Attitude Analysis in Process Conflict for C919 Aircraft Manufacturing

Xu Haiyan *, *Xu Peng*, *Sharafat Ali*

College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing 211106, P. R. China

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Abstract: Based on the option prioritization in graph model for conflict resolution of two decision makers (DMs), new logical and matrix representations of four stability concepts for DMs' attitude are proposed. The logical representation of attitude is defined, and converted to the matrix form in order to develop a decision support system (DSS) efficiently. Compared with existing definitions of DMs' attitude based on states, the proposed definitions of attitude based on options are convenient and more effective to generate preferences since that of states can be significantly larger than that of options in a large conflict. In addition, it is easier to obtain the information of the prioritization of option statements than to obtain preference of states for users. The proposed representations are applied to the process conflict during aircraft manufacturing to demonstrate the efficiency of the new approach.

Key words: graph model for conflict resolution; attitudes; option prioritization; process conflict for aircraft manufacturing

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0 Introduction

A number of game forms such as normal form[1], option form[2], and the graph model form[3] for the analysis of strategic conflicts have been introduced. In 1944, the normal form was put forward by Neumann and Morgenstern[1]. In 1971, Howard et al.[2] proposed the option form that is more convenient than other game models for large games. Then Fraser and Hipel[4,5] proposed option form's enhancements in 1979 and 1984, respectively. In 1987, the graph model for conflict resolution (GMCR) was put forward by Kilgour et al.[3], which expanded conflict analysis and conflict resolution significantly. In order to depict a variety of behaviors for decision makers(DMs) in a conflict, some definitions of stabilities for a state in graph model, including Nash stability[6,7], general metarationality (GMR)[2], symmetric metarationality (SMR)[2] and sequential stability (SEQ)[4], were developed and applied to many fields. However, those stabilities have not considered attitudes of DMs in conflicts.

In 1993, a GMCR with DMs' attitude was presented by Fang et al[8]. In a conflict, a DM's attitude often affects the preferences of other DMs or his own so that the outcome of conflict might be changed. In 2007, the four stabilities of Nash, GMR, SMR, and SEQ based on attitude were proposed by Inohara[9] using a logical form. But it is difficult to develop a decision support system(DSS) based on the logical form. Subsequently, the matrix representation of attitude in conflicts was proposed by Walker et al.[10] in 2013. The matrix form is amenable for coding and the development of a DSS. However, the corresponding definitions[9,10] of attitude are based on states, which may lead to dimensionality issue. Fortunately, the dimensionality issue can be resolved by using an option form. From the definition of a state, the option form is particularly

* Corresponding author, E-mail address: xuhaiyan@nuaa.edu.cn.

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useful for defining states in a wide range of real world conflicts that can be readily investigated within the paradigm of the graph model. Let the numbers of states and options be m and k, respectively. Then, m and k satisfy the equation ($m = 2^k$), which shows that of states is much bigger than that of options. Therefore, for a large model, preferences based on DM′s attitude generated from option statements are more convenient than those generated from states.

We propose a logical representation of attitude based on option prioritization[11] for GMCR. Compared with the existing logical representation of attitude based on states, the new logical approach is more convenient for users to obtain preferences including DMs′ attitude. Then, the logical definitions are converted to a matrix representation, which makes the definitions of attitude easier to be encoded into a DSS. In addition, previous definitions of attitude for GMCR[9,10] are updated in this paper, therefore they will be more flexible to degenerate to the definitions without attitude. In fact, the stabilities based on attitude expand the four basic forms stabilities for the Nash, GMR, SMR, and SEQ.

With the rapid development of aircraft manufacturing, there are many studies on material properties[12], propulsion system[13], noise mechanism[14] and so on, but the research on process conflict in aircraft manufacturing is scarce, while the process conflict often occurs. Therefore, we analyze the process conflict in aircraft manufacturing using the proposed theory.

1 Updated Stabilities of Attitude Based on States

1.1 Graph model for conflict resolution

A graph model for conflict resolution is a four-tuple (N , S , $(A_i)_{i\in N}$, $(>_i, \sim_i)_{i\in N}$), where N is the set of DMs ($|N|\geqslant 2$), S the set of all states in the conflict ($|S|\geqslant 2$), (S, A_i) the graph of DM i(S: the set of all vertices, $A_i \in S\times S$: the set of all arcs such that $(s,s)\notin A_i$ for all $s\in S$ and all $i \in N$), and ($>_i$, $\sim_i$): DM i′s preferences on S.

For $s,t \in S$, $s >_i t$ means that DM i prefers state s to t, while $s\sim_i t$ indicates that DM i is indifferent between s and t. Relative preferences are assumed to satisfy the following properties:

(1) $>_i s$ is asymmetric; hence, for all $s,t\in S$, $s>_i t$ and $t>_i s$ cannot be true simultaneously.

(2) $\sim_i$ is reflective; therefore, for any $s\in S$, $s\sim_i s$.

(3) $\sim_i$ is symmetric; hence, for any $s,t\in S$, if $s\sim_i t$, $t\sim_i s$.

