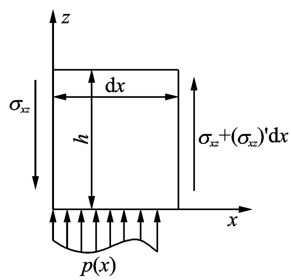

Fig. 5 Vertical stress distribution of an infinitesimal length of the wheel outer ring

Obviously, this pressure distribution does not lead to rolling resistance, because in the contact area, contact pressures are symmetric. Therefore, we need to amend Eq. (16) and then establish the following equation

$$p(x)=\frac{GhR}{R^2+x^2}\left(1+k_p\frac{x}{a}\right) \quad (17)$$

where k_p is the pressure distribution coefficient, related to the wheel speed, temperature, longitudinal stiffness and ground stiffness.

2.2.2 Rolling resistance of non-pneumatic elastic wheel

According to Eq. (17), rolling resistance function F_r can be obtained by $p(x)$ integration in the contact patch.

$$F_r=\frac{\int_{-a}^{a}p(x)x\mathrm{d}x}{R}=\frac{\int_{-a}^{a}\dfrac{GhR}{R^2+x^2}\left(1+k_p\dfrac{x}{a}\right)x\mathrm{d}x}{R}=\frac{2Ghk_P}{a}\left[a-R\arctan\left(\frac{a}{R}\right)\right] \quad (18)$$

where a is the longitudinal length of wheel contact patch.

2.3 Driving force model of non-pneumatic elastic wheel

From Eqs. (10),(18), we can obtain

$$F_{\mathrm{dri}}=F_x+F_r=\frac{b\mu F_z\cos\theta}{4a}\left(3x_s-\frac{x_s^3}{a^2}+2a\right)+bs_xk_x\left(\frac{a^2}{2}-ax_s+\frac{x_s^2}{2}\right)-\frac{2Ghk_P}{a}\left[a-R\arctan\left(\frac{a}{R}\right)\right] \quad (19)$$

where F_{dri} is the wheel driving force and F_x the wheel longitudinal force, as shown in Fig. 6.

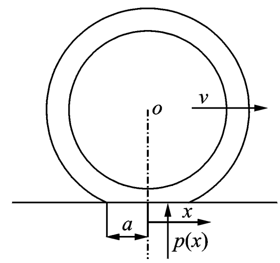

Fig. 6 Rolling resistance

3 Experiment Verification by CarSim

CarSim is designed for vehicle dynamics simulation, and CarSim models run 3—6 times faster than the real-time. It can simulate responses of the vehicle to the driver, road conditions and aerodynamics inputs. And it is usually used to predict and simulate handling stability, braking performance, riding comfort, dynamic property and economy, as well as in the development of modern automotive control systems. CarSim can define the test environment and procedures conveniently, and give detailed definition of the vehicle characteristic parameters and properties.

The vehicle "E-Class, SUV: High CG", and "Full Throttle Acceleration (6 spd.)" were chosen, and the specific parameters are lised in Table 1, where $c_{z,\mathrm{tre}}$ and $c_{z,\mathrm{sup}}$ are the tread vertical stiffness and the elastic support vertical stiffness, respectively, c_z is the wheel vertical stiffness, and their relationship is

Table 1 Parameter in carsim

No.	Subject	Value
1	$c_{z,\mathrm{tre}}/(\mathrm{N\cdot mm^{-1}})$	300
2	$c_{z,\mathrm{sup}}/(\mathrm{N\cdot mm^{-1}})$	300
3	$c_z/(\mathrm{N\cdot mm^{-1}})$	150
4	R/mm	385
5	h/mm	135
6	r/mm	250
7	b/mm	255

$$\frac{1}{c_z}=\frac{1}{c_{z,\mathrm{tre}}}+\frac{1}{c_{z,\mathrm{sup}}} \tag{20}$$

Simulation results of the rolling resistance F_r, longitudinal force F_x, vertical force F_z and vehicle velocity are shown in Fig. 7—10. Simulation and theoretical analysis results of driving force F_{dri} with slip rate ranging from 0 to 0.6 are shown in Fig. 11. When slip rate ranged from 0 to 0.27, the error between them is small(less than 4%). When slip rate ranged from 0.27 to 0.6, the error increases. This is because in large longitudinal slip rates, the elastic support longitudinal deformation reaches the limit, and the tread is squeezed into the rigid wheel hub, and the longitudinal stiffness and vertical stiffness of the wheel change dramatically in the geometric nonlinearity.

