Aeroengine Nonlinear Sliding Mode Control Based on Artificial Bee Colony Algorithm

Lu Binbin, *Xiao Lingfei* *, *Chen Yuhan*

College of Energy and Power, Jiangsu Province Key Laboratory of Aerospace Power System, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, P. R. China

(Received 7 November 2015; revised 22 May 2016; accepted 3 June 2016)

Abstract: For a class of aeroengine nonlinear systems, a novel nonlinear sliding mode controller (SMC) design method based on artificial bee colony (ABC) algorithm is proposed. In view of the strong nonlinearity and uncertainty of aeroengines, sliding mode control strategy is adopted to design controller for the aeroengine. On basis of exact linearization approach, the nonlinear sliding mode controller is obtained conveniently. By using ABC algorithm, the parameters in the designed controller can be tuned to achieve optimal performance, resulting in a closed-loop system with satisfactory dynamic performance and high steady accuracy. Simulation on an aeroengine verifies the effectiveness of the presented method.

Key words: aeroengine; nonlinear control; sliding mode control (SMC); artificial bee colony (ABC) algorithm

CLC number: TP273 **Document code**: A **Article ID**: 1005-1120(2017)02-0152-11

0 Introduction

Aeroengines are a kind of strong nonlinear systems of complicated thermodynamic, while their working conditions are very poor generally. In the actual aeroengine system, there exist nonlinear unknown disturbances and nonlinear parameter perturbations, so the system needs to be controlled in order to reach good dynamic performance and robustness [1-2]. The design of controller is an essential problem in the field of aeroengines. There are several common control methods, such as PID (Proportion-intergral-derivactive) control[3], LQR (Linear quadratic regulator) control [4], H_∞ control [5], adaptive control[6], robust control[7], etc. The general linear control method cannot meet the requirement of performance in large deviation of aeroengine due to its complicated thermodynamic process and nonlinear factors. Therefone, it is necessary to study the aeroengine nonlinear control[8-9].

The designed controller based on approximate linearization cannot guarantee the system stability in large initial deviation, and this will have a negative effect on aeroengines. While the exact linearization theory based on the differential geometry can exactly solve the control problem of nonlinear system in large deviation under certain conditions [10]. Sliding mode control (SMC), also known as variable structure control (VSC) or sliding mode variable structure control (SMVSC), purposely changes the control structure according to the current system state, and then forces the system's trajectories to reach the sliding mode in finite time[11-13]. SMC has invariability when the parameters change and the disturbances exist, so it possesses great research value. Artificial bee colony (ABC) algorithm is an optimization algorithm on basis of the intelligent behavior of honey bee swarm[14]. This algorithm has advantages of easiness, simple calculation, good optimization performance and strong robustness, so it can be used to tune the parameters of the nonlinear controller of aeroengines to

* Corresponding author, E-mail address: lfxiao@nuaa.edu.cn.

How to cite this article: Lu Binbin, Xiao Lingfei, Chen Yuhan. Aeroengine nonlinear sliding mode control based on artificial bee colony algorithm[J]. Trans. Nanjing Univ. Aero. Astro., 2017,34(2):152-162.
http://dx.doi.org/10.16356/j.1005-1120.2017.02.152

obtain the optimal performance.

In recent years, many advanced design methods of nonlinear control system have been applied to aeroengines, such as nonlinear predictive control[15], nonlinear back-stepping control[16], intelligent control[17-19], nonlinear control based on a generalized Gronwall-Bellman Lemma[20], etc. At present, ABC algorithm has been used in aerospace domain. In Ref. [21], ABC algorithm is used for parametric optimization of spacecraft attitude tracking controller. In Ref. [22], ABC algorithm is used for path planning of unmanned air vehicle. For a class of affine nonlinear mathematical model of aeroengines, a nonlinear sliding mode controller based on ABC algorithm is designed.

1 Preliminaries

1.1 Exact linearization theory

Only an affine nonlinear system

$$\dot{\boldsymbol{x}} = \dot{\boldsymbol{f}}(x) + \boldsymbol{B}(x)\boldsymbol{u} + \boldsymbol{d}(t) \quad (1)$$

can be exactly linearized, where $\dot{\boldsymbol{x}}$ is a n-dimension state vector, $\boldsymbol{f}(x)$ the smooth n-dimension vector function and $\boldsymbol{f}=[f_1,\cdots,f_n]^{\mathrm{T}}$, $\boldsymbol{B}(x)$ the $n\times m$ function matrix and $\boldsymbol{B}=[b_1,\cdots,b_m]$, $\boldsymbol{u}$ the m-dimension control vector, and $\boldsymbol{d}(t)$ the n-dimension interference vector. $\boldsymbol{d}(t)$ satisfies the matching condition

$$\mathrm{rank}(\boldsymbol{B},\boldsymbol{d}) = \mathrm{rank}(\boldsymbol{B}) \quad (2)$$

Exact linearization theory is based on the differential geometry. Some definitions are clarified as follows:

(1) Lie derivative and Lie bracket

Lie derivative of two functions, $\boldsymbol{q}$ and $\boldsymbol{f}$ can be defined as

$$L_f\boldsymbol{q} = (\nabla \boldsymbol{q})\boldsymbol{f} \quad (3)$$

where $\boldsymbol{q}$ is a continuous and smooth scalar function, $\boldsymbol{q}=\boldsymbol{q}(x)$, $\boldsymbol{f}$ the smooth n-dimension vector function and $\boldsymbol{f}=[f_1(x),\cdots,f_n(x)]^{\mathrm{T}}$, $\nabla\boldsymbol{q}$ the Jacobian matrix of $\boldsymbol{q}$ and $L_f^0\boldsymbol{q}=\boldsymbol{q}$, $L_f^i\boldsymbol{q}=L_f(L_f^{i-1}\boldsymbol{q})=(\nabla L_f^{i-1}\boldsymbol{q})\boldsymbol{f}$, $i\geqslant 2$. Besides, Lie bracket of $\boldsymbol{q}$ and $\boldsymbol{f}$ can be defined as $ad_f\boldsymbol{b}=[\boldsymbol{f},\boldsymbol{b}]=(\nabla\boldsymbol{b})\boldsymbol{f}-(\nabla\boldsymbol{f})\boldsymbol{b}$, $ad_f^0\boldsymbol{b}=\boldsymbol{b}$, $ad_f^i\boldsymbol{b}=[\boldsymbol{f},ad_f^{i-1}\boldsymbol{b}]$, $i\geqslant 1$.