(4) ($\sim_i$, $>_i$) is complete; therefore, for all $s,t\in S$, one of $s>_i t$, $t>_i$ and $s\sim_i t$ are true.

Definition 1 (Reachable list) For $i\in \mathbf{N}, s\in S$, reachable list of DM i from state s is the set $\{t\in S\mid (s,t)\in A_i\}$, denoted by $R_i(s)\subset S$. The reachable list is a record of all the states that a given DM can reach from a specified starting state in one step.

Matrix representation of reachable list is

$$J_i(s,t)=\begin{cases}1 & (s,t)\in A_i\\ 0 & \text{otherwise}\end{cases}$$

In addition, in order to calculate the solution concepts in matrix representation, a series of preference matrices (Table 1) had been presented by Xu et al[15-17].

Table 1 Preference matrices

Preference	$P_i^+(s,t)$	$P_i^-(s,t)$	$P_i^=(s,t)$
$t>_i s$	1	0	0
$t<_i s$	0	1	0
$t\sim_i s$	0	0	1

1.2 Logical representation and matrix representation of attitude based on states

Definition 2 (Attitudes) For DMs $i,j\in N$,

if i has a positive attitude towards DM j, the attitude is denoted by $e_{ij}=+$;

if i has a negative attitude towards DM j, the attitude is denoted by $e_{ij}=-$;

if i has an indifferent attitude towards DM j, the attitude is denoted by $e_{ij}=0$.

Definition 3 (Devoting preference (DP)) DP of DM $i\in N$ with respect to DM $j\in N$ is $>_j$, denoted by DP_{ij}, such that for $s,t\in S$, $t\ \mathrm{DP}_{ij}\ s$ if and only if $t>_j s$.

Definition 4 (Aggressive preference (AP)) AP of DM $i\in N$ with respect to DM $j\in N$ is $<_j$, denoted by AP_{ij} , where $<_j$ is defined as follows: $s, t \in S$, $t\ AP_{ij}\ s$ if and only if $t <_j s$.

Definition 5 (Indifferent preference (IP)) IP of DM $i \in N$ with respect to DM $j \in N$ is denoted by IP_{ij} , which denotes that DM i is indifferent with respect to preference of j.

Definition 6 (Relational preference (RP))

$$RP(e)_{ij} = \begin{cases} DP_{ij} & e_{ij} = + \\ AP_{ij} & e_{ij} = - \\ IP_{ij} & e_{ij} = 0 \end{cases}$$

where the types of preferences are matched with the three different attitudes. What this means is that if DM i has a positive attitude towards DM j, DM i will have a devoting preference with respect to DM j. If DM i has a negative attitude towards DM j, DM i will have an aggressive preference with respect to DM j. Thus, a DM behaves according to his or her attitudes.

Matrix representation of relational preference is

$$\boldsymbol{P}_{ij}^{R} = \begin{cases} \boldsymbol{P}_j^{+} & e_{ij} = + \\ \boldsymbol{P}_j^{-} & e_{ij} = - \\ \boldsymbol{E} & e_{ij} = 0 \end{cases}$$

where $\boldsymbol{E}$ denotes an $|S| \times |S|$ matrix with each element 1.

Definition 7 (Total relational preference (TRP)) TRP of DM i at e for state s is defined as the set $\{t \mid t \in TRP(e)_i(s)\}$ if and only if $t\ RP(e)_{ij}\ s$ for $\forall j \in N$.

According to attitudes of DM i for all DMs in conflict, we can obtain n (the quantity of DMs) RP sets. DM i prefers the states in their intersection to initial state. For instance, in Fig. 1, we know that there are three DMs (i, j, k). According to attitudes of every DM, we can obtain three RP sets. Then, the states in their intersection are preference of DM i.

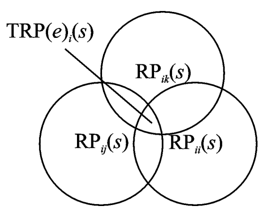

Fig. 1 Venn diagram for TRP

Matrix representation of total relational preference is

$$\boldsymbol{P}_i^{RT} = \boldsymbol{P}_{ii}^{R} \circ \boldsymbol{P}_{ij}^{R} \circ \boldsymbol{P}_{ik}^{R}$$

where "$\circ$" denotes the Hadamard product.

Definition 8 (Total relational reply (TRR)) TRP list of DM i at e for state s is defined as the set $\{t \mid t \in TRR(e)_i(s)\}$ if and only if $t \in R_i(s)$ and $t\ TRP(e)_i\ s$ (See Fig. 2).

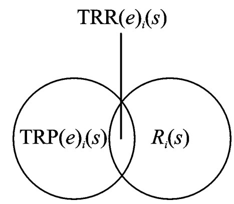

Fig. 2 Venn diagram for TRR

Matrix representation of total relational reply is

$$\boldsymbol{M}_i^{R+} = \boldsymbol{J}_i \circ \boldsymbol{P}_i^{RT}$$

Definition 9 (Relational less preferred or equally preferred states) For $s, t \in S$, if $t \notin TRP(e)_i(s)$, $t \in R\varphi^{\simeq}(e)_i(s)$. Here, DM i prefer state s to the states in $R\varphi^{\simeq}(e)_i(s)$.

