Dynamic Modeling and Adaptive Fast Nonsingular Terminal Sliding Mode Control for Satellite with Double Rotary Payloads

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Abstract: A robust attitude control methodology is proposed for satellite system with double rotary payloads. The dynamic model is built by the Newton-Euler method and then the dynamic interconnection between satellite′s main body and payloads is described precisely. A nonlinear disturbance observer is designed for satellite′s main body to estimate disturbance torque acted by motion of payloads. Meanwhile, the adaptive fast nonsingular terminal sliding-mode attitude stabilization controller is proposed for satellite′s main body to quicken convergence speed of state variables. Similarly, the adaptive fast nonsingular terminal sliding-mode attitude maneuver controller is designed for each payload to weaken the disturbance effect of motion of satellite′s main body. Simulation results verify the effectiveness of the proposed method.

Key words: satellite with double rotary payloads; multi-rigid-body system; sliding mode control; disturbance observer; attitude control

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0 Introduction

Satellite often carries various of rotary payloads such as antenna or mobile crane to meet the application demand, and attitude control of this system has gained extensive interests in recent years, motivated by benefits in space application[1-6]. The motions of satellite′s main body and payloads act as disturbance to each other, thus increasing the difficulty of attitude control for both satellite′s main body and payloads. Therefore, dynamical coupling becomes a major problem of designing the attitude controller of such a multi-rigid-body satellite system, which requires a strong robustness. In Ref. [7], an active disturbance rejection controller (ADRC) is proposed to compensate the disturbances induced by solar array driving. In Ref. [8] the motor driving torque of antenna was calculated from a dynamical model and compensated by a satellite attitude controller based on the decoupling mechanism. In Ref. [9], residual disturbing torque of the rotary payload was calculated from a dynamic balance test. In Ref. [10] the robust gain-scheduled control was applied for spacecraft with mobile appendages, and the coupling torque was reckoned based on a dynamic model. However, the coupling torque is hard to acquire accurately in practice because some parameters such as motor current is difficult to measure. Besides, it is also hard to obtain coupling torque precisely based on calculation from dynamical model because of model uncertainty, or the state variables, such as attitude angle and angular velocity of payload due to electrical failure. When double rotary payloads work together, the coupling torque between satellite′s main body and payloads should be determined as possible as we can for high precision attitude control. Based on the consideration, the nonlinear disturbance ob-

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server is designed for satellite's main body to estimate disturbance torque exerted by payloads' motion in this work.

Sliding-mode control (SMC) has been widely used in many applications because of its strong robustness. Robotics, spacecraft control are some typical applications[11-12]. One of SMC is the terminal SMC[13], which can converge the control system in finite time. However, the singular problem limits the application in practice. Then, nonsingular terminal SMC[14] is proposed, which is able to avoid the singular problem completely. To achieve fast finite-time convergence, fast nonsingular terminal SMC[15-16] has been suggested recently. In Ref. [15], an adaptive nonsingular fast terminal SMC was proposed, containing an adaptive control term and a robust control term for electromechanical actuator. Ref. [16] investigated a nonsingular fast terminal SMC for a class of nonlinear dynamical systems. Therefore, we propose an adaptive fast nonsingular terminal SMC for satellite with double rotary payloads working simultaneously in this paper.

1 Dynamic Model of Satellite with Double Rotary Payloads

Dynamic model of satellite with double rotary payloads (Fig. 1) is established by the Newton-Euler method in this section. The dynamic interconnection between satellite's main body and the payloads is described accurately.

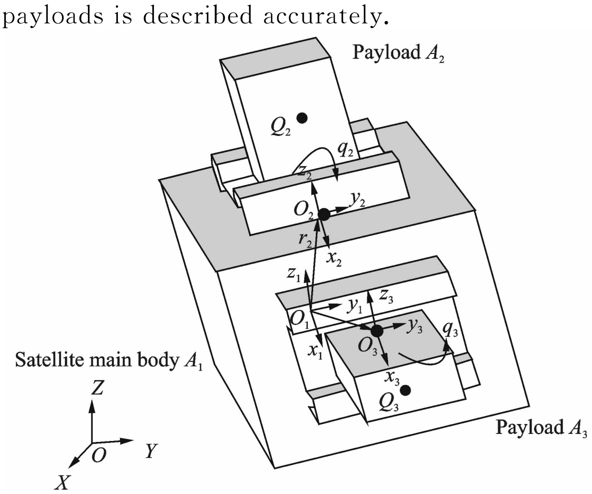

Fig. 1 Satellite with double rotary payloads

For satellite's main body A_1 shown in Fig. 1, the kinematic equation is described in the coordinate $O_1x_1y_1z_1$ as follows

$$\dot{\boldsymbol{q}}_1 = \begin{bmatrix} \cos\psi_1/\cos\theta_1 & -\sin\psi_1/\cos\theta_1 & 0 \\ \sin\psi_1 & \cos\psi_1 & 0 \\ -\cos\psi_1 \cdot \tan\theta_1 & \sin\psi_1 \cdot \tan\theta_1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \omega_{1x} \\ \omega_{1y} \\ \omega_{1z} \end{bmatrix} = \boldsymbol{H}_1 \cdot \boldsymbol{\omega}_1 \tag{1}$$

where $O_1x_1y_1z_1$ is the body-fixed coordinate of A_1, whose origin is at the mass center of A_1 and axes are paralleled to the principal axes of inertia. $\boldsymbol{q}_1 = [\phi_1 \quad \theta_1 \quad \psi_1]^{\mathrm{T}}$ is the Euler attitude angle and $\boldsymbol{\omega}_1 = [\omega_{1x} \quad \omega_{1y} \quad \omega_{1z}]^{\mathrm{T}}$ the angular velocity.

The dynamic equations of A_1 described in the coordinate $O_1x_1y_1z_1$ are

$$m_1\boldsymbol{a}_1 = \boldsymbol{F}_1 \tag{2}$$

$$\boldsymbol{F}_1 = -\boldsymbol{A}_{12}\boldsymbol{F}_2 - \boldsymbol{A}_{13}\boldsymbol{F}_3 \tag{3}$$

$$\boldsymbol{J}_1\dot{\boldsymbol{\omega}}_1 + \boldsymbol{\omega}_1 \times \boldsymbol{J}_1\boldsymbol{\omega}_1 = \boldsymbol{T}_1 - \boldsymbol{A}_{12}\boldsymbol{T}_2 - \boldsymbol{A}_{13}\boldsymbol{T}_3 - \boldsymbol{r}_2^{\times}\boldsymbol{A}_{12}\boldsymbol{F}_2 - \boldsymbol{r}_3^{\times}\boldsymbol{A}_{13}\boldsymbol{F}_3 \tag{4}$$

where $\boldsymbol{a}_1$ is the mass center acceleration of A_1, $\boldsymbol{F}_1$ the hinged constrained force applied to A_1, and $\boldsymbol{T}_1 \in \mathbf{R}^{3\times1}$ the control torque. $\boldsymbol{T}_i$ and $\boldsymbol{F}_i\,(i=2,3)$ are the control torque and hinged constrained force defined in the coordinate $O_ix_iy_iz_i\,(i=2,3)$ and exerted by the payloads A_2 and A_3. Here, $O_ix_iy_iz_i$ are the body-fixed coordinates of A_2, A_3, whose origins are at hinged points O_2 and O_3, respectively and axes are paralleled to the principal axes of inertia of A_2 and A_3. $\boldsymbol{A}_{1i}\,(i=2,\ 3)$ are the coordinate transformation matrices from $O_ix_iy_iz_i$ to $O_1x_1y_1z_1$. $\boldsymbol{J}_1 = \mathrm{diag}(J_{1x},\ J_{1y},\ J_{1z})$ is the inertia matrix of A_1, $\boldsymbol{r}_1 = \overrightarrow{O_2Q_2}$, $\boldsymbol{r}_2 = \overrightarrow{O_1O_2}$, $\boldsymbol{r}_3 = \overrightarrow{O_1O_3}$, $\boldsymbol{r}_4 = \overrightarrow{O_3Q_3}$, Q_2 and Q_3 are the mass centers of A_2 and A_3. $(\cdot)^{\times}$ is the skew matrix defined as

$$\boldsymbol{a}^{\times} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

For the appendage A_2, the kinematic equations are described in the coordinate $O_2x_2y_2z_2$.