(2) Involutiveness

A collection of linearly independent vector functions $\{G_1,\cdots,G_m\}$ on $\Omega\subset R^n$ is involutive, if it satisfies

$$[G_i,G_j]\in \mathrm{span}\{G_1,\cdots,G_m\} \qquad i,j=1,\cdots,m \quad (4)$$

Combining the conditions for exact linearization of nonlinear system[10], we analyze sufficient and necessary conditions for exact linearization of the affine nonlinear system (Eq. (1)) as follows (defining a set $G_i(x)=\{ad_f^k b_j: 0\leqslant k\leqslant i, 1\leqslant j\leqslant m, 0\leqslant i\leqslant n-1\}$):

(1) Rank$(\boldsymbol{B},\boldsymbol{d})$=rank$(\boldsymbol{B})$, namely the matching condition of interference;

(2) When $0\leqslant i\leqslant n-1$, the rank of $G_i(x)$ does not change with x;

(3) When $0\leqslant i\leqslant n-2$, each pair of $G_i(x)$ is are involutive;

(4) Rank$(G_{n-1}(x))=n$.

1.2 ABC algorithm

ABC algorithm is a random search algorithm based on group cooperation. It solves the optimization of target problems by imitating the honey process of bee colony. ABC algorithm model is systematically proposed by a Turkish academic Karaboga D in 2005, and used to solve the optimal value of the objective multivariate function. It can be found that bees have high efficiency in honey process and show an intelligent behavior. In the honey process, the scouts in the bee colony are in charge of searching the food sources, and become the employed bees after finding a food source. The employed bees fly to the waiting area, and share the information with the onlooker bees by their own dance movements. The range of the dance movements reflects the quality of the food sources. Onlooker makes decisions to choose a food source by comparing with the dance movements. The efficiency of gathering honey can be greatly increased by the mechanism that food sources of high quality can attract more onlookers. In ABC algorithm, a solution of optimization problem corresponds to a food source, it means that the higher quality the food source has, the better the solution will be. The solution of problem is optimized by ABC, which will search better solutions.

Main feature of ABC algorithm is that special information of problems is not necessary, instead, this algorithm only needs to compare with the advantages and disadvantages of the problems, enables each individual to have local optimization-oriented operation, and finds the global optimum value in the colony.

ABC algorithm model contains three elements of food source, employed bees, and unemployed bees. And the unemployed bees can be divided into onlookers and scouts. In this paper, employed bees account for half of the colony size. ABC algorithm flowchart is shown in Fig. 1.

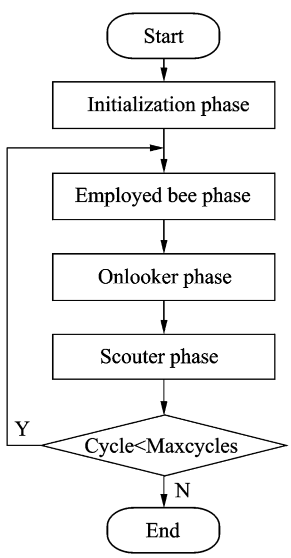


Fig. 1 Flowchart of ABC algorithm

ABC algorithm process includes initialization phase, employed bee phase, onlooker phase, and scouter phase.

In the initialization phase, scouts initialize the solution vectors by

$$X_{mi} = \text{lower}(i) + \text{rand} \times (\text{upper}(i) - \text{lower}(i)) \quad (5)$$

where rand is a random variable and rand $\in [0, 1]$, m the integer between 1 and half of the colony size, and i the integer between 1 and the dimension of solution vector, lower(i) and upper(i) are the lower limit and upper limit of X_{mi}, respectively.

In the employed phase, the employed bees find the previous food source or solution vector by memory, and conduct neighborhood search by

$$V_{mj} = X_{mj} + \varphi_{mj}(X_{mj} - X_{kj}) \quad (6)$$

where k is the random variable and $k \neq m$; φ_{mj} the random variable, $\varphi_{mj} \in [-a, a]$ and $a = 1$ in this paper; and j the random integer, $j \in [1, D_{im}]$ and D_{im} the number of parameters or the dimension of solution vector. If V_{mj} exceeds the value range, it is substituted by the nearest limit value. Next, the greedy selection mechanism is used to choose better solution vector, that is, if the new solution vector is better than the old one, the solution vector is updated.

In the onlooker phase, the probability of an employed bee being selected is calculated by

$$P_m = \frac{\text{fit}(\boldsymbol{X}_m)}{\sum_{i=1}^{\text{Colony size}/2} \text{fit}(\boldsymbol{X}_i)} \quad (7)$$

where P_m are the probability of selection of employed bee, fit ($\boldsymbol{X}_m$) the fitness values for individual $\boldsymbol{X}_m$, and colony size the individual number. The onlookers conduct neighborhood search by Eq. (6) after selecting a food source.

In the scouter phase, If the number of updates for a employed bee′s solution vector reaches the limit, the employed bee transforms into a scouter again, and initializes the solution vector by Eq. (5).

The optimal solution is obtained while ABC algorithm execute the loops for the pre-set max times. ABC algorithm possesses advantages of less control parameters, easiness, simple calculation, good optimization performance and strong robustness, so it can self-tune the parameters of aeroengine nonlinear controller to obtain a optimal performance.

2 ABC-Based Aeroengine Nonlinear Sliding Mode Controller Design

There are lots of aeroengines that can be described by the affine nonlinear model (Eq. (1))[9,16,20], For example, the aeroengine nonlinear model given in Ref. [20] is