Matrix representation of relational less preferred or equally preferred states is

$$\boldsymbol{P}_i^{R-} = \boldsymbol{E} - \boldsymbol{P}_i^{RT}$$

1.3 Logical representation of solution concepts

Definition 10 (Relational Nash stability (RNash)) If $TRR(e)_i(s) = \varnothing$, $s \in S_i^{RNash(e)}$.

When the set of total relational reply is empty for DM, the initial state is stable according to RNash.

Definition 11 (Relational general metarationality (RGMR)) If for all $x \in TRR(e)_i(s)$, and $R_j(x) \cap R\varphi^{\simeq}(e)_i(s) \neq \varnothing$, $s \in S_i^{RGMR(e)}$. Here, if all possible total relational replies of DM are sanctioned by opposing DM's moves, the initial state is stable according to RGMR.

Definition 12 (Relational symmetric metarationality (RSMR)) If for all $x \in TRR(e)_i(s)$, there exist $y \in R_j(x) \cap R\varphi^{\simeq}(e)_i(s)$ and $z \in R\varphi^{\simeq}(e)_i(s)$ for all $z \in R_i(y)$, $s \in S_i^{RSMR(e)}$. Here, SMR is based on GMR. If all possible total relational replies for DM are sanctioned by opposing DM's moves and DM cannot escape from it, the initial state is stable according to RSMR.

Definition 13 (Relational sequential stability (RSEQ)) If for all $x \in TRR(e)_i(s)$ and

$\mathrm{TRR}(e)_j(x) \cap R\varphi^{\simeq}(e)_i(s) \neq \varnothing, s \in S_i^{\mathrm{RSEQ}(e)}$. Here, if all possible total relational replies for DM are sanctioned by opposing DM′s total relational reply, the initial state is stable according to RSEQ.

1.4 Matrix representation of solution concepts

Detail proofs of below four theorems for four basic stabilities are in Ref. [10].

Theorem 1 (Matrix form of relational Nash Stability): State s is stable for DM i if and only if $\boldsymbol{e}_s^{\mathrm{T}} \cdot \boldsymbol{M}_i^{R+} = \boldsymbol{0}^{\mathrm{T}}$. Here T denotes matrix transpose and $\boldsymbol{e}_s^{\mathrm{T}}$ is the transpose of the sth standard basis vector of the m-dimensional Euclidean space.

Theorem 2 (Matrix form of relational general metarationality) Let $\boldsymbol{M}_i^{\mathrm{RGMR}} = \boldsymbol{M}_i^{R+}(\boldsymbol{E} - \mathrm{sign}(\boldsymbol{J}_j \cdot (\boldsymbol{P}_i^{R-})^{\mathrm{T}}))$, if $M_i^{\mathrm{RGMR}}(s,s) = 0$, $s \in S_i^{\mathrm{RGMR}}$.

Theorem 3 (Matrix form of relational symmetric metarationality) Let $\boldsymbol{M}_i^{\mathrm{RSMR}} = \boldsymbol{M}_i^{R+}(\boldsymbol{E} - \mathrm{sign}(\boldsymbol{J}_j \cdot \boldsymbol{G}))$, where $\boldsymbol{G} = (\boldsymbol{P}_i^{R-})^{\mathrm{T}} \circ (\boldsymbol{E} - \mathrm{sign}(\boldsymbol{J}_i \cdot (\boldsymbol{P}_i^{R+})^{\mathrm{T}})$, and if $M_i^{\mathrm{RSMR}}(s,s) = 0$, $s \in S_i^{\mathrm{RSMR}}$.

Theorem 4 (Matrix form of relational Sequential stability) Let $\boldsymbol{M}_i^{\mathrm{RSEQ}} = \boldsymbol{M}_i^{R+}(\boldsymbol{E} - \mathrm{sign}(\boldsymbol{M}_j^{R+} \cdot (\boldsymbol{P}_i^{R-})^{\mathrm{T}}))$, if $M_i^{\mathrm{RSEQ}}(s,s) = 0$, $s \in S_i^{\mathrm{RSEQ}}$.

Corollary 1 The correlation definitions without attitudes among DMs are a special case of the definitions of attitudes among DMs. If DM has a positive attitude for himself and an indifferent attitude for other DMs, the correlation stability definitions of attitude degenerate to the general stability definitions.

In general, the UI list is a subset of the reachable list and includes all states which are more preferred than the starting state for DM i (See Fig. 3).