$$\boldsymbol{\omega}_2 = \boldsymbol{A}_{21} \cdot \boldsymbol{\omega}_1 + \boldsymbol{\omega}_{12} \tag{5}$$

$$\dot{\boldsymbol{\omega}}_2 = \boldsymbol{A}_{21} \cdot \dot{\boldsymbol{\omega}}_1 + \mathring{\boldsymbol{\omega}}_{12} + \boldsymbol{A}_{21} \cdot \boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_{12} \tag{6}$$

$$\boldsymbol{v}_2 = \boldsymbol{A}_{21} \cdot (\boldsymbol{v}_1 + \boldsymbol{\omega}_1 \times \boldsymbol{r}_2) + \boldsymbol{\omega}_{12} \times \boldsymbol{r}_1 \tag{7}$$

where $\boldsymbol{\omega}_2=[\omega_{2x} \quad \omega_{2y} \quad \omega_{2z}]^{\mathrm{T}}$ is the angular velocity of A_2, $\boldsymbol{\omega}_{12}=[\dot{q}_2 \quad 0 \quad 0]^{\mathrm{T}}$ the angular velocity of A_2 relative to A_1, $\mathring{\boldsymbol{\omega}}_{12}=[\ddot{q}_2 \quad 0 \quad 0]^{\mathrm{T}}$ the angular acceleration of A_2 relative to A_1, $\boldsymbol{A}_{21}$ the coordinate transformation matrix from $O_1x_1y_1z_1$ to $O_2x_2y_2z_2$, $\boldsymbol{v}_1$ the mass center velocity of A_1 designed in $O_1x_1y_1z_1$, and $\boldsymbol{v}_2$ the mass center velocity of A_2 designed in $O_2x_2y_2z_2$.

Furthermore, the payload A_2 just rotates around the axis O_2x_2, which is a motion with one degree of freedom. We can obtain a simpler kinematic equation based on Eq. (5).

$$\omega_{2x}=\omega_{1x}+\dot{q}_2 \tag{8}$$

Let $\boldsymbol{v}_2^*=\boldsymbol{A}_{21}\cdot(\boldsymbol{v}_1+\boldsymbol{\omega}_1\times\boldsymbol{r}_2)$. The dynamic equations of the payload A_2 described in the coordinate $O_2x_2y_2z_2$ are

$$m_2\boldsymbol{a}_2=\boldsymbol{F}_2 \tag{9}$$

$$\boldsymbol{a}_2=\boldsymbol{A}_{21}(\boldsymbol{a}_1+\dot{\boldsymbol{\omega}}_1^{\times}\boldsymbol{r}_2+\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_1^{\times}\boldsymbol{r}_2)+\dot{\boldsymbol{\omega}}_2^{\times}\boldsymbol{r}_1+\boldsymbol{\omega}_2^{\times}\boldsymbol{\omega}_2^{\times}\boldsymbol{r}_1 \tag{10}$$

$$\begin{aligned}&\boldsymbol{J}_2\dot{\boldsymbol{\omega}}_2+\boldsymbol{\omega}_2\times\boldsymbol{J}_2\boldsymbol{\omega}_2+m_2\boldsymbol{v}_2^*\times(\boldsymbol{v}_2^*+\boldsymbol{\omega}_2\times\boldsymbol{r}_1)=\\&\quad\boldsymbol{J}_2\dot{\boldsymbol{\omega}}_2+\boldsymbol{\omega}_2\times\boldsymbol{J}_2\boldsymbol{\omega}_2+m_2\boldsymbol{v}_2^*\times(\boldsymbol{\omega}_2\times\boldsymbol{r}_1)=\boldsymbol{T}_2\end{aligned} \tag{11}$$

where $\boldsymbol{J}_2=\mathrm{diag}(J_{2x},J_{2y},J_{2z})$ is the inertia matrix of A_2 relative to O_2, and $\boldsymbol{T}_2=[T_{2x} \quad T_{2y} \quad T_{2z}]^{\mathrm{T}}$ the control torque.

From Eqs. (6), (11), we have

$$\begin{aligned}&\boldsymbol{J}_2(\boldsymbol{A}_{21}\cdot\dot{\boldsymbol{\omega}}_1+\mathring{\boldsymbol{\omega}}_{12}+\boldsymbol{A}_{21}\cdot\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_{12})+\boldsymbol{\omega}_2\times\boldsymbol{J}_2\boldsymbol{\omega}_2+\\&\quad m_2\boldsymbol{v}_2^*\times(\boldsymbol{\omega}_2\times\boldsymbol{r}_1)=\boldsymbol{T}_2\end{aligned} \tag{12}$$

Likewise, the motion of the payload A_2 just has a single degree of freedom. We can thus also deduce the simpler dynamic equation based on Eq. (11).

$$\begin{aligned}&J_{2x}\dot{\omega}_{2x}+\omega_{2y}\omega_{2z}(J_{2z}-J_{2y})+[m_2\boldsymbol{v}_2^*\times(\boldsymbol{\omega}_2\times\boldsymbol{r}_1)]_x=\\&\quad T_{2x}\end{aligned} \tag{13}$$

where $[m_2\boldsymbol{v}_2^*\times(\boldsymbol{\omega}_2\times\boldsymbol{r}_1)]_x$ is the first line of $m_2\boldsymbol{v}_2^*\times(\boldsymbol{\omega}_2\times\boldsymbol{r}_1)$. From Eqs. (6), (13), we obtain

$$\begin{aligned}&\ddot{q}_2=J_{2x}^{-1}T_{2x}-J_{2x}^{-1}\omega_{2y}\omega_{2z}(J_{2z}-J_{2y})-[m_2\boldsymbol{v}_2^*\times\\&\quad(\boldsymbol{\omega}_2\times\boldsymbol{r}_1)+\boldsymbol{A}_{21}\cdot\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_{12}]_x-\dot{\omega}_{1x}\end{aligned}$$

For appendage A_3, the kinematic equations are described in the coordinate $O_3x_3y_3z_3$ as follow

$$\boldsymbol{\omega}_3=\boldsymbol{A}_{31}\cdot\boldsymbol{\omega}_1+\boldsymbol{\omega}_{13} \tag{14}$$

$$\dot{\boldsymbol{\omega}}_3=\boldsymbol{A}_{31}\cdot\dot{\boldsymbol{\omega}}_1+\mathring{\boldsymbol{\omega}}_{13}+\boldsymbol{A}_{31}\cdot\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_{13} \tag{15}$$

$$\boldsymbol{v}_3=\boldsymbol{A}_{31}\cdot(\boldsymbol{v}_1+\boldsymbol{\omega}_1\times\boldsymbol{r}_3)+\boldsymbol{\omega}_{13}\times\boldsymbol{r}_4 \tag{16}$$

where $\boldsymbol{\omega}_3=[\omega_{3x} \quad \omega_{3y} \quad \omega_{3z}]^{\mathrm{T}}$ is the angular velocity of A_3, $\boldsymbol{\omega}_{13}=[0 \quad 0 \quad \dot{q}_3]^{\mathrm{T}}$ the angular velocity of A_3 relative to A_1, $\mathring{\boldsymbol{\omega}}_{13}=[0 \quad 0 \quad \ddot{q}_3]^{\mathrm{T}}$ the angular acceleration of A_3 relative to A_1, $\boldsymbol{A}_{31}$ the coordinate transformation matrix from $O_1x_1y_1z_1$ to $O_3x_3y_3z_3$, and v_3 the mass center velocity of A_3 designed in $O_3x_3y_3z_3$.

Furthermore, the payload A_3 just rotates around the axis O_3z_3 to perform one degree of freedom motion. We can obtain the simpler kinematic equation based on Eq. (14).

$$\omega_{3z}=\omega_{1z}+\dot{q}_3 \tag{17}$$

Let $\boldsymbol{v}_3^*=\boldsymbol{A}_{31}\cdot(\boldsymbol{v}_1+\boldsymbol{\omega}_1\times\boldsymbol{r}_3)$. The dynamic equations of the payload A_2 described in the coordinate $O_2x_2y_2z_2$ are

$$m_3\boldsymbol{a}_3=\boldsymbol{F}_3 \tag{18}$$

$$\boldsymbol{a}_3=\boldsymbol{A}_{31}(\boldsymbol{a}_1+\dot{\boldsymbol{\omega}}_1^{\times}\boldsymbol{r}_3+\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_1^{\times}\boldsymbol{r}_3)+\dot{\boldsymbol{\omega}}_3^{\times}\boldsymbol{r}_4+\boldsymbol{\omega}_3^{\times}\boldsymbol{\omega}_3^{\times}\boldsymbol{r}_4 \tag{19}$$

$$\boldsymbol{J}_3\dot{\boldsymbol{\omega}}_3+\boldsymbol{\omega}_3\times\boldsymbol{J}_3\boldsymbol{\omega}_3+m_3\boldsymbol{v}_3^*\times(\boldsymbol{\omega}_3\times\boldsymbol{r}_4)=\boldsymbol{T}_3 \tag{20}$$

where $\boldsymbol{J}_3=\mathrm{diag}(J_{3x},J_{3y},J_{3z})$ is the inertia matrix of A_3 and $\boldsymbol{T}_3$ the control torque.