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -4.1476 & 1.4108 \\ 0.2975 & -3.1244 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 12x_1^2 - x_2^2 \\ -1.7x_1^2 + x_2^2 \end{bmatrix} + \begin{bmatrix} 0.2491 \\ 0.2336 \end{bmatrix} \boldsymbol{u}$$

$$\boldsymbol{y}=\begin{bmatrix}8.7379 & 0\\ 3.3033 & 3.8025\\ 2.1940 & 2.5749\end{bmatrix}\boldsymbol{x} \tag{8}$$

where $\boldsymbol{u}$ is the increment of fuel flow and $\boldsymbol{u}=\Delta W_f$; $\boldsymbol{x}=[x_1,x_2]^{\mathrm{T}}$, and x_1 and x_2 are the intermediate variables; $\boldsymbol{y}=[PCN2R\ \ P_{56}/P_{25}\ \ P_{16}/P_{56}]^{\mathrm{T}}$, and each component of $\boldsymbol{x}$ and $\boldsymbol{y}$ is a normalized relative increment and saves the increment sign Δ. In $\boldsymbol{y}$, $PCN2R$ indicates the percent corrected fan speed, P_{56} the high-pressure turbine exit pressure, P_{25} the compressor inlet pressure, and P_{16} the bypass duct pressure. Let

$$\boldsymbol{f}(x)=\begin{bmatrix}-4.1476 & 1.4108\\ 0.2975 & -3.1244\end{bmatrix}\boldsymbol{x}+\begin{bmatrix}12x_1^2-x_2^2\\ -1.7x_1^2+x_2^2\end{bmatrix}=\begin{bmatrix}12x_1^2-x_2^2-4.1476x_1+1.4108x_2\\ -1.7x_1^2+x_2^2+0.2975x_1-3.1244x_2\end{bmatrix} \tag{9}$$

and

$$\boldsymbol{B}=\begin{bmatrix}0.2491\\ 0.2336\end{bmatrix} \tag{10}$$

then

$$\dot{\boldsymbol{x}}=\boldsymbol{f}(x)+\boldsymbol{Bu} \tag{11}$$

On this basis, add an interference $d=Dg(t)$ into it, where $g(t)$ is a scalar function. Therefore, the aeroengine nonlinear system with interference can be represented as

$$\dot{\boldsymbol{x}}=\boldsymbol{f}(x)+\boldsymbol{Bu}+\boldsymbol{d}(t) \tag{12}$$

just like Eq. (1).

In the following, a novel nonlinear sliding mode controller design method for aeroengines which are in the form of Eq. (1), will be given based on ABC algorithm.

First of all, a smooth m-dimension vector function $\boldsymbol{h}$ is introduced, where $\boldsymbol{h}=[h_1,\cdots,h_m]^{\mathrm{T}}$. The derivative of h_i ($1\leqslant i\leqslant m$) along system (Eq. (1)) is represented as

$$\dot{h}_i=\nabla h_i\cdot(\boldsymbol{f}+\boldsymbol{Bu}+\boldsymbol{d})=L_f h_i+\sum_{j=1}^{m}L_{b_j}h_i u_j+L_d h_i \tag{13}$$

When $L_{b_j}h_i=0$, by the matching condition (Eq. (2)), it can be concluded that $L_d h_i=0$, and $L_d(L_f^k h_i)=0$ as long as $L_{b_j}(L_f^k h_i)=0$. At this point, the second derivative of h_i along system (Eq. (1)) is expressed as

$$\ddot{h}_i=L_f^2 h_i+\sum_{j=1}^{m}L_{b_j}(L_f h_i)u_j+L_d(L_f h_i) \tag{14}$$

When there exists a minimum integer r_i which satisfies

$$L_{b_j}(L_f^k h_i)=0$$
$$1\leqslant i,j\leqslant m;0\leqslant k\leqslant r_i-1 \tag{15}$$

and j satisfies

$$L_{b_j}(L_f^{r_i-1}h_i)\neq 0 \tag{16}$$

then system (Eq. (1)) has relative orders $\{r_1,\cdots,r_m\}$ and total relative order $r=r_1+\cdots+r_m$.

For each h_i

$$\begin{cases}h_i^{(k)}=L_f^k h_i\\ h_i^{(r_i)}=L_f^{r_i}h_i+\displaystyle\sum_{j=1}^{m}L_{b_j}(L_f^{r_i-1}h_i)u_j+L_d(L_f^{r_i-1}h_i)\end{cases} \tag{17}$$

The coefficient matrix of input vector $\boldsymbol{u}$ in Eq. (17), namely the decoupling matrix, can be represented as

$$\boldsymbol{E}(x)=\begin{bmatrix}L_{b_1}(L_f^{r_1-1}h_1) & \cdots & L_{b_m}(L_f^{r_1-1}h_1)\\ \vdots & \ddots & \vdots\\ L_{b_1}(L_f^{r_m-1}h_m) & \cdots & L_{b_m}(L_f^{r_m-1}h_m)\end{bmatrix} \tag{18}$$

Assuming $\boldsymbol{E}(x)$ is reversible, $\boldsymbol{P}(x)=[L_f^{r_1}h_1,\cdots,L_f^{r_m}h_m]^{\mathrm{T}}$ and $\boldsymbol{Q}(x)=[L_d(L_f^{r_1-1}h_1),\cdots,L_d(L_f^{r_m-1}h_m)]^{\mathrm{T}}$, the nonlinear state transformation

$$\begin{cases}z_i^j=T_i^k(x)=L_f^j h_i & 0\leqslant i\leqslant m;0\leqslant j\leqslant r_i-1\\ z_k=T_k(x) & r+1\leqslant k\leqslant n\end{cases} \tag{19}$$

and the input transformation

$$\boldsymbol{u}=\boldsymbol{E}^{-1}(x)[\boldsymbol{v}-\boldsymbol{P}(x)-\boldsymbol{Q}(x)] \tag{20}$$

are applied, then the system described by Eq. (1) can be represented as

$$\begin{cases}\dot{\tilde{\boldsymbol{x}}}=\tilde{\boldsymbol{A}}\tilde{\boldsymbol{x}}+\tilde{\boldsymbol{B}}\boldsymbol{v}\\ \dot{\boldsymbol{\zeta}}=\alpha(\boldsymbol{x},\boldsymbol{\zeta})+\beta(\boldsymbol{x},\boldsymbol{\zeta})\boldsymbol{v}\end{cases} \tag{21}$$

where $\tilde{\boldsymbol{x}}=[z_1^0,\cdots,z_1^{r_1-1},\cdots z_m^0,\cdots,z_m^{r_m-1}]^{\mathrm{T}}$, $\boldsymbol{\zeta}=(z_{r+1},\cdots,z_n)^{\mathrm{T}}$, $\boldsymbol{v}=[v_1,\cdots,v_m]^{\mathrm{T}}$, $\tilde{\boldsymbol{A}}=\mathrm{diag}(\tilde{\boldsymbol{A}}_1,\cdots,\tilde{\boldsymbol{A}}_m)$, $\tilde{\boldsymbol{B}}=\mathrm{diag}(\tilde{\boldsymbol{b}}_1,\cdots,\tilde{\boldsymbol{b}}_m)$, and $v_i=\sum_{k=1}^{m}L_{b_k}(L_f^{r_i-1}h_1)\cdot u_k+L_f^{r_i}h_i+L_d(L_f^{r_i-1}h_i)$, $\tilde{\boldsymbol{A}}_i=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\ \vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\cdots&1\\0&0&0&\cdots&0\end{bmatrix}_{r_i\times r_i}$, $\tilde{\boldsymbol{b}}_i=\begin{bmatrix}0\\0\\ \vdots\\0\\1\end{bmatrix}_{r_i\times 1}$, $1\leqslant i\leqslant m$.