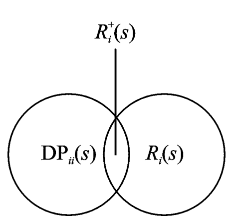

Fig. 3 Venn diagram for UI

Proof In Definition 14, because DM has a positive attitude for himself and an indifferent attitude for others, $\mathrm{TRP}(e)_i(s) = \mathrm{DP}_{ii}(s) \cap I_{ij}(s) \cap I_{ik}(s) \cdots = \mathrm{DP}_{ii}(s)$, then $R\varphi^{\simeq}(e)_i(s) = \varphi_i^{\simeq}(s)$, $\mathrm{TRR}(e)_i(s) = R_i^+(s)$. Therefore, all the stability definitions above degenerate to the definitions without attitude.

Definition 14 (Unilateral improvement (UI) list for a DM) The UI list of DM i is defined as the set $\{t \mid t \in R_i^+(s)\}$ if and only if $t \in R_i(s)$, $t\ \mathrm{DP}_{ii}\ s$.

2 Stabilities of Attitude Based on Options

2.1 Logical representation and matrix representation of attitude based on options

According to option prioritization[11], if there are n DMs, every DM has some options. And option statement of DM i is denoted by $L_i (i=1,2,\cdots,n)$. According to L_i, we can obtain the preference of DM i, denoted by $P_i (i=1,2,\cdots,n)$.

Definition 15 (Positive attitude option statements) If DM i has a positive attitude for DM j, DM i likes option statements of DM j, denoted by $L_i(e_{ij}=+)=L_j$. Here, DM i has a positive attitude for DM j, and option statements of DM j are good for DM j. So option statements of DM i in positive attitude for DM j are same to option statements of DM j.

Definition 16 (Negative attitude option statements) If DM i has a negative attitude for DM j, DM i likes the opposite of option statements of DM j, denoted by $L_i(e_{ij}=-)=-L_j$. Here, DM i has a negative attitude for DM j, and the opposite of option statements of DM j are bad for DM j, so option statements of DM i in negative attitude for DM j are same to the opposite of option statements of DM j.

Definition 17 (Indifferent attitude option statements) If DM i has an indifferent attitude for DM j, DM i does not care his option statements in this attitude, denoted by $L_i(e_{ij}=0)=I$.

Definition 18 (Attitude option statements)

$$L_{ij} = \begin{cases} L_j & e_{ij}=+ \\ -L_j & e_{ij}=- \\ I & e_{ij}=0 \end{cases}$$

where L_{ij} denotes the option statements of DM i

at corresponding attitude.

Definition 19 (Attitude preference) According to L_{ij}, we can obtain the preference of DM i, denoted by T_{ij}. For $s,t \in S$ and $i \in N$, $t \in T_{ij}(s)$ if and only if $t >_i s$ satisfies T_{ij}.

Matrix representation of attitude preference: For $i \in N$, T_{ij} is a $|S| \times |S|$ 0-1 matrix defined by

$$T_{ij}(s,t) = \begin{cases} 1 & t >_i s \\ 0 & \text{otherwise} \end{cases}$$

where $|S|$ is the quantity of states.

Definition 20 (Total attitude preference): For $s,t \in S$, and $i \in N$, $t \in T_i^+(s)$ if and only if $t \in T_{ij}(s)$ for all $j \in N$, then we call total attitude preference.

Matrix representation of total attitude preference is

$$\boldsymbol{T}_i^+ = \boldsymbol{T}_{ii} \circ \boldsymbol{T}_{ij} \circ \boldsymbol{T}_{ik} \cdots$$

If we want to obtain the set of preferred states, we need to consider all preference for DM in different attitudes. The states in intersection of them are what DM prefers.

If there are three DMs (i, j, k), the intersection of all attitude preferences of DM i is the total attitude preference for DM i. Of course, if there are n DMs, the total attitude preference for DM i can be obtained in the same way.

Definition 21 (Set of less or equally preferred states at total attitude) For $s,t \in S$ and $i \in N$, $t \in T_i^{-=}(s)$ if and only if $t \notin T_i^+(s)$.

Matrix representation of set of less or equally preferred states at total attitude is

$$\boldsymbol{T}_i^{-=} = \boldsymbol{E} - \boldsymbol{T}_i^+$$

According to the definition, we can know that $T_i^{-=}(s)$ is the supplementary set of $T_i^+(s)$.

Definition 22 (Unilateral improvement list for a DM at attitude) For $s,t \in S$ and $i \in N$, $t \in T_i^*(s)$ if and only if $t \in R_i(s)$ and $t \in T_i^+(s)$.

Matrix representation of unilateral improvement list for a DM at attitude is

$$\boldsymbol{T}_i^* = \boldsymbol{J}_i \circ \boldsymbol{T}_i^+$$

where $\boldsymbol{J}_i$ is reachable matrix. And the intersection of reachable list and total attitude preference is the reachable and improvement list at attitude, then matrix representation

$$\boldsymbol{T}_i^* = \boldsymbol{J}_i \circ \boldsymbol{T}_i^+$$

2.2 Logical representation and matrix representation of solution concepts

Definition 23 (Relational Nash stability (RNash)) If $T_i^*(s) = \varnothing$, $s \in S_i^{\mathrm{RNash}}$. Here, if the unilateral improvement list for DM at attitude is empty, the initial state is RNash stable for DM.