From Eqs. (15), (20), we get

$$\begin{aligned}&\boldsymbol{J}_3(\boldsymbol{A}_{31}\cdot\dot{\boldsymbol{\omega}}_1+\mathring{\boldsymbol{\omega}}_{13}+\boldsymbol{A}_{31}\cdot\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_{13})+\boldsymbol{\omega}_3\times\boldsymbol{J}_3\boldsymbol{\omega}_3+\\&\quad m_3\boldsymbol{v}_3^*\times(\boldsymbol{\omega}_3\times\boldsymbol{r}_4)=\boldsymbol{T}_3\end{aligned} \tag{21}$$

Likewise, the payload A_3 just moves with a single DOF, so we can also gain the simpler dynamic equation based on Eq. (20)

$$\begin{aligned}&J_{3z}\dot{\omega}_{3z}+\omega_{3x}\omega_{3y}(J_{3y}-J_{3x})+[m_3\boldsymbol{v}_3^*\times(\boldsymbol{\omega}_3\times\boldsymbol{r}_4)]_z=\\&\quad T_{3z}\end{aligned} \tag{22}$$

where $[m_3\boldsymbol{v}_3^*\times(\boldsymbol{\omega}_3\times\boldsymbol{r}_4)]_z$ is the third line of $m_3\boldsymbol{v}_3^*\times(\boldsymbol{\omega}_3\times\boldsymbol{r}_4)$. From Eqs. (15), (22), we obtain

$$\begin{aligned}&\ddot{q}_3=J_{3z}^{-1}T_{3z}-J_{3z}^{-1}\omega_{3x}\omega_{3y}(J_{3y}-J_{3x})-[m_3\boldsymbol{v}_3^*\times\\&\quad(\boldsymbol{\omega}_3\times\boldsymbol{r}_4)+\boldsymbol{A}_{31}\cdot\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_{13}]_z-\dot{\omega}_{1z}\end{aligned} \tag{23}$$

Substituting Eqs. (3), (9), (10), (18), (19) into Eq. (2), it yields

$$\begin{aligned}m_1\boldsymbol{a}_1=&-\boldsymbol{A}_{12}m_2[\boldsymbol{A}_{21}(\boldsymbol{a}_1+\dot{\boldsymbol{\omega}}_1^{\times}\boldsymbol{r}_2+\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_1^{\times}\boldsymbol{r}_2)+\\&\dot{\boldsymbol{\omega}}_2^{\times}\boldsymbol{r}_1+\boldsymbol{\omega}_2^{\times}\boldsymbol{\omega}_2^{\times}\boldsymbol{r}_1]-\boldsymbol{A}_{13}m_3[\boldsymbol{A}_{31}(\boldsymbol{a}_1+\dot{\boldsymbol{\omega}}_1^{\times}\boldsymbol{r}_3+\\&\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_1^{\times}\boldsymbol{r}_3)+\dot{\boldsymbol{\omega}}_3^{\times}\boldsymbol{r}_4+\boldsymbol{\omega}_3^{\times}\boldsymbol{\omega}_3^{\times}\boldsymbol{r}_4]\end{aligned} \tag{24}$$

Substituting Eqs. (6), (15) into Eq. (24), we get

$$\begin{aligned}\boldsymbol{a}_1=&(m_1+m_2+m_3)^{-1}(m_2\boldsymbol{r}_1^{\times}+m_2\boldsymbol{r}_2^{\times}+m_3\boldsymbol{r}_3^{\times}+\\&m_3\boldsymbol{r}_4^{\times})\dot{\omega}_1+\boldsymbol{A}_{12}(m_1+m_2+m_3)^{-1}m_2\boldsymbol{r}_1^{\times}\mathring{\boldsymbol{\omega}}_{12}+\\&\boldsymbol{A}_{13}(m_1+m_2+m_3)^{-1}m_3\cdot\boldsymbol{r}_4^{\times}\mathring{\boldsymbol{\omega}}_{13}+\boldsymbol{f}_{a1}\end{aligned} \tag{25}$$

where

$$\boldsymbol{f}_{a1}=-\boldsymbol{A}_{12}(m_1+m_2+m_3)^{-1}m_2(\boldsymbol{A}_{21}\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_1^{\times}\boldsymbol{r}_2-$$

$$\boldsymbol{r}_1^{\times}\boldsymbol{A}_{21}\cdot\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_{12}+\boldsymbol{\omega}_2^{\times}\boldsymbol{\omega}_2^{\times}\boldsymbol{r}_1)-\boldsymbol{A}_{13}(m_1+m_2+m_3)^{-1}m_3\cdot(\boldsymbol{A}_{31}\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_1^{\times}\boldsymbol{r}_3-\boldsymbol{r}_4^{\times}\boldsymbol{A}_{31}\cdot\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_{13}+\boldsymbol{\omega}_3^{\times}\boldsymbol{\omega}_3^{\times}\boldsymbol{r}_4)$$

Substituting Eqs. (6), (25) into Eq. (10), we obtain

$$\boldsymbol{a}_2=\boldsymbol{A}_{21}[(m_1+m_2+m_3)^{-1}(m_2\boldsymbol{r}_1^{\times}+m_2\boldsymbol{r}_2^{\times}+m_3\boldsymbol{r}_3^{\times}+m_3\boldsymbol{r}_4^{\times})\dot{\boldsymbol{\omega}}_1+\boldsymbol{A}_{12}(m_1+m_2+m_3)^{-1}m_2\boldsymbol{r}_1^{\times}\mathring{\boldsymbol{\omega}}_{12}+\boldsymbol{A}_{13}(m_1+m_2+m_3)^{-1}m_3\cdot\boldsymbol{r}_4^{\times}\mathring{\boldsymbol{\omega}}_{13}+\boldsymbol{f}_{a1}-\boldsymbol{r}_2^{\times}\dot{\boldsymbol{\omega}}_1+\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_1^{\times}\boldsymbol{r}_2]-\boldsymbol{r}_1^{\times}(\boldsymbol{A}_{21}\cdot\dot{\boldsymbol{\omega}}_1+\mathring{\boldsymbol{\omega}}_{12}+\boldsymbol{A}_{21}\cdot\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_{12})+\boldsymbol{\omega}_2^{\times}\boldsymbol{\omega}_2^{\times}\boldsymbol{r}_1 \tag{26}$$

Substituting Eqs. (15), (25) into Eq. (19), one can obtain

$$\boldsymbol{a}_3=\boldsymbol{A}_{31}[(m_1+m_2+m_3)^{-1}(m_2\boldsymbol{r}_1^{\times}+m_2\boldsymbol{r}_2^{\times}+m_3\boldsymbol{r}_3^{\times}+m_3\boldsymbol{r}_4^{\times})\dot{\boldsymbol{\omega}}_1+\boldsymbol{A}_{12}(m_1+m_2+m_3)^{-1}m_2\boldsymbol{r}_1^{\times}\mathring{\boldsymbol{\omega}}_{12}+\boldsymbol{A}_{13}(m_1+m_2+m_3)^{-1}m_3\cdot\boldsymbol{r}_4^{\times}\mathring{\boldsymbol{\omega}}_{13}+\boldsymbol{f}_{a1}-\boldsymbol{r}_3^{\times}\dot{\boldsymbol{\omega}}_1+\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_1^{\times}\boldsymbol{r}_3]-r_4^{\times}(\boldsymbol{A}_{31}\cdot\dot{\boldsymbol{\omega}}_1+\mathring{\boldsymbol{\omega}}_{13}+\boldsymbol{A}_{31}\cdot\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_{13})+\boldsymbol{\omega}_3^{\times}\boldsymbol{\omega}_3^{\times}\boldsymbol{r}_4 \tag{27}$$