Evidently, $\tilde{x}$ and ζ are an n-dimension state vector and an n-r dimension state vector, respectively. Therefore, only when $r=n$, ζ will not exist, and then system (Eq. (1)) can be represented as

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}v \tag{22}$$

If sufficient and necessary conditions for exact linearization of Eq. (1) can be satisfied, by referring to Section 1.1, the existence of h and the equation $r=n$ will be satisfied.

Aimed at Eq. (22) this paper designs a nonlinear sliding mode controller and optimizes the parameters by the ABC algorithm. Obviously, Eq. (22) can be partitioned into m subsystems as follows

$$\dot{z}_i = \tilde{A}_i z_i + \tilde{b}_i v_i \quad i=1,\cdots,m \tag{23}$$

where $z_i = [z_i^0,\cdots,z_i^{r_i-1}]^{\mathrm{T}}$. For each subsystem, the switching function of SMC is set as

$$s_i = C_i z_i = \sum_{j=0}^{r_i-2} c_i^j z_i^j + z_i^{r_i-1} \quad i=1,\cdots,m \tag{24}$$

where $C_i = [c_i^0,\cdots,c_i^{r_i-2},1]$ and each c_i^j is a design parameter which ensures that the polynomial $p^{r_i-1} + c_i^{r_i-2}p^{r_i-2} + \cdots + c_i^1 p + c_i^0$ satisfies Hurwitz stability (p is Laplace operator). Thus, the system is asymptotic stable after reaching each sliding surface $s_i=0$.

By the "reaching law approach", the reaching law for each subsystem can be designed as

$$\dot{s}_i = -k_i s_i - \varepsilon_i \mathrm{sgn}(s_i) \quad i=1,\cdots,m \tag{25}$$

where $\varepsilon_i > 0$ and $k_i \geqslant 0$. By Lyapunov theory, a Lyapunov function is defined as $V_i = \frac{1}{2}s_i^2$, then $\dot{V}_i = s_i \dot{s}_i$, namely

$$\dot{V}_i = -k_i s_i^2 - \varepsilon_i \mathrm{sgn}(s_i)\cdot s_i = -k_i s_i^2 - \varepsilon_i |s_i| \quad i=1,\cdots,m \tag{26}$$

Obviously, $\dot{V}_i \leqslant 0$, so the system can reach the sliding surface $s_i=0$ in finite time. Because

$$\dot{s}_i = C_i \dot{z}_i = C_i \tilde{A}_i z_i + C_i \tilde{b}_i v_i \quad i=1,\cdots,m \tag{27}$$

$C_i\tilde{b}_i = 1$, $C_i\tilde{b}_i$ is reversible, thus

$$v_i = -(C_i\tilde{b}_i)^{-1}[C_i\tilde{A}_i z_i + k_i s_i + \varepsilon_i \mathrm{sgn}(s_i)] \quad i=1,\cdots,m \tag{28}$$

Substituting Eq. (28) into Eq. (20), the designed nonlinear sliding mode controller can be represented as

$$u = E^{-1}(x)\cdot \left\{ \begin{bmatrix} -(C_1\tilde{b}_1)^{-1}[C_1\tilde{A}_1 z_1 + k_1 s_1 + \varepsilon_1 \mathrm{sgn}(s_1)] \\ \vdots \\ -(C_m\tilde{b}_m)^{-1}[C_m\tilde{A}_m z_m + k_m s_m + \varepsilon_m \mathrm{sgn}(s_m)] \end{bmatrix} - P(x) - Q(x) \right\} \tag{29}$$

In order to eliminate the chattering phenomenon in SMC, this paper uses a quasi-sliding mode method, namely adopting the saturation function sat(s) to replace the sign function sgn(s). The saturation function sat(s) can be represented as

$$\mathrm{sat}(s) = \begin{cases} 1 & s > \Delta \\ (1/\Delta)\cdot s & |s| \leqslant \Delta \\ -1 & s < -\Delta \end{cases} \tag{30}$$

where Δ is the boundary layer thickness. To avoid the degradation of the anti-jamming capability due to large boundary layer thickness, set $\Delta = 0.001$. Then the designed controller will be

$$u = E^{-1}(x)\cdot \left\{ \begin{bmatrix} -(C_1\tilde{b}_1)^{-1}[C_1\tilde{A}_1 z_1 + k_1 s_1 + \varepsilon_1 \mathrm{sat}(s_1)] \\ \vdots \\ -(C_m\tilde{b}_m)^{-1}[C_m\tilde{A}_m z_m + k_m s_m + \varepsilon_m \mathrm{sat}(s_m)] \end{bmatrix} - P(x) - Q(x) \right\} \tag{31}$$

Next, the controller parameters are optimized by using ABC algorithm to obtain optimal performance. In Eq. (31), the design parameters are $c_i^0,\cdots, c_i^{r_i-2}$, ε_i and k_i ($\varepsilon_i > 0$ & $k_i \geqslant 0, i=1,\cdots,m$).

Therefore, the solution vector of the designed controller in ABC algorithm is an $n+m$ dimensional vector

$$[c_1^0,\cdots,c_1^{r_1-2},\varepsilon_1,k_1,\cdots,c_m^0,\cdots,c_m^{r_m-2},\varepsilon_m,k_m]$$

The objective performance function for ABC algorithm is designed based on the systems' state responses as follows

$$J = \int_0^\infty [w_1|e_1(t)| + \cdots + w_n|e_n(t)|]\,\mathrm{d}t \tag{32}$$

where $e_1(t),\cdots,e_n(t)$ are the errors of $x_1,\cdots,x_n$ at time t, respectively. $w_1,\cdots,w_n$ are the weights, forming the equation $w_1+\cdots+w_n=1$.