Theorem 5 (Matrix form of relational Nash stability) For $i \in N, s \in S$, if $\boldsymbol{e}_s^{\mathrm{T}} \cdot \boldsymbol{T}_i^* = \boldsymbol{0}^{\mathrm{T}}$, $s \in S_i^{\mathrm{RNash}}$.

Proof It is obvious that $T_i^*(s) = \varnothing$, for i if and only if $\boldsymbol{e}_s^{\mathrm{T}} \cdot \boldsymbol{T}_i^* = \boldsymbol{0}^{\mathrm{T}}$.

Definition 24 (Relational general metarationality (RGMR)) If for all $h \in T_i^*(s)$, and $R_j(h) \cap T_i^{-=}(s) \neq \varnothing$, $s \in S_i^{\mathrm{RGMR}}$. Here, if all possible unilateral improvement moves for DM at attitude are sanctioned by opposing DM′s moves, the initial state is RGMR stable for DM.

Let $V_i^{\mathrm{RGMR}}(h)$ denote RGMR stability function of DM i, then the following theorem provides an efficient tool for calculating RGMR stability.

Theorem 6 (Matrix form of relational general metarationality) For $i,j \in N$, and $s,h \in S$, let

$$V_i^{\mathrm{RGMR}}(h) = 1 - \mathrm{sign}[(\boldsymbol{e}_h^{\mathrm{T}} \boldsymbol{J}_j) \cdot (\boldsymbol{e}_s^{\mathrm{T}} \boldsymbol{T}_i^{-=})^{\mathrm{T}}]$$

$$(\boldsymbol{e}_s^{\mathrm{T}} \cdot \boldsymbol{T}_i^*) \circ \boldsymbol{V}_i^{\mathrm{RGMR}} = \boldsymbol{0}^{\mathrm{T}} \quad s \in S_i^{\mathrm{RGMR}} \tag{1}$$

Proof h denotes the h-st element of $\boldsymbol{V}_i^{\mathrm{RGMR}}$ with the m-dimensional Euclidean space (m denotes the quantity of states).

From Definition 24, we can know that Eq. (1) is equivalent to

$$V_i^{\mathrm{RGMR}}(h) = 0 \quad \forall h \in T_i^*(s) \tag{2}$$

It′s obvious that (2) is equivalent to $(\boldsymbol{e}_h^{\mathrm{T}} \boldsymbol{J}_j) \cdot (\boldsymbol{e}_s^{\mathrm{T}} \boldsymbol{T}_i^{-=})^{\mathrm{T}} \neq 0$ for $\forall h \in T_i^*(s)$, which implies that for all $h \in T_i^*(s)$, and $R_j(h) \cap T_i^{-=}(s) \neq \varnothing$.

Definition 25 (Relational symmetric metarationality (RSMR)) If for all $h \in T_i^*(s)$, there exist $y \in R_j(h) \cap T_i^{-=}(s)$ and $z \in T_i^{-=}(s)$ for all $z \in R_i(y)$, $s \in S_i^{\mathrm{RSMR}}$. Here, If all possible unilateral improvement moves for DM at attitude are sanctioned by opposing DM′s moves and DM cannot escape from it, the initial state is RSMR stable for DM.

Theorem 7 (Matrix form of relational symmetric metarationality) For $i,j \in N$, and $s,h \in S$, let

$$V_i^{\mathrm{RSMR}}(h)=1-\mathrm{sign}([(\boldsymbol{e}_h^{\mathrm{T}}\boldsymbol{J}_j)\circ(\boldsymbol{e}_s^{\mathrm{T}}\boldsymbol{T}_i^{-=})]\cdot G)$$

$$G=[\boldsymbol{e}_s^{\mathrm{T}}\boldsymbol{E}-\mathrm{sign}((\boldsymbol{e}_s^{\mathrm{T}}\boldsymbol{T}_i^{+})\cdot \boldsymbol{j}_i^{\mathrm{T}})]^{\mathrm{T}}$$

$$(\boldsymbol{e}_s^{\mathrm{T}}\cdot \boldsymbol{T}_i^{*})\circ \boldsymbol{V}_i^{\mathrm{RSMR}}=\boldsymbol{0}^{\mathrm{T}}\quad s\in S_i^{\mathrm{RSMR}} \tag{3}$$

Proof h denotes the h-st element of $\boldsymbol{V}_i^{\mathrm{RSMR}}$ with the m-dimensional Euclidean space.

According to Definition 25, we can know that Eq. (3) is equivalent to

$$V_i^{\mathrm{RSMR}}(h)=0\quad \forall h\in T_i^{*}(s) \tag{4}$$

It is obvious that Eq. (3) is equivalent to $[(\boldsymbol{e}_h^{\mathrm{T}}\boldsymbol{J}_j)\circ(\boldsymbol{e}_s^{\mathrm{T}}\boldsymbol{T}_i^{-=})]\cdot[\boldsymbol{e}_s^{\mathrm{T}}\boldsymbol{E}-\mathrm{sign}((\boldsymbol{e}_s^{\mathrm{T}}\boldsymbol{T}_i^{+})\cdot \boldsymbol{j}_i^{\mathrm{T}})]^{\mathrm{T}} \neq 0$ for $\forall h\in T_i^{*}(s)$.