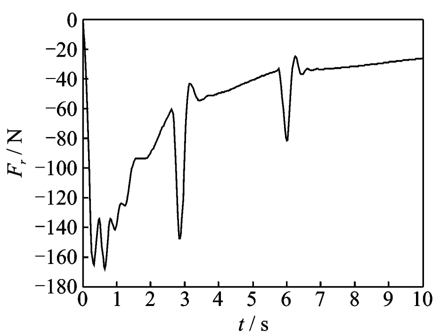

Fig. 7 Rolling resistance

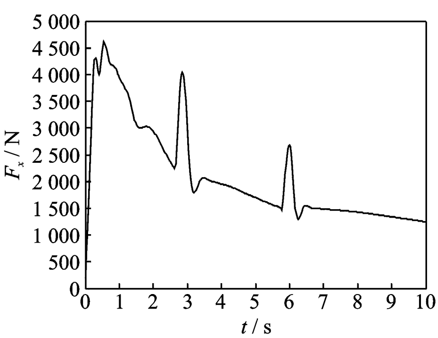

Fig. 8 Longitudinal force

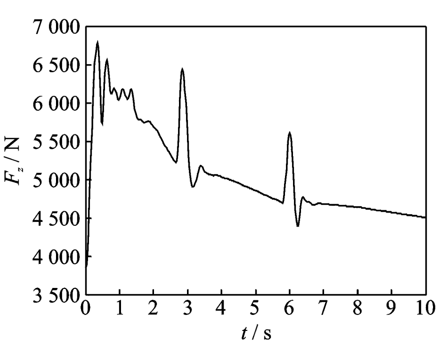

Fig. 9 Vertical force

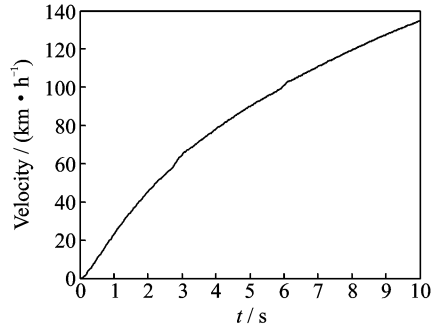

Fig. 10 Vehicle velocity

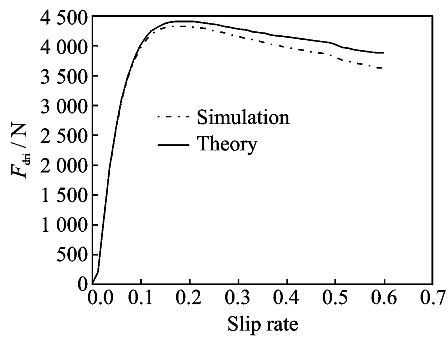

Fig. 11 Theoretical and simulation model at different slip rates

4 Conclusions

(1) Brush model is introduced into the driving force analysis of the non-pneumatic elastic wheel. Ground-wheel interaction model is founded based on the structural characteristics of non-pneumatic elastic wheel. In the modeling process, with a full consideration of elastic wheel deformation, the effective thickness of elastic support and tread is involved for slip rate as a variable.

(2) According to the pressure distribution characteristics of the non-pneumatic elastic wheel under vertical loads, considering external conditions, including temperature, stiffness and speed, the pressure distribution coefficient is set and the wheel longitudinal rolling resistance function is established.

(3) Specific parameters of the non-pneumatic elastic wheel are discovered for simulating and validating the theoretical model by CarSim. The results shows a good fit, especially at low longitudinal slip rate.

Acknowledgements

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