The steps for parameter optimization of aero-engine nonlinear sliding mode controller based on ABC algorithm are shown as:

Step 1 Set $n+m$ dimensional solution vectors corresponding to the $n+m$ design parameters ($c_1^0,\cdots,c_1^{r_1-2},\varepsilon_1,k_1,\cdots,c_m^0,\cdots,c_m^{r_m-2},\varepsilon_m,k_m$) of the designed aeroengine nonlinear sliding mode controller, and initialize all the vectors by Eq. (5).

Step 2 Calculate the objective Eq. (32) of all the solution vectors, then set the minimum function value and the corresponding optimum vector as the global minimum function value and global optimum vector.

Step 3 Startthe cycles, and set the numbers of the no-updated times for employed bees as 0.

Step 4 The employed bees execute neighborhood search by Eq. (6) to find new solution vectors, and choose better vectors through the greedy selection mechanism.

Step 5 If some employed bees do not find a better solution vector, the number of the no-updated times for it will be the old value plus 1.

Step 6 Calculate the probabilities of all the employed bees being selected by Eq. (7), and the onlookers execute random selection, the employed bees which have larger probability are more likely to be selected.

Step 7 The onlookers execute neighborhood search, choose better vectors through the greedy selection mechanism, and calculate the numbers of the no-updated times with method in Step 5.

Step 8 Confirm the minimum function value in this cycle, if it is less than the value in the last cycle, set it as the global minimum function value, and set the corresponding solution vector as the global optimum vector.

Step 9 If the number of the no-updated times for some employed bees is larger than the pre-set limit times, this employed bee turns into a scouter, and initialize the solution vector by Eq. (5).

Step 10 If the cycle times do not achieve the pre-set maximum cycle times, go to Step 2, otherwise, the cycle ends and obtains the optimum controller parameters.

3 Simulation

Aimed at the aeroengine system (Eq. (12)), supposing $D=B$, the interference matching condition(Eq. (12)) will be satisfied. In the simulation, $g(t)=50\sin(10\pi t)$ is set. As for the system(Eq. (12)), $m=1$ and $\boldsymbol{B}=[\boldsymbol{b}]$. Then

$$G_0(x)=\{\boldsymbol{b}\}=\left\{\begin{bmatrix}0.2491\\0.2336\end{bmatrix}\right\}$$

$$G_1(x)=\{\boldsymbol{b},ad_f\boldsymbol{b}\}=$$

$$\left\{\begin{bmatrix}0.2491\\0.2336\end{bmatrix},-\begin{bmatrix}5.806x_1-0.4672x_2-0.7391\\0.4672x_2-0.8225x_1-0.6579\end{bmatrix}\right\} \tag{33}$$

Obviously, sufficient and necessary conditions for exact linearization of Eq. (12) can be satisfied, by reference to Section 1.1.

Then, the smooth m-dimension vector function $\boldsymbol{h}(x)$ is introduced, and

$$L_b\boldsymbol{h}=0$$
$$L_b(L_f\boldsymbol{h})\neq 0 \tag{34}$$

such that

$$\frac{\partial \boldsymbol{h}}{\partial x}\boldsymbol{b}=0 \tag{35}$$

Let

$$\boldsymbol{\varphi}(x)=\frac{\partial \boldsymbol{h}}{\partial x} \tag{36}$$

where $\boldsymbol{\varphi}(x)$ is a 1×2 function matrix, so

$$\boldsymbol{\varphi}(x)\boldsymbol{b}=0 \tag{37}$$

If $\boldsymbol{h}(x)$ is smooth sufficiently, $\boldsymbol{h}(x)$ has second partial derivatives and they are equal, namely

$$\frac{\partial^2\boldsymbol{h}}{\partial x_1\partial x_2}=\frac{\partial^2\boldsymbol{h}}{\partial x_2\partial x_1} \tag{38}$$

then

$$\frac{\partial\varphi_1(x)}{\partial x_2}=\frac{\partial\varphi_2(x)}{\partial x_1} \tag{39}$$

If $\boldsymbol{\varphi}(x)=[\varphi_1(x),\varphi_2(x)]$ exists and satisfies Eqs. (37) and (39), $\boldsymbol{h}(x)$ can be solved using integration method by substituting $\boldsymbol{\varphi}(x)$ into Eq. (36). Here, a solution of $\boldsymbol{h}(x)$ is obtained

$$h=0.02728448x_1^2-0.029257805x_2^2-0.05650784x_1x_2 \tag{40}$$

then

$$L_fh=0.75089085x_1^3-0.57861754x_1^2x_2-0.25787492x_1^2-0.1110768x_1x_2^2-0.48575962x_1x_2-0.00200777x_2^3+0.10310491x_2^2 \tag{41}$$

$$L_b(L_fh)=0.40975643x_1^2-0.33183025x_1x_2-0.011286438x_1-0.028276523x_2^2+0.16567587x_2 \tag{42}$$

$$L_f^2h=28.015720351x_1^4-13.50915992x_1^3x_2-17.1383344x_1^3-4.15397205x_1^2x_2^2+$$

$$15.510\,572\,89x_1^2x_2 + 2.422\,889\,98x_1^2 + 0.935\,081\,48x_1x_2^3 + 0.551\,879\,85x_1x_2^2 + 4.329\,871\,54x_1x_2 + 0.105\,053\,48x_2^4 - 0.417\,437\,7213x_2^3 + 0.041\,027\,70x_2^2 \quad (43)$$

$$L_d(L_fh) = -g(t)(0.409\,756\,43x_1^2 - 0.331\,830\,25x_1x_2 - 0.011\,286\,438x_1 - 0.028\,276\,523x_2^2 + 0.165\,675\,87x_2) \quad (44)$$

By the method proposed in Section 2, $\boldsymbol{E}(x) = L_b(L_fh)$, $\boldsymbol{E}(x)$ is reversible, $\boldsymbol{P}(x) = L_f^2h$, $\boldsymbol{Q}(x) = L_d(L_fh)$. Then the nonlinear state transformation

$$\tilde{\boldsymbol{x}} = [h \quad L_fh]^{\mathrm{T}} \quad (45)$$

and the input transformation

$$\boldsymbol{v} = L_b(L_fh) \cdot u + L_f^2h + L_d(L_fh) \quad (46)$$

are applied. Therefore, the original system $\dot{\boldsymbol{x}} = \boldsymbol{f}(x) + \boldsymbol{B}u + \boldsymbol{d}(t)$ is transformed into

$$\dot{\tilde{\boldsymbol{x}}} = \tilde{\boldsymbol{A}}\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{B}}\boldsymbol{v} \quad (47)$$

where $\tilde{\boldsymbol{A}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $\tilde{\boldsymbol{B}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