Obviously, for all $h\in T_i^{*}(s)$, there exists $y\in R_j(h)\cap T_i^{-=}(s)$ and $z\in T_i^{-=}(s)$ for all $z\in R_i(y)$.

Definition 26 (Relational sequential stability (RSEQ)) If for all $h\in T_i^{*}(s)$, and $T_j^{*}(h)\cap T_i^{-=}(s)\neq\varnothing$, $s\in S_i^{\mathrm{RSEQ}}$. Here, if all possible unilateral improvement moves for DM at attitude are sanctioned by opposing DM′s unilateral improvement moves at attitude, the initial state is RSEQ stable for DM.

Theorem 8 (Matrix form of relational sequential stability) For $i,j\in N$, and $s,h\in S$, let

$$V_i^{\mathrm{RSEQ}}(h)=1-\mathrm{sign}[(\boldsymbol{e}_h^{\mathrm{T}}\boldsymbol{T}_j^{*})\cdot(\boldsymbol{e}_s^{\mathrm{T}}\boldsymbol{T}_i^{-=})^{\mathrm{T}}]$$

$$(\boldsymbol{e}_s^{\mathrm{T}}\cdot \boldsymbol{T}_i^{*})\circ \boldsymbol{V}_i^{\mathrm{RSEQ}}=\boldsymbol{0}^{\mathrm{T}}\quad s\in S_i^{\mathrm{RSEQ}} \tag{5}$$

Proof h denotes the h-st element of $\boldsymbol{V}_i^{\mathrm{RSEQ}}$ with the m-dimensional Euclidean space.

From Definition 26, we can know that Eq. (5) is equivalent to

$$V_i^{\mathrm{RSEQ}}(h)=0\quad \forall h\in T_i^{*}(s) \tag{6}$$

It′s obvious that (6) is equivalent to $(\boldsymbol{e}_h^{\mathrm{T}}\boldsymbol{T}_j^{*})\cdot(\boldsymbol{e}_s^{\mathrm{T}}\boldsymbol{T}_i^{-=})^{\mathrm{T}}\neq 0$ for $\forall h\in T_i^{*}(s)$, which implies that for all $h\in T_i^{*}(s)$, and $T_j^{*}(h)\cap T_i^{-=}(s)\neq\varnothing$.

3 Process Conflict for Aircraft Manufacturing

Commercial Aircraft Corporation of China Ltd. (COMAC) is the main company responsible for large-scale passenger aircraft projects within the major national large-scale aircraft projects. The C919 aircraft developed by COMAC is the second large domestic passenger aircraft. Due to the complexity of C919′s parts supply, the conflict among technology, resources and process will inevitably occur in the development process. For instance, the process conflict for the delivery of parts on time between supplier and manufacturer often happens.

Chengdu Aircraft Corporation(CAC), one of the main suppliers of COMAC, is mainly responsible for the production of large aircraft flying nose and the whole machine signs. Due to the impact of the earthquake, the supply chain of CAC was broken and the CAC could not follow the original plan for commercial flight delivery. Consequently, the overall production schedule was delayed because of the delay in the delivery of flying nose. While COMAC still expected CAC to deliver the flight nose on time, CAC decided to postpone the delivery. Finally, the two sides deadlocked.

3.1 Decision makers, options and feasible states

As shown in Table 2, there are two decision makers: CAC (DM1) and COMAC (DM2). There are four options: Delayed delivery (Labeled 1), Overtime production (Labeled 2), Requirements on time delivery (Labeled 3) and Allowed to delay delivery (Labeled 4). Lastly, we obtain 9 feasible states after eliminating the unreasonable states.

Table 2 Feasible states in the process conflict of aircraft manufacturing

DM	Option	Feasible state								
1	(1) Delayed delivery	N	N	N	N	N	N	Y	Y	Y
	(2) Overtime production	N	N	N	Y	Y	Y	N	N	N
2	(3) Requirements on time delivery	N	N	Y	N	N	Y	N	N	Y
	(4) Allowed to delay delivery	N	Y	N	N	Y	N	N	Y	N
	Label	S1	S2	S3	S4	S5	S6	S7	S8	S9

3.2 Graph model of conflict

In Fig. 4, the CAC's moves and COMAC's moves are presented in Fig. 4, which depicts the movements that DM unilaterally controls between two states. According to the graph model, we obtain the reachable list for every DM(Table 3).