Substituting Eqs. (26), (27) into Eq. (4), we get

$$[\boldsymbol{J}_1+(m_1+m_2+m_3)^{-1}(\boldsymbol{r}_2^{\times}m_2+\boldsymbol{r}_3^{\times}m_3)(m_2\boldsymbol{r}_1^{\times}+m_2\boldsymbol{r}_2^{\times}+m_3\boldsymbol{r}_3^{\times}+m_3\boldsymbol{r}_4^{\times})-m_2\boldsymbol{r}_1^{\times}\boldsymbol{r}_2^{\times}-m_2\boldsymbol{r}_2^{\times}\cdot\boldsymbol{r}_2^{\times}-m_3\boldsymbol{r}_3^{\times}\boldsymbol{r}_3^{\times}-m_3\boldsymbol{r}_3^{\times}\boldsymbol{r}_4^{\times}]\dot{\boldsymbol{\omega}}_1=\boldsymbol{T}_1-\boldsymbol{A}_{12}\boldsymbol{T}_2-\boldsymbol{A}_{13}\boldsymbol{T}_3-\boldsymbol{\omega}_1\times\boldsymbol{J}_1\boldsymbol{\omega}_1-\boldsymbol{r}_2^{\times}\boldsymbol{A}_{12}m_2\cdot[\boldsymbol{A}_{21}(\boldsymbol{f}_{a1}+\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_1^{\times}\boldsymbol{r}_2)-\boldsymbol{r}_1^{\times}(\mathring{\boldsymbol{\omega}}_{12}+\boldsymbol{A}_{21}\cdot\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_{12})+\boldsymbol{\omega}_2^{\times}\boldsymbol{\omega}_2^{\times}\boldsymbol{r}_1]-\boldsymbol{r}_3^{\times}\boldsymbol{A}_{13}m_3\cdot[\boldsymbol{A}_{31}(\boldsymbol{f}_{a1}+\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_1^{\times}\boldsymbol{r}_3)-\boldsymbol{r}_4^{\times}(\mathring{\boldsymbol{\omega}}_{13}+\boldsymbol{A}_{31}\cdot\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_{13})+\boldsymbol{\omega}_3^{\times}\boldsymbol{\omega}_3^{\times}\boldsymbol{r}_4]-(\boldsymbol{r}_2^{\times}m_2+\boldsymbol{r}_3^{\times}m_3)[\boldsymbol{A}_{12}(m_1+m_2+m_3)^{-1}m_2\boldsymbol{r}_1^{\times}\mathring{\boldsymbol{\omega}}_{12}+\boldsymbol{A}_{13}(m_1+m_2+m_3)^{-1}m_3\cdot\boldsymbol{r}_4^{\times}\mathring{\boldsymbol{\omega}}_{13}] \tag{28}$$

Substituting Eqs. (12), (21) into Eq. (28), we can obtain

$$\dot{\boldsymbol{\omega}}_1=\boldsymbol{f}_j\cdot(\boldsymbol{T}_1-\boldsymbol{\omega}_1\times\boldsymbol{J}_1\boldsymbol{\omega}_1)-\boldsymbol{f}_j\cdot\boldsymbol{f}_{D1} \tag{29}$$

where

$$\boldsymbol{f}_j=[\boldsymbol{J}_1+\boldsymbol{J}_2+\boldsymbol{J}_3+(m_1+m_2+m_3)^{-1}(\boldsymbol{r}_2^{\times}m_2+\boldsymbol{r}_3^{\times}m_3)(m_2\boldsymbol{r}_1^{\times}+m_2\boldsymbol{r}_2^{\times}+m_3\boldsymbol{r}_3^{\times}+m_3\boldsymbol{r}_4^{\times})-m_2\boldsymbol{r}_1^{\times}\boldsymbol{r}_2^{\times}-m_2\boldsymbol{r}_2^{\times}\boldsymbol{r}_2^{\times}-m_3\boldsymbol{r}_3^{\times}\boldsymbol{r}_3^{\times}-m_3\boldsymbol{r}_3^{\times}\boldsymbol{r}_4^{\times}]^{-1}$$

$$\boldsymbol{f}_{D1}=\boldsymbol{A}_{12}[\boldsymbol{J}_2\boldsymbol{A}_{21}\cdot\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_{12}+\boldsymbol{\omega}_2\times\boldsymbol{J}_2\boldsymbol{\omega}_2+m_2\boldsymbol{v}_2^{*}\times(\boldsymbol{\omega}_2\times\boldsymbol{r}_1)]+\boldsymbol{A}_{13}[\boldsymbol{J}_3\boldsymbol{A}_{31}\cdot\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_{13}+\omega_3\times\boldsymbol{J}_3\boldsymbol{\omega}_3+m_3\boldsymbol{v}_3^{*}\times(\boldsymbol{\omega}_3\times\boldsymbol{r}_4)]+\boldsymbol{r}_2^{\times}\boldsymbol{A}_{12}m_2\boldsymbol{f}_{\omega a1}+\boldsymbol{r}_3^{\times}\boldsymbol{A}_{13}m_3\boldsymbol{f}_{\omega a2}+\boldsymbol{F}_{\omega12}\mathring{\boldsymbol{\omega}}_{12}+\boldsymbol{F}_{\omega13}\mathring{\boldsymbol{\omega}}_{13}$$

$$\boldsymbol{f}_{\omega a1}=\boldsymbol{A}_{21}(\boldsymbol{f}_{a1}+\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_1^{\times}\boldsymbol{r}_2)-\boldsymbol{r}_1^{\times}\boldsymbol{A}_{21}\cdot\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_{12}+\boldsymbol{\omega}_2^{\times}\boldsymbol{\omega}_2^{\times}\boldsymbol{r}_1$$

$$\boldsymbol{f}_{\omega a2}=\boldsymbol{A}_{31}(\boldsymbol{f}_{a1}+\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_1^{\times}\boldsymbol{r}_3)-\boldsymbol{r}_4^{\times}\boldsymbol{A}_{31}\cdot\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_{13}+\boldsymbol{\omega}_3^{\times}\boldsymbol{\omega}_3^{\times}r_4$$

$$\boldsymbol{F}_{\omega12}=(\boldsymbol{r}_2^{\times}m_2+\boldsymbol{r}_3^{\times}m_3)\boldsymbol{A}_{12}(m_1+m_2+m_3)^{-1}m_2\boldsymbol{r}_1^{\times}+\boldsymbol{A}_{12}\boldsymbol{J}_2-m_2\boldsymbol{r}_2^{\times}\boldsymbol{A}_{12}\boldsymbol{r}_1^{\times}$$

$$\boldsymbol{F}_{\omega13}=(\boldsymbol{r}_2^{\times}m_2+\boldsymbol{r}_3^{\times}m_3)\boldsymbol{A}_{13}(m_1+m_2+m_3)^{-1}m_3\boldsymbol{r}_4^{\times}+\boldsymbol{A}_{13}\boldsymbol{J}_3-m_3\boldsymbol{r}_3^{\times}\boldsymbol{A}_{13}\boldsymbol{r}_4^{\times}$$

Define the state variables as follows: $\boldsymbol{x}_1=\boldsymbol{v}_1$, $\boldsymbol{x}_2=\boldsymbol{q}_1$, $\boldsymbol{x}_3=\dot{\boldsymbol{q}}_1$, $x_4=q_2$, $x_5=\dot{q}_2$, $x_6=q_3$, $x_7=\dot{q}_3$, $\boldsymbol{u}_1=\boldsymbol{T}_1$, $u_2=T_{2x}$, $u_3=T_{3z}$. From Eqs. (25), (29), we can obtain

$$\dot{\boldsymbol{x}}_1=(m_1+m_2+m_3)^{-1}(m_2\boldsymbol{r}_2+m_2\boldsymbol{r}_1+m_3\boldsymbol{r}_3+m_3\boldsymbol{r}_4)[\boldsymbol{f}_j\cdot[\boldsymbol{u}_1-\boldsymbol{H}_1(\boldsymbol{H}_1^{-1}\boldsymbol{x}_3)^{\times}\boldsymbol{J}_1\boldsymbol{H}_1^{-1}\boldsymbol{x}_3]-\boldsymbol{f}_j\cdot\boldsymbol{f}_{D1}]+\boldsymbol{A}_{12}(m_1+m_2+m_3)^{-1}m_2\boldsymbol{r}_1^{\times}\mathring{\boldsymbol{\omega}}_{12}+\boldsymbol{A}_{13}(m_1+m_2+m_3)^{-1}m_3\cdot\boldsymbol{r}_4^{\times}\mathring{\boldsymbol{\omega}}_{13}+\boldsymbol{f}_{a1}$$

$$\begin{cases}\dot{\boldsymbol{x}}_2=\boldsymbol{x}_3\\ \dot{\boldsymbol{x}}_3=[\dot{\boldsymbol{H}}_1\boldsymbol{H}_1^{-1}-\boldsymbol{H}_1\boldsymbol{f}_j(\boldsymbol{H}_1^{-1}\boldsymbol{x}_3)^{\times}\boldsymbol{J}_1\boldsymbol{H}_1^{-1}]\boldsymbol{x}_3+\boldsymbol{H}_1(\boldsymbol{f}_j\cdot\boldsymbol{u}_1-\boldsymbol{f}_j\cdot\boldsymbol{f}_{D_1})\end{cases} \tag{30}$$

$$\begin{cases}\dot{x}_4=x_5\\ \dot{x}_5=J_{2x}^{-1}u_2-J_{2x}^{-1}\omega_{2y}\omega_{2z}(J_{2z}-J_{2y})-\dot{\omega}_{1x}-[m_2\boldsymbol{v}_2^{*}\times(\boldsymbol{\omega}_2\times\boldsymbol{r}_1)+\boldsymbol{A}_{21}\cdot\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_{12}]_x\end{cases} \tag{31}$$

$$\begin{cases}\dot{x}_6=x_7\\ \dot{x}_7=J_{3z}^{-1}u_3-J_{3z}^{-1}\omega_{3x}\omega_{3y}(J_{3y}-J_{3x})-\dot{\omega}_{1z}-[m_3\boldsymbol{v}_3^{*}\times(\boldsymbol{\omega}_3\times\boldsymbol{r}_4)+\boldsymbol{A}_{31}\cdot\boldsymbol{\omega}_1^{\times}\boldsymbol{\omega}_{13}]_z\end{cases} \tag{32}$$

All state variables can be solved from Eqs. (29)—(32).