In Eq. (47), $\dot{\tilde{x}}_1 = \tilde{x}_2$, thus the switching control function can be designed as

$$s(\tilde{\boldsymbol{x}}) = \tilde{\boldsymbol{C}}\tilde{\boldsymbol{x}} = \tilde{c}\tilde{x}_1 + \tilde{x}_2 \quad (48)$$

where $\tilde{\boldsymbol{C}} = [\tilde{c}, 1]$ and $\tilde{c}$ is a design parameter which ensures that $p + \tilde{c}$ satisfies Hurwitz stability (p is Laplace operator). By Hurwitz criterion, the root of $p + \tilde{c} = 0$ should possess a negative real part, namely $\tilde{c} > 0$. Substituting $\tilde{\boldsymbol{x}} = [h \quad L_fh]^{\mathrm{T}}$ into $s = s(\tilde{\boldsymbol{x}})$, then

$$s(x) = s(\tilde{\boldsymbol{x}})\,|_{\tilde{\boldsymbol{x}}=[h \quad L_fh]^{\mathrm{T}}} = \tilde{c}h(x) + L_fh(x) \quad (49)$$

By Section 2, the reaching law is designed as

$$\dot{s} = -ks - \varepsilon \mathrm{sat}(s) \quad (50)$$

$$\dot{s} = \frac{\partial s}{\partial \tilde{x}}\dot{\tilde{\boldsymbol{x}}} = \tilde{\boldsymbol{C}}(\tilde{\boldsymbol{A}}\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{B}}v) = \tilde{\boldsymbol{C}}\tilde{\boldsymbol{A}}\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{C}}\tilde{\boldsymbol{B}}v \quad (51)$$

and $\tilde{\boldsymbol{C}}\tilde{\boldsymbol{B}} = 1$ which means $\tilde{\boldsymbol{C}}\tilde{\boldsymbol{B}}$ is reversible, so

$$v = -(\tilde{\boldsymbol{C}}\tilde{\boldsymbol{B}})^{-1}(\tilde{\boldsymbol{C}}\tilde{\boldsymbol{A}}\tilde{\boldsymbol{x}} + ks + \varepsilon \mathrm{sat}(s)) \quad (52)$$

Substituting Eq. (52) into Eq. (46), the designed nonlinear sliding mode controller can be represented as

$$u = -[L_b(L_fh)]^{-1}\{(\tilde{\boldsymbol{C}}\tilde{\boldsymbol{B}})^{-1}(\tilde{\boldsymbol{C}}\tilde{\boldsymbol{A}}\tilde{\boldsymbol{x}} + ks + \varepsilon \mathrm{sat}(s)) - [L_f^2h + L_d(L_fh)]\} \quad (53)$$

In Eq. (8), $\boldsymbol{y} = \begin{bmatrix} 8.737\,9 & 0 \\ 3.303\,3 & 3.802\,5 \\ 2.194\,0 & 2.574\,9 \end{bmatrix} \boldsymbol{x}$, so x_1 has greater impact than x_2 on average. Therefore, set $w_1 = 0.9$ and $w_2 = 0.1$ in this paper. In the simulation, the ranges of the design parameters are defined around experiential values, which are listed in Table 1 and parameter settings of ABC algorithm are shown in Table 2.

Table 1 Range of design parameters

Parameter	Minimum	Maximum
$\tilde{c}$	0.5	5
k	3	30
ε	10	60

Table 2 Parameter settings of ABC algorithm

Parameter	Value
Colony size	10
Maximal cycle times	50
Maximal no-updated times	10

The ABC-based aeroengine nonlinear sliding mode control algorithm is firstly simulated under the same initial condition ($\boldsymbol{x}(0) = [-0.5 \ -1]^{\mathrm{T}}$) as in Ref. [20] to be compared with the controller designed by the Gronwall-Bellman Lemma approach. At $\boldsymbol{x}(0) = [-0.5 \ -1]^{\mathrm{T}}$, Fig. 2 shows the convergence of the global optimal value in ABC algorithm, obviously, and the ABC algorithm converges very fast. In the end, the global optimal value is 0.0 107 377 and the global optimal solution is $[\tilde{c}, k, \varepsilon] = [3.565\,2, 14.945\,8, 50.678\,8]$. Figs. 3,4 show the response of the designed controller using ABC-based aeroengine nonlinear sliding mode control algorithm. Since the sliding surface expressed in the original state

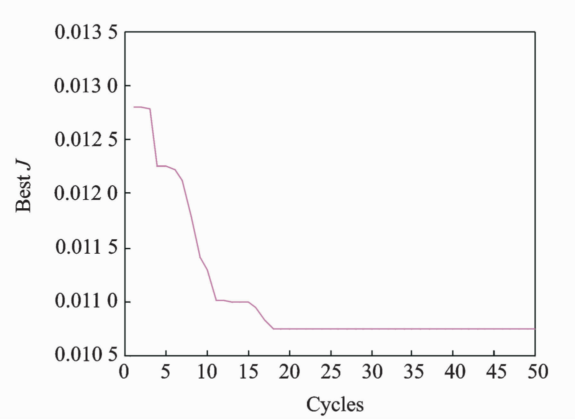
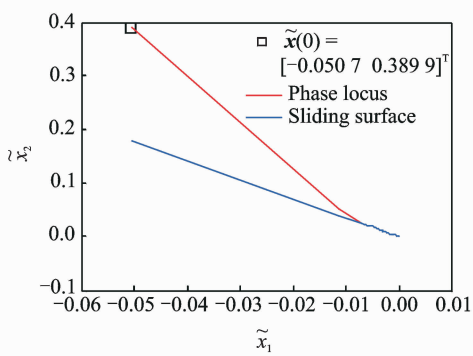