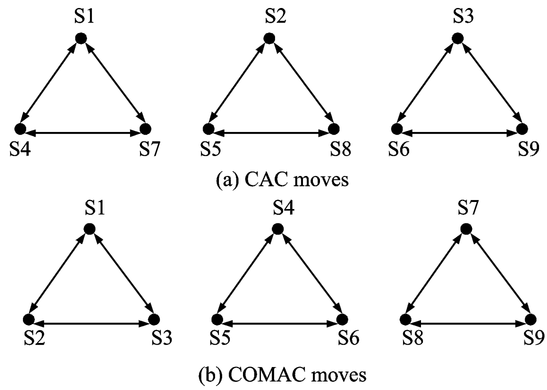

Fig. 4 Graph model in process conflict of aircraft manufacturing

Table 3 Reachable list of DMs in process conflict of aircraft manufacturing

State	$R_1(s)$	$R_2(s)$
S1	S4, S7	S2, S3
S2	S5, S8	S1, S3
S3	S6, S9	S1, S2
S4	S1, S7	S5, S6
S5	S2, S8	S4, S6
S6	S3, S9	S4, S5
S7	S1, S4	S8, S9
S8	S2, S5	S7, S9
S9	S3, S6	S7, S8

3.3 Option statements for DMs

For CAC, he hopes COCMC can allow to delay delivery, because he needs to spend a huge price for overtime production. Therefore, the option statements from most preferred to least preferred for CAC is 4, 1, −2, −3 (See the left of Table 4). Here, 4 means that CAC likes option 4, and −2 means that CAC likes the opposite of option 2.

COMAC hopes CAC can deliver on time, because if the flight cannot finish in the appointed time, COMAC needs to pay a huge amount of liquidated damages for client. Therefore, the option statements from the most preferred to the least preferred for COMAC is 2, 3, −4, −1(See the right of Table 4).

Table 4 Option statements of DMs in the process conflict of aircraft manufacturing

DM	L_1	DM	L_2
COMAC	4(Y)	CAC	2(Y)
	1(Y)		3(Y)
	−2(N)		−4(N)
	−3(N)		−1(N)

3.4 Attitude and preference

Attitude (Table 5): In order to maintain the friendly corporation with COMAC, CAC may spend a great price to deliver on time. So CAC has a positive attitude for himself and a positive attitude for COMAC. For COMAC, out of the consideration of the huge compensation for client, COMAC hopes CAC can deliver on time. So COMAC has an indifferent attitude for CAC and a positive attitude for himself.

Table 5 Attitude among DMs in process conflict of aircraft manufacturing

DM	CAC	COMAC
CAC	+	+
COMAC	0	+

Attitude option statements and attitude preference:

From Table 5, CAC has a positive attitude for himself and a positive attitude for COMAC. So CAC's option statements at positive attitude for himself are the same as his own option statements that is beneficial for him. And CAC's option statements at positive attitude for COMAC are the same as COMAC's option statements that are good for COMAC.

COMAC has an indifferent attitude for CAC and a positive attitude for himself. So COMAC does not care his option statements at an indifferent attitude for CAC. And COMAC's option statements at positive attitude for himself are the same as his own option statements that are beneficial for himself.

According to the attitudes among DMs, we obtain the attitude option statements for DMs shown in Table 6. Then we also obtain the attitude preferences for DMs by attitude option statements displayed in Table 7.

Table 6 Attitude option statements for DMs in process conflict of aircraft manufacturing

DM	$L_{11} = L_1$	$L_{12} = L_2$
CAC	4(Y)	2(Y)
	1(Y)	3(Y)
	−2(N)	−4(N)
	−3(N)	−1(N)
DM	$L_{21} = I$	$L_{22} = L_2$
COMAC	Null	2(Y)
		3(Y)
		−4(N)
		−1(N)

Table 7 Attitude preference for DMs in process conflict of aircraft manufacturing

DM	Attitude	Attitude preference (T_{ij})
CAC	$e_{11} = + \quad T_{11}$	S8>S2>S5>S7>S9>S1>S3>S4>S6
	$e_{12} = + \quad T_{12}$	S6>S4>S5>S3>S9>S1>S7>S2>S8
COMAC	$e_{21} = 0 \quad T_{21}$	Null
	$e_{22} = + \quad T_{22}$	S6>S4>S5>S3>S9>S1>S7>S2>S8

Total attitude preference: if we want to obtain the total attitude preference for DMs, we need to consider all attitude preferences. The intersection of all attitude preferences is total attitude preference shown in Table 8.

3.5 Stability analysis

According to the above reachable list, total attitude preference and the definitions of RNash, RGMR, RSMR, RSEQ, we can obtain the equilibrium of this conflict shown in Table 9, in which "√"denotes that the state is stable for DM under the corresponding stability, and Eq the equilibrium for this conflict.