2 Controller Design of Satellite with Double Rotary Payloads

In this section, the nonlinear disturbance observer is designed for satellite's main body to estimate and compensate disturbance torque acted by payloads' motion, and the adaptive fast nonsingular terminal attitude stabilization SMC is designed for satellite's main body. Meanwhile, the adaptive fast nonsingular terminal attitude maneuver SMC is planned for payloads' attitude maneuver along the reference trajectory in same way.

2.1 Nonlinear disturbance observer design

The main task in this subsection is to design disturbance observer for satellite's main body A_1 to estimate the disturbance part $-\boldsymbol{H}_1\boldsymbol{f}_j\cdot\boldsymbol{f}_{D_1}$ in Eq. (30). From Eq. (30), we have

$$\begin{bmatrix}\dot{\boldsymbol{x}}_3\\ \dot{\boldsymbol{E}}_1\end{bmatrix}=\begin{bmatrix}\boldsymbol{K}_1 & \boldsymbol{I}\\ 0 & 0\end{bmatrix}\cdot\begin{bmatrix}\boldsymbol{x}_3\\ \boldsymbol{E}_1\end{bmatrix}+\begin{bmatrix}\boldsymbol{H}_1\boldsymbol{f}_j\\ 0\end{bmatrix}\cdot\boldsymbol{u}_1+\begin{bmatrix}0\\ \dot{\boldsymbol{E}}_1\end{bmatrix} \tag{33}$$

where $\boldsymbol{K}_1=\dot{\boldsymbol{H}}_1\boldsymbol{H}_1^{-1}-\boldsymbol{H}_1\boldsymbol{f}_j(\boldsymbol{H}_1^{-1}\boldsymbol{x}_3)^{\times}\boldsymbol{J}_1\boldsymbol{H}_1^{-1}$ and

$\boldsymbol{E}_1 = -\boldsymbol{H}_1 \boldsymbol{f}_j \cdot \boldsymbol{f}_{D_1}$. The observer is given by

$$\begin{bmatrix} \dot{\hat{\boldsymbol{x}}}_3 \\ \dot{\hat{\boldsymbol{E}}}_1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{K}_1 & \boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \cdot \begin{bmatrix} \hat{\boldsymbol{x}}_3 \\ \hat{\boldsymbol{E}}_1 \end{bmatrix} + \begin{bmatrix} \boldsymbol{H}_1 \boldsymbol{f}_j \\ \boldsymbol{0} \end{bmatrix} \cdot \boldsymbol{u}_1 + \begin{bmatrix} \boldsymbol{R}_1 \\ \boldsymbol{R}_2 \end{bmatrix} \cdot \tilde{\boldsymbol{x}}_3 \tag{34}$$

where $\tilde{\boldsymbol{x}}_3 = \hat{\boldsymbol{x}}_3 - \boldsymbol{x}_3$, and gain $\boldsymbol{R}_1 \in 3\times 3$, $\boldsymbol{R}_2 \in 3\times 3$. Let $\widetilde{\boldsymbol{E}}_1 = \hat{\boldsymbol{E}}_1 - \boldsymbol{E}_1$, from Eqs. (33), (34), we get

$$\begin{bmatrix} \dot{\tilde{\boldsymbol{x}}}_3 \\ \dot{\widetilde{\boldsymbol{E}}}_1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{K}_1 + \boldsymbol{R}_1 & \boldsymbol{I} \\ \boldsymbol{R}_2 & \boldsymbol{0} \end{bmatrix} \cdot \begin{bmatrix} \tilde{\boldsymbol{x}}_3 \\ \widetilde{\boldsymbol{E}}_1 \end{bmatrix} - \begin{bmatrix} \boldsymbol{0} \\ \dot{\boldsymbol{E}}_1 \end{bmatrix} \tag{35}$$

Let $\boldsymbol{R}_1 = -\boldsymbol{K}_1 + \boldsymbol{N}_1 + \boldsymbol{N}_2$, $\boldsymbol{R}_2 = -\boldsymbol{N}_2 \boldsymbol{N}_1$, $\boldsymbol{Q}_1 = [\boldsymbol{I} \quad \boldsymbol{0}; \boldsymbol{N}_2 \quad \boldsymbol{I}]$, $\boldsymbol{s} = [\boldsymbol{s}_1^{\mathrm{T}}, \boldsymbol{s}_2^{\mathrm{T}}]^{\mathrm{T}} = \boldsymbol{Q}_1 \cdot [\tilde{\boldsymbol{x}}_3^{\mathrm{T}}, \widetilde{\boldsymbol{E}}_1^{\mathrm{T}}]^{\mathrm{T}}$ where $\boldsymbol{N}_1 \in 3\times 3$, $\boldsymbol{N}_2 \in 3\times 3$ will be designed later, then Lemma 1 is obtained.

Lemma 1[17] Considering $\| \dot{\boldsymbol{E}}_1 \| \leqslant a_1$ and $a_2 > 0$, where a_1 and a_2 are given positive scalars. If there exist matrices $\boldsymbol{P}_1 > 0$, $\boldsymbol{P}_2 > 0$, $\boldsymbol{M}_1$, $\boldsymbol{M}_2$ and scalar $a_3 > 0$ that the following linear matrix inequality (LMI) holds

$$\begin{bmatrix} \boldsymbol{M}_1 + \boldsymbol{M}_1^{\mathrm{T}} + 2a_2 \boldsymbol{P}_1 & \boldsymbol{P}_1 & \boldsymbol{0} \\ \boldsymbol{P}_1^{\mathrm{T}} & \boldsymbol{M}_2 + \boldsymbol{M}_2^{\mathrm{T}} + 2a_2 \boldsymbol{P}_2 & \boldsymbol{P}_2 \\ \boldsymbol{0} & \boldsymbol{P}_2^{\mathrm{T}} & -a_3 \boldsymbol{I} \end{bmatrix} < 0 \tag{36}$$

Then $\lim\limits_{t\to\infty} \| \boldsymbol{s}_1 \| \leqslant a_1 \sqrt{\dfrac{a_3}{2a_2 \lambda_{\min}(\boldsymbol{P}_1)}}$, $\lim\limits_{t\to\infty} \| \boldsymbol{s}_2 \| \leqslant a_1 \sqrt{\dfrac{a_3}{2a_2 \lambda_{\min}(\boldsymbol{P}_2)}}$. Moreover, $\boldsymbol{N}_1 = \boldsymbol{P}_1^{-1} \boldsymbol{M}_1$, $\boldsymbol{N}_2 = \boldsymbol{P}_2^{-1} \boldsymbol{M}_2$.

2.2 Nonsingular fast terminal attitude stabilization SMC design

The main task in this subsection is to design attitude stabilization controller for satellite′s main body A_1.

The sliding surface is defined by[16]

$$\boldsymbol{C}_1 = \boldsymbol{x}_2 + \boldsymbol{\phi}_{11} \boldsymbol{x}_2^{r_{11}} + \boldsymbol{\phi}_{12} \boldsymbol{x}_3^{r_{12}} \tag{37}$$

where $\boldsymbol{x}_2$ and $\boldsymbol{x}_3$ are the state variables defined in Eq. (30), $\boldsymbol{\phi}_{11} = \mathrm{diag}\,[\phi_{11}^1, \phi_{11}^2, \phi_{11}^3]$, $\boldsymbol{\phi}_{12} = \mathrm{diag}\,[\phi_{12}^1, \phi_{12}^2, \phi_{12}^3]$, $\phi_{1i}^j > 0\,(i=1,2;\ j=1,2,3)$, $1 < r_{12} = \dfrac{m_{12}}{l_{12}} < 2$, $r_{12} < r_{11} = \dfrac{m_{11}}{l_{11}}$. Here, $l_{11}, m_{11}, l_{12}, m_{12}$ are the positive odds to be designed.