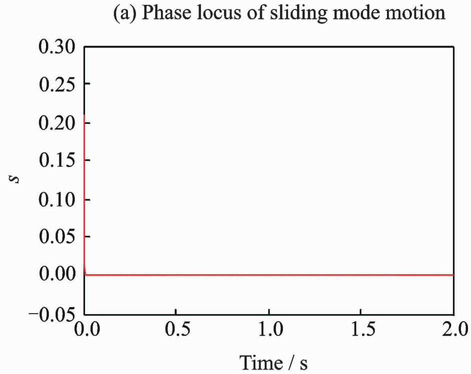
Fig. 2 J functions of ABC algorithm at $\boldsymbol{x}(0) = [-0.5 \ -1]^{\mathrm{T}}$

vector $\boldsymbol{x}(\boldsymbol{x}=[x_1 \quad x_2]^{\mathrm{T}})$ is $s(x)=\tilde{c}h(x)+L_f h(x)=0$ and the analytic solutions of this function is not unique, this paper uses the sliding surface after state transformation $s(\tilde{\boldsymbol{x}})=\tilde{c}\tilde{x}_1+\tilde{x}_2=0$ and response of $\tilde{x}_1$ and $\tilde{x}_2$ to draw phase diagram (Fig. 3(a)). The phase diagram and s response (See Fig. 3(b)) show that the system can reach the sliding surface in a short time. The u response shows that the designed controller can resist disturbance well (Fig. 3(c)). The state response (Fig. 3(d)) and the output response (Fig. 4) show that the ABC-based aeroengine nonlinear sliding mode control algorithm has faster responses than the Gronwall-Bellman Lemma Approach in Ref. [20] (Figs. 5,6).

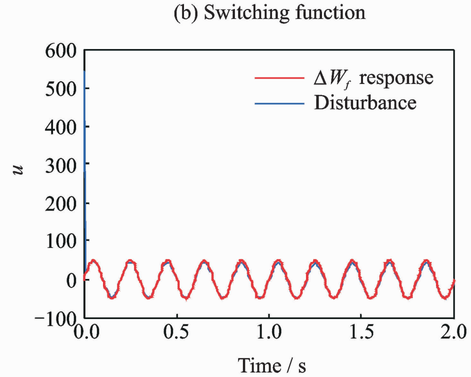
To verify the adaptive ability in large deviation of the designed controller, a simulation at another initial condition $\boldsymbol{x}(0)=[0.3 \quad 0.5]^{\mathrm{T}}$ is given. Finally, the global optimal value is 0.004 928 9 and the global optimal solution is $[\tilde{c}, k, \varepsilon]=[3.506\,4, 14.843\,1, 42.047\,09]$. The results show that the ABC-based aeroengine nonlinear sliding mode control algorithm has a satisfying control effect as well (Figs. 7—9).



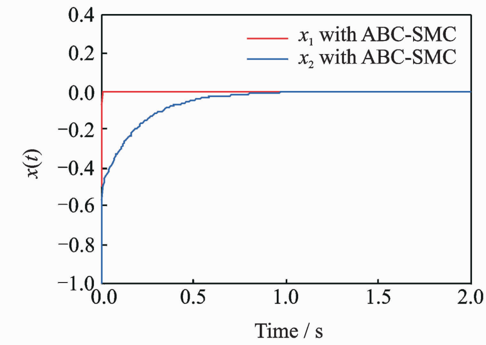
(a) Phase locus of sliding mode motion



(b) Switching function



(c) u response



(d) State response

Fig. 3 Response at $\boldsymbol{x}(0)=[-0.5 \quad -1]^{\mathrm{T}}$ using ABC-based aeroengine nonlinear sliding mode control algorithm

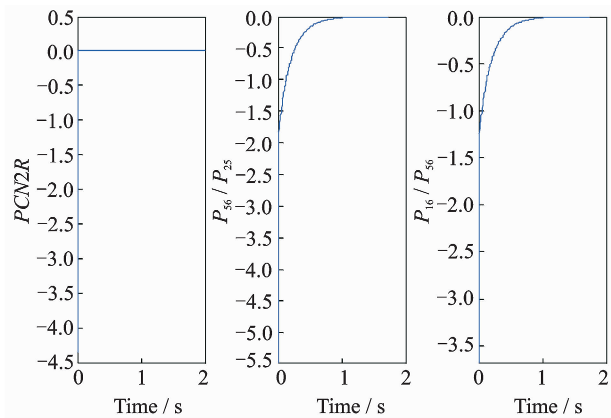


Fig. 4 Output responses at $\boldsymbol{x}(0)=[-0.5 \quad -1]^{\mathrm{T}}$ using ABC-based aeroengine nonlinear sliding mode control algorithm

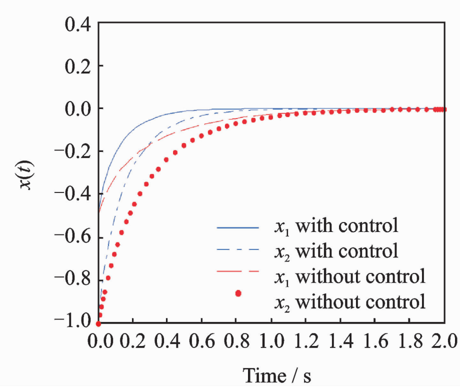


Fig. 5 State responses at $\boldsymbol{x}(0)=[-0.5 \quad -1]^{\mathrm{T}}$ using Gronwall-Bellman Lemma approach[20]

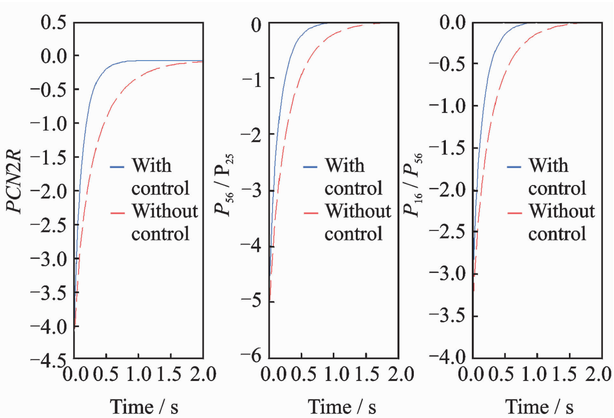


Fig. 6 Output responses at $\boldsymbol{x}(0)=[-0.5 \quad -1]^{\mathrm{T}}$ using Gronwall-Bellman Lemma approach[20]