Table 8 Total attitude preference for DMs in process conflict of aircraft manufacturing

State	$\boldsymbol{T}_1^+ = \boldsymbol{T}_{11} \circ \boldsymbol{T}_{12}$	$\boldsymbol{T}_2^+ = \boldsymbol{T}_{21} \circ \boldsymbol{T}_{22}$
S1	S5, S9	S3, S4, S5, S6, S9
S2	NULL	S1, S3, S4, S5, S6, S7, S9
S3	S5	S4, S5, S6
S4	NULL	S6
S5	NULL	S4, S6
S6	NULL	NULL
S7	S5	S1, S3, S4, S5, S6, S9
S8	NULL	S1, S2, S3, S4, S5, S6, S7, S9
S9	S5	S3, S4, S5, S6

From Table 9, we can find that S3, S6 and S9 are equilibrium for the four stabilities. At S3, COMAC requests CAC to deliver on time and CAC does not take any action. Obviously, S3 is a temporary equilibrium, because CAC will take some actions with the conflict evolution. At S9, COMAC requires CAC to deliver on time, but CAC chooses to delay delivery, because COMAC or CAC will change the strategy in order to maintain their friendly relations, S9 also is a temporary equilibrium. So S6 is a finally equilibrium for this conflict. At S6, CAC chooses overtime production in order to keep the friendly relationship of corporation.

We also calculate the stability without attitude called general stability shown in Table 10 in which the equilibrium is S9.

Table 9 Attitude stability analysis in process conflict of aircraft manufacturing

State	RNash			RGMR			RSMR			RSEQ		
	DM1	DM2	Eq	DM1	DM2	Eq	DM1	DM2	Eq	DM1	DM2	Eq
S1	√			√			√			√		
S2	√			√			√			√		
S3	√	√	√	√	√	√	√	√	√	√	√	√
S4	√			√	√	√	√	√	√	√		
S5	√			√	√	√	√	√	√	√		
S6	√	√	√	√	√	√	√	√	√	√	√	√
S7	√			√			√					
S8	√			√			√			√		
S9	√	√	√	√	√	√	√	√	√	√	√	√

Table 10 General stability analysis in process conflict of aircraft manufacturing

State	Nash			GMR			SMR			SEQ		
	DM1	DM2	Eq	DM1	DM2	Eq	DM1	DM2	Eq	DM1	DM2	Eq
S1												
S2				√			√			√		
S3		√			√			√			√	
S4					√			√			√	
S5				√	√	√	√	√	√	√	√	√
S6		√			√			√			√	
S7	√			√			√			√		
S8	√			√			√			√		
S9	√	√	√	√	√	√	√	√	√	√	√	√

The fact of this conflict is that CAC chooses overtime production in order to deliver on time. From this fact, it is clear to see that the equilibrium of attitude stability is much closer to the reality comparing with the equilibrium of general stability. It demonstrates that the proposed model including attitude is very effective and accurate for solving conflict.

4 Conclusions

We propose the definitions of attitude based on options in the graph model for two DMs conflicts which provide a new view of generating DM′spreference including attitude and are more convenient for users. In order to implement the stabilities efficiently, we also convert the logical definitions of attitude to a matrix representation similar to that in Xu et al[15-17]. Finally, the theory developed in this paper is used to analyze the process conflict for aircraft manufacturing, which helps manufacturers and suppliers resolve the process conflict for aircraft manufacturing, and ensures that aircrafts can be produced on time.

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Prof. **Xu Haiyan** received the B. A. Sc. degree in pure mathematics from Nanjing University, Nanjing, China, and the M. Sc. degree in combinatorics and optimization and Ph. D. in systems design engineering from University of Waterloo, Waterloo, Ontario, Canada, in 1984, 2006, and 2009, respectively. She is a professor with the Faculty of Economics and Management, Nanjing University of Aeronautics and Astronautics. She has over 30 academic papers published in foreign periodicals and important international conferences, more than 10 papers have been published in SCI/SSCI journals, such as European Journal of Operational Research, Discrete Applied Mathematics, and IEEE Transactions on Systems, Man, and Cybernetics, Part A: Systems and Humans. Her current research interests in designing mathematical models for tackling complex systems problems include the development of methods in portfolio optimization and normal algebraic techniques for conflict resolution with applications in finance, environmental management, and elsewhere.

Mr. **Xu Peng** received the B. Admin degrees in management from Tianjin University of Technology, China, in 2011 and 2015. In September 2015, he studied in the Economic and Management of Nanjing University of Aeronautics and Astronautics (NUAA) for the degree of master, and joined the Intelligent Decision and Risk Analysis Institute of NUAA. He has some published papers about the attitude analysis for the big power conflict. His research interests in analyzing the attitude of decision makers based on option prioritization and the conflict evolution in finance, warfare, and elsewhere.

Mr. **Sharafat Ali** received the M. A. Economics and MPhil in Economics in 2001 and 2015 from Bahauddin Zakariya University Multan, Pakistan and Allama Iqbal Open University Islamabad, Pakistan, respectively. He has worked as Lecturer in Economics from 2002 to 2011 and as an Assistant Professor of Economics in Higher Education Department, Government of the Punjab, Pakistan, from 2011 to 2015. He has research publications regarding financial capital flows and their impact on investment, economic growth, poverty and income inequality in Pakistan. Currently, he is a Ph. D. candidate at College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing, P. R. China. His research interest is decision and conflict analysis regarding domestic policy issues in regional economic perspective.

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