Assumption 1 The disturbance observer error $\widetilde{\boldsymbol{E}}_1$ is assumed to be bounded and satisfies the following condition

$$\| \widetilde{\boldsymbol{E}}_1 \| < f_1$$

where f_1 is the unknown upper bound of disturbance observer error.

For the system described by Eq. (30), the controller $\boldsymbol{u}_1$ is designed as follows

$$\boldsymbol{u}_1 = \boldsymbol{u}_{11} + \boldsymbol{u}_{12} \tag{38}$$

$$\boldsymbol{u}_{11} = -(\boldsymbol{H}_1 \boldsymbol{f}_j)^{-1} [(\boldsymbol{\phi}_{12} r_{12})^{-1} \cdot (\boldsymbol{I} + \boldsymbol{\phi}_{11} r_{11} \mathrm{diag}\,(\boldsymbol{x}_2^{r_{11}-1})) \cdot \boldsymbol{x}_3^{2-r_{12}} + \hat{\boldsymbol{E}}_1 + \mathrm{diag}\,(\mathrm{sgn}(\boldsymbol{C}_1)\boldsymbol{k}_1 + \boldsymbol{K}_1 \boldsymbol{x}_3]$$

$$\boldsymbol{u}_{12} = \begin{cases} -(\boldsymbol{H}_1 \boldsymbol{f}_j)^{-1} \dfrac{(r_{12} \boldsymbol{C}_1^{\mathrm{T}} \boldsymbol{\phi}_{12} \mathrm{diag}\,(\boldsymbol{x}_3^{r_{12}-1}))^{\mathrm{T}}}{\| r_{12} \boldsymbol{C}_1^{\mathrm{T}} \boldsymbol{\phi}_{12} \mathrm{diag}\,(\boldsymbol{x}_3^{r_{12}-1}) \|} \hat{f}_1 & \text{if } \boldsymbol{x}_3 \neq 0 \\ 0 & \text{if } \boldsymbol{x}_3 = 0 \end{cases}$$

and the adaptation update law is

$$\dot{\hat{f}}_1 = \rho_0 \| r_{12} \boldsymbol{C}_1^{\mathrm{T}} \boldsymbol{\phi}_{12} \mathrm{diag}\,(\boldsymbol{x}_3^{r_{12}-1}) \|$$

where $\dot{\hat{f}}_1$ is used to estimate upper bound of f_1 and $\rho_0 > 0$, $\boldsymbol{k}_1 = [k_{11}, k_{12}, k_{13}]^{\mathrm{T}}$ and $k_{1i} > 0$, $\boldsymbol{I}_{3\times 3}$ is the identity matrix. Then Theme 1 is obtained.

Theme 1 For satellite′s main body system Eq. (30), if the sliding surface is designed as Eq. (37) and the controller is defined as Eq. (38), then system Eq. (30) is in asymptotic stability.

Proof Considering the Lyapunov function $V_1 = \dfrac{1}{2} \boldsymbol{C}_1^{\mathrm{T}} \boldsymbol{C}_1 + \rho_0^{-1} \tilde{f}_1^2$ where $\tilde{f}_1 = f_1 - \hat{f}_1$, then the time derivative of V_1 is

$$\begin{aligned} \dot{V}_1 &= \boldsymbol{C}_1^{\mathrm{T}} \dot{\boldsymbol{C}}_1 - \rho_0^{-1} \tilde{f}_1 \dot{\hat{f}}_1 = \boldsymbol{C}_1^{\mathrm{T}} [\boldsymbol{x}_3 + r_{11} \boldsymbol{\phi}_{11} \cdot \\ &\mathrm{diag}\,(\boldsymbol{x}_2^{r_{11}-1}) \boldsymbol{x}_3 + r_{12} \boldsymbol{\phi}_{12} \mathrm{diag}\,(\boldsymbol{x}_3^{r_{12}-1}) \dot{\boldsymbol{x}}_3] - \\ &\rho_0^{-1} \tilde{f}_1 \dot{\hat{f}}_1 = \boldsymbol{C}_1^{\mathrm{T}} [\boldsymbol{x}_3 + r_{11} \boldsymbol{\phi}_{11} \mathrm{diag}\,(\boldsymbol{x}_2^{r_{11}-1}) \boldsymbol{x}_3 + \\ &r_{12} \boldsymbol{\phi}_{12} \mathrm{diag}\,(\boldsymbol{x}_3^{r_{12}-1}) \cdot (\boldsymbol{K}_1 \boldsymbol{x}_3 + \boldsymbol{H}_1 \boldsymbol{f}_j \boldsymbol{u}_1 + \boldsymbol{E}_1)] - \\ &\rho_0^{-1} \tilde{f}_1 \dot{\hat{f}}_1 = \boldsymbol{C}_1^{\mathrm{T}} \{ \boldsymbol{x}_3 + r_{11} \boldsymbol{\phi}_{11} \mathrm{diag}\,(\boldsymbol{x}_2^{r_{11}-1}) \boldsymbol{x}_3 + \\ &r_{12} \boldsymbol{\phi}_{12} \mathrm{diag}(\boldsymbol{x}_3^{r_{12}-1}) \cdot [-(r_{12} \boldsymbol{\phi}_{12})^{-1} (\boldsymbol{I} + \\ &\boldsymbol{\phi}_{11} r_{11} \mathrm{diag}(\boldsymbol{x}_2^{r_{11}-1})) \cdot \boldsymbol{x}_3^{2-r_{12}} - \widetilde{\boldsymbol{E}}_1 - \mathrm{diag}\,(\mathrm{sgn} \\ &(\boldsymbol{C}_1) \boldsymbol{k}_1 + \boldsymbol{H}_1 \boldsymbol{f}_j \boldsymbol{u}_{12}]\} - \rho_0^{-1} \tilde{f}_1 \dot{\hat{f}}_1 = \boldsymbol{C}_1^{\mathrm{T}} [r_{12} \boldsymbol{\phi}_{12} \cdot \\ &\mathrm{diag}\,(\boldsymbol{x}_3^{r_{12}-1})(-\widetilde{\boldsymbol{E}}_1 - \mathrm{diag}\,(\mathrm{sgn}(\boldsymbol{C}_1) \boldsymbol{k}_1 + \\ &\boldsymbol{H}_1 \boldsymbol{f}_j \boldsymbol{u}_{12})] - \rho_0^{-1} \tilde{f}_1 \dot{\hat{f}}_1 \leqslant \boldsymbol{C}_1^{\mathrm{T}} [r_{12} \boldsymbol{\phi}_{12} \mathrm{diag}\,(\boldsymbol{x}_3^{r_{12}-1}) \\ &(-\widetilde{\boldsymbol{E}}_1 - \mathrm{diag}\,(\mathrm{sgn}(\boldsymbol{C}_1) \boldsymbol{k}_1)] - \| r_{12} \boldsymbol{C}_1^{\mathrm{T}} \boldsymbol{\phi}_{12} \\ &\mathrm{diag}\,(\boldsymbol{x}_3^{r_{12}-1}) \| \hat{f}_1 - \rho_0^{-1} \tilde{f}_1 \dot{\hat{f}}_1 < \boldsymbol{C}_1^{\mathrm{T}} [r_{12} \boldsymbol{\phi}_{12} \\ &\mathrm{diag}\,(\boldsymbol{x}_3^{r_{12}-1})(-\mathrm{diag}\,(\mathrm{sgn}(\boldsymbol{C}_1) \boldsymbol{k}_1)] - \| r_{12} \\ &\boldsymbol{C}_1^{\mathrm{T}} \boldsymbol{\phi}_{12} \mathrm{diag}\,(\boldsymbol{x}_3^{r_{12}-1}) \| (-f_1 + \hat{f}_1 + \tilde{f}_1) \leqslant 0 \end{aligned} \tag{39}$$

When $\dot{V}_1 < 0$, it is easy to obtain that $\boldsymbol{x}_2$ and $\boldsymbol{x}_3$ approach zero ultimately. When $\dot{V}_1 = 0$ we get $\boldsymbol{x}_3 = 0$ from Eq. (39), and we obtain $\dot{\boldsymbol{x}}_3 = -\text{diag}(\text{sgn}(\boldsymbol{C}_1))\boldsymbol{k}_1 - \widetilde{\boldsymbol{E}}$ from Eqs. (30), (38) when $\boldsymbol{x}_3 = 0$. It is obviously that $\dot{\boldsymbol{x}}_3 = 0$ cannot be hold on in practice and $\boldsymbol{x}_3 = 0$ cannot also be hold on. So $\dot{V}_1$ will be less than zero at last and $\boldsymbol{x}_2$, $\boldsymbol{x}_3$ go to zero finally, and system Eq. (30) is asymptotic stability from reduction above. The proof is complete.

To avoid chattering problem, $\boldsymbol{u}_{12}$ can be changed into the following form

$$\boldsymbol{u}_{12} = \begin{cases} -(\boldsymbol{H}_1\boldsymbol{f}_j)^{-1}\dfrac{(r_{12}\boldsymbol{C}_1^{\mathrm{T}}\boldsymbol{\phi}_{12}\,\text{diag}\,(\boldsymbol{x}_3^{r_{12}-1}))^{\mathrm{T}}}{\|\, r_{12}\boldsymbol{C}_1^{\mathrm{T}}\boldsymbol{\phi}_{12}\,\text{diag}\,(\boldsymbol{x}_3^{r_{12}-1})\,\|}\hat{f}_1 \\ \quad \text{if } \|\, r_{12}\boldsymbol{C}_1^{\mathrm{T}}\boldsymbol{\phi}_{12}\,\text{diag}(\boldsymbol{x}_3^{r_{12}-1})\,\| > \lambda \\ -(\boldsymbol{H}_1\boldsymbol{f}_j)^{-1}\dfrac{(r_{12}\boldsymbol{C}_1^{\mathrm{T}}\boldsymbol{\phi}_{12}\,\text{diag}\,(\boldsymbol{x}_3^{r_{12}-1}))^{\mathrm{T}}}{\lambda}\hat{f}_1 \\ \quad \text{if } \|\, r_{12}\boldsymbol{C}_1^{\mathrm{T}}\boldsymbol{\phi}_{12}\,\text{diag}\,(\boldsymbol{x}_3^{r_{12}-1})\,\| < \lambda \end{cases}$$

where λ is the small positive real number and $\lambda \leqslant 1$.

2.3 Nonsingular fast terminal attitude maneuver SMC design

In this subsection we will design adaptive nonsingular fast terminal attitude maneuver SMC for payloads disturbed by motion of satellite′s main body.

The sliding surface is defined by[16]

$$C_2 = e_{21} + \phi_{21}e_{21}^{r_{21}} + \phi_{22}e_{22}^{r_{22}} \tag{40}$$

where $e_{21} = q_2 - q_{2r}$ and $\dot{e}_{21} = e_{22}$, q_{2r} is the reference trajectory. $\phi_{21} > 0$ and $\phi_{22} > 0$, $1 < r_{22} = \dfrac{m_{22}}{l_{22}} < 2$, $r_{22} < r_{21} = \dfrac{m_{21}}{l_{21}}$, $l_{21}, m_{21}, l_{22}, m_{22}$ are positive odds to be designed.

Assumption 2 The disturbance E_2 is assumed to be bounded and satisfy the following condition

$$E_2 < f_2$$

where $E_2 = -J_{2x}^{-1} \cdot \omega_{2y}\omega_{2z}(J_{2z} - J_{2y}) - \dot{\omega}_{1x}$, f_2 is unknown upper bound of disturbance.

In same way, the controller for A_2 is designed by

$$u_2 = -J_{2x}[(\phi_{22}r_{22})^{-1} \cdot (1 + \phi_{21}r_{21}e_{21}^{r_{21}-1}) \cdot e_{22}^{2-r_{22}} + \text{sgn}(C_2)k_2 + \hat{f}_2 - \ddot{x}_{2r}] \tag{41}$$

and the adaptation update law is

$$\dot{\hat{f}}_2 = \rho_1 r_{22} C_2 \phi_{22} x_{22}^{r_{22}-1}$$

Then Theme 2 is given.

Theme 2 For system Eq. (31), if the sliding surface is designed as Eq. (40) and the controller is defined as Eq. (41), then $e_{21} \to 0$ and $e_{22} \to 0$ when $t \to \infty$.

Proof Considering the Lyapunov function $V_2 = \dfrac{1}{2}\boldsymbol{C}_2^{\mathrm{T}}\boldsymbol{C}_2 + \rho_1^{-1}\widetilde{f}_2^2$ where $\widetilde{f}_2 = f_2 - \hat{f}_2$, then the time derivative of V_2 is

$$\begin{aligned}\dot{V}_2 &= \boldsymbol{C}_2^{\mathrm{T}}\dot{\boldsymbol{C}}_2 - \rho_1^{-1}\widetilde{f}_2\dot{\hat{f}}_2 = \boldsymbol{C}_2^{\mathrm{T}}(e_{22} + r_{21}\boldsymbol{\phi}_{21}e_{21}^{r_{21}-1}e_{22} + r_{22}\boldsymbol{\phi}_{22}e_{22}^{r_{22}-1}\dot{e}_{22}) - \rho_1^{-1}\widetilde{f}_2\dot{\hat{f}}_2 = \boldsymbol{C}_2^{\mathrm{T}}[e_{22} + r_{21}\boldsymbol{\phi}_{21}e_{21}^{r_{21}-1}e_{22} + r_{22}\boldsymbol{\phi}_{22}e_{22}^{r_{22}-1} \cdot (\boldsymbol{J}_{2x}^{-1}u_2 + \boldsymbol{E}_2 - \ddot{x}_{2r})] - \\ &\rho_1^{-1}\widetilde{f}_2\dot{\hat{f}}_2 = \boldsymbol{C}_2^{\mathrm{T}}\{e_{22} + r_{21}\boldsymbol{\phi}_{21}e_{21}^{r_{21}-1}e_{22} - r_{22}\boldsymbol{\phi}_{22} \cdot e_{22}^{r_{22}-1} \cdot [(\boldsymbol{\phi}_{22}r_{22})^{-1} \cdot (1 + \boldsymbol{\phi}_{21}r_{21}e_{22}^{r_{22}-1}) \cdot e_{22}^{2-r_{22}} + \text{sgn}(\boldsymbol{C}_2)\boldsymbol{k}_2 + \hat{f}_2 - \boldsymbol{E}_2]\} - \rho_1^{-1}\widetilde{f}_2\dot{\hat{f}}_2 = \\ &\boldsymbol{C}_2^{\mathrm{T}}[-r_{22}\boldsymbol{\phi}_{22}e_{22}^{r_{22}-1}(\text{sgn}(\boldsymbol{C}_2)\boldsymbol{k}_2 + \hat{f}_2 - \boldsymbol{E}_2)] - \rho_1^{-1}\widetilde{f}_2\dot{\hat{f}}_2 < \boldsymbol{C}_2^{\mathrm{T}}[-r_{22}\boldsymbol{\phi}_{22}e_{22}^{r_{22}-1}(\text{sgn}(\boldsymbol{C}_2)k_2 + \hat{f}_2 - f_2)] - \widetilde{f}_2 r_{22}\boldsymbol{C}_2\boldsymbol{\phi}_{22}e_{22}^{r_{22}-1} = -\boldsymbol{C}_2^{\mathrm{T}}(r_{12}\boldsymbol{\phi}_{22} \cdot e_{22}^{r_{22}-1}\text{sgn}(\boldsymbol{C}_2)\boldsymbol{k}_2) \leqslant 0\end{aligned}$$

When $\dot{V}_2 < 0$, it′s easy to obtain that e_{21} and e_{22} go to zero ultimately. The proof is complete.

The procedure to design attitude maneuver controller of payload A_3 is similar to A_2, and we do not repeat it here for lack of space.

3 Simulations and Results

To illustrate the effectiveness of the proposed method, we conduct simulations for satellite system with double rotary appendages in Matlab/Simulink environment. The main parameters are listed as follows. The inertia matrices of A_1, A_2 and A_3 are $\boldsymbol{J}_1 = \text{diag}[97, 83, 76]$ kg · m ^{2}, $\boldsymbol{J}_2 = \text{diag}[19, 15, 8]$ kg · m ^{2} and $\boldsymbol{J}_3 = \text{diag}[16, 18, 9]$ kg · m ^{2}. The mass of A_1, A_2 and A_3 are $m_1 = 95$ kg, $m_2 = 9$ kg, $m_3 = 11$ kg, $\boldsymbol{r}_1 = [0, 0, 0.2]$ m, $\boldsymbol{r}_2 = [0, 0, 0.6]$ m, $\boldsymbol{r}_3 = [0.6, 0, 0]$ m, $\boldsymbol{r}_4 = [0.15, 0, 0]$ m. For A_1, the initial value of attitude angle is $\boldsymbol{q}_1 = [0, 0, 0]^{\mathrm{T}}$ rad. The parameters of disturbance observer are $\boldsymbol{N}_1 = \text{diag}[-8.669\ 9,$

$-8.6699, -8.6699]$, $\boldsymbol{N}_2=\mathrm{diag}[-35.2341, -35.2341, -35.2341]$, and of controller (38) are $\boldsymbol{\phi}_{11}=\mathrm{diag}[2,2,2]$, $\boldsymbol{\phi}_{12}=\mathrm{diag}[0.1,0.1,0.1]$, $r_{11}=5/3$, $r_{12}=7/5$, $\boldsymbol{k}_1=1.2\times[1,1,1]^{\mathrm{T}}$, $\rho_0=0.01$.

Considering the actual situation, saturation function sat(•) is used instead of sign(•) to avoid the chattering problem in the controller Eq. (38). Simulation results are shown in Figs. 2,3,4.

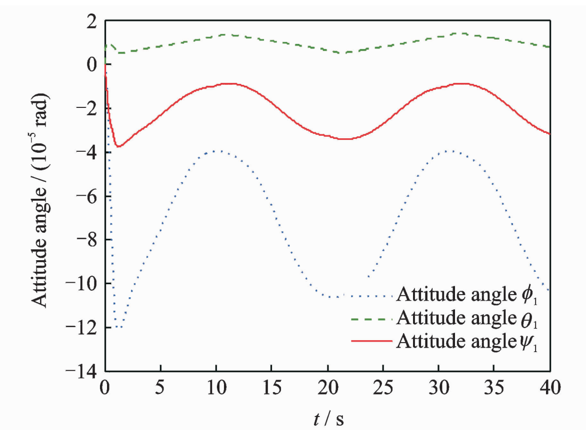

Fig. 2 Attitude angles of satellite′s main body A_1

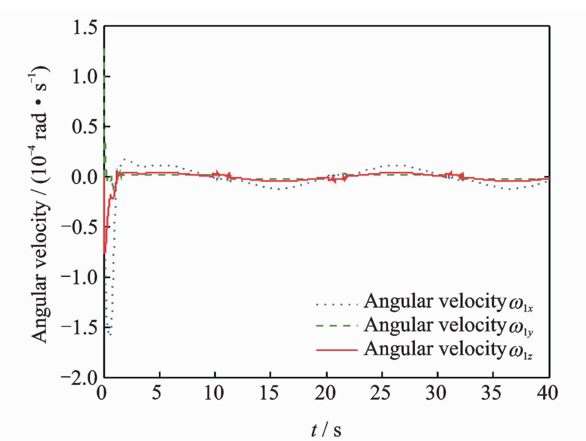

Fig. 3 Angular velocities of satellite′s main body A_1

In Figs. 2, 3, the Euler attitude angle and angular velocity of satellite′s main body A_1 are illustrated. From Fig. 1, the control precision of attitude angle is about 2×10^{-4}. Control accuracy of attitude angle will be improved when $\boldsymbol{k}_1$ in Eq. (38) is enlarged, however, control torque will be also enlarged. The control torque is given in Fig. 4. The observer error of disturbance torque acted on A_1 is shown in Fig. 5, and we

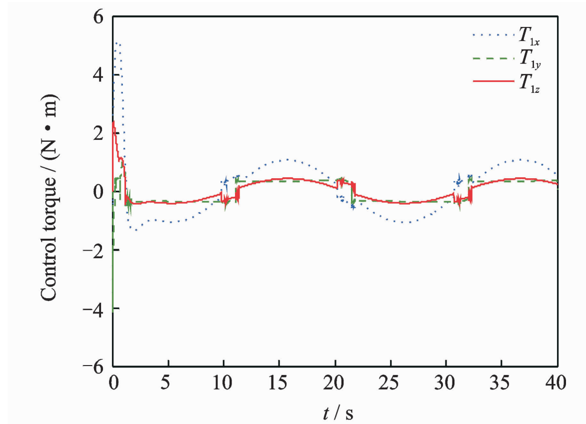

Fig. 4 Control torques of satellite′s main body A_1

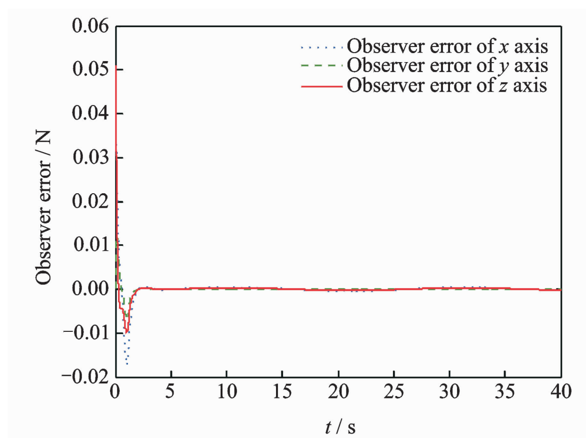

Fig. 5 Disturbance observer error $\widetilde{\boldsymbol{E}}_1$

conclude that the disturbance observer designed in this paper is effective.

For the payloads A_2 and A_3, the inertia matrices are $\boldsymbol{J}_2=\mathrm{diag}[19,15,8]$ kg • m^2 and $\boldsymbol{J}=\mathrm{diag}[16,18,9]$ kg • m^2. The initial value of attitude angles are $q_2=q_3=0$ rad. The target attitude angles are $q_{2d}=\pi/5\cdot\sin(0.3t)$ and $q_{3d}=\pi/6\cdot\sin(0.3t)$. The control parameters are $r_{21}=r_{31}=5/3$, $r_{22}=r_{32}=7/5$, $\phi_{21}=\phi_{31}=2$, $\phi_{22}=\phi_{32}=3$ and $k_2=k_3=2$, $\rho_1=\rho_2=0.003$。

In Figs. 6, 7, the Euler attitude angles of payloads A_2 and A_3 are shown. We can obtain that the payloads A_2 and$_{A3}$ track target attitude angles accurately even when A_1 introduces a disturbance.

It is worth mentioning that the observer error of observer Eq. (34) is relative to $\|\dot{\boldsymbol{E}}_1\|$ and the robustness of controllers Eqs. (38) and (41) are relative to $\boldsymbol{k}_1$ and k_2. So when more than two

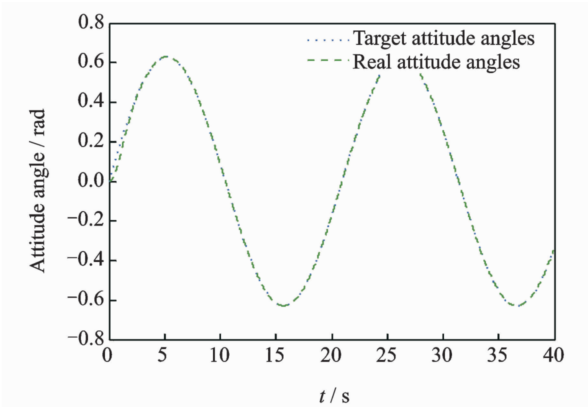

Fig. 6 Attitude angles of payload A_2

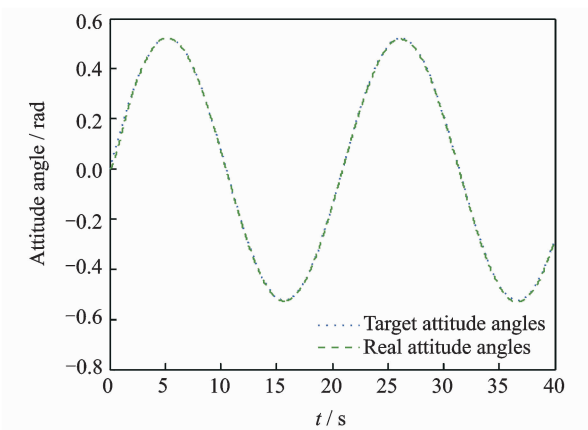

Fig. 7 Attitude angles of payload A_3

rotary payloads working together, the control results are mainly effected by the variation rate of coupling torque and amplitude of control actuator.

4 Conclusions

The attitude control of satellite system with double rotary payloads is proposed in this paper. Firstly, system dynamic model is established by the Newton-Euler method so that the system dynamic interconnection is described accurately. Secondly, nonlinear disturbance observer is designed for satellite's main body A_1 to estimate disturbance torque exerted by A_2 and A_3 and then compensated by the controller of A_1. Meanwhile, the adaptive fast nonsingular terminal attitude stabilization SMC is proposed for A_1, and the attitude of A_1 thus has a fast convergence speed. In same way, the adaptive fast nonsingular terminal attitude maneuver SMC controller is designed for each payload of A_2 and A_3. Finally, simulation results prove the effectiveness of the proposed control strategy, thereby providing reference values in practice.

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