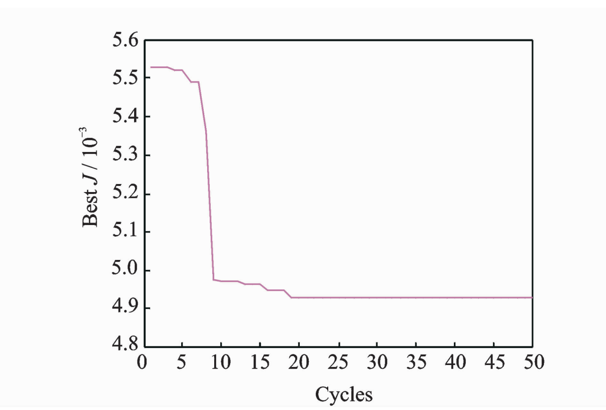
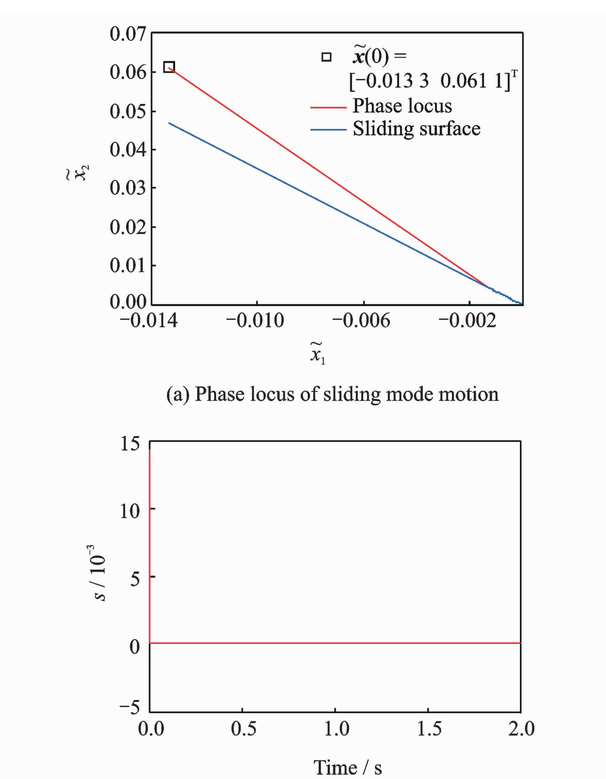
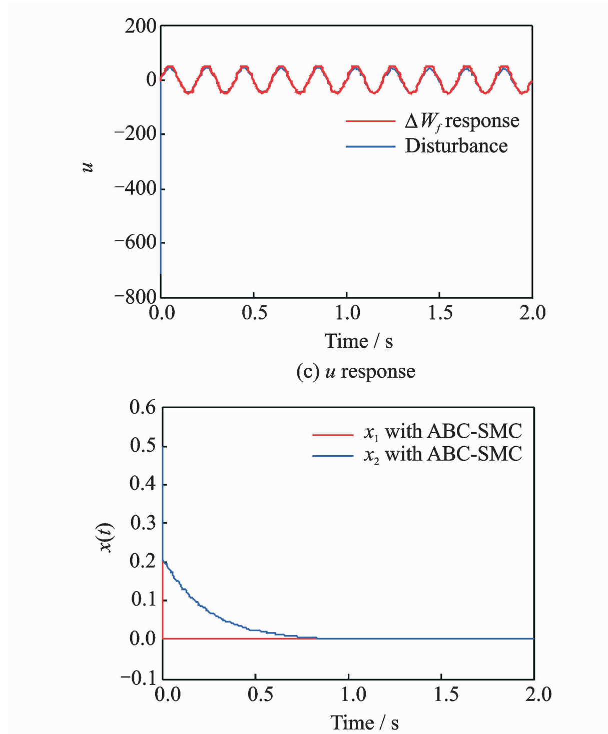


Fig. 7 J function of ABC algorithm at $\boldsymbol{x}(0)=[0.3 \quad 0.5]^{\mathrm{T}}$



(a) Phase locus of sliding mode motion

(b) Switching function



(c) u response

(d) State response

Fig. 8 Responses at $\boldsymbol{x}(0)=[0.3 \quad 0.5]^{\mathrm{T}}$ using ABC-based aeroengine nonlinear sliding mode control algorithm

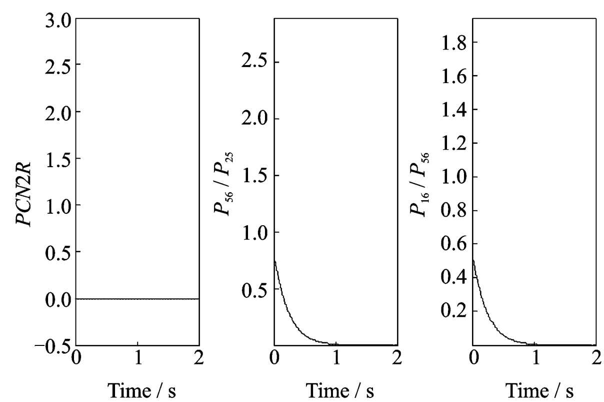


Fig. 9 Output responses at $\boldsymbol{x}(0)=[0.3 \quad 0.5]^{\mathrm{T}}$ using ABC-based aeroengine nonlinear sliding mode control algorithm

4 Conclusions

A novel design approach of aeroengine nonlinear sliding mode controller based on ABC algorithm is presented. The designed nonlinear controller fully utilizes the advantages of sliding mode control strategy and the optimization capacity of ABC algorithm. The ABC-based aeroengine nonlinear sliding mode controller can implement optimal control performance for the aeroengine.

Simulation results show that the system has a quick response, good anti-interference ability and strong robustness, and can obtain optimal performance owing to the designed control algorithm.

Acknowledgement

This work was supported by the Fundamental Research Funds for the Central Universities(NS2016027).

References:

[1] WANG C, LI D, LI Z, et al. Optimization of controllers for gas turbine based on probabilistic robustness[J]. Journal of Engineering for Gas Turbines and Power, 2009,131(5):054502-1-054502-5.

[2] BROWN H, ELGIN J A. Aircraft engine control mode analysis[J]. Journal of Engineering for Gas Turbines and Power,1985,107(4):838-844.

[3] LIANG B B, JIANG J, ZHEN Z Y, et al. Improved shuffled frog leaping algorithm optimizing integral separated PID control for unmanned hypersonic vehicle[J]. Transactions of Nanjing University of Aeronautics and Astronautics, 2015, 32(1): 110-114.

[4] LIU J X, LI Y H, CHEN Y G, et al. Aeroengine LQR control based on fuzzy-neural networks [J]. Journal of Aerospace Power, 2004,19(6):839-843.

[5] HAREFORS M. Application of h_∞ robust control to the RM12 jet engine[J]. Control Engineering Practice, 1997,5(9):1189-1201.

[6] PU H Z, ZHEN Z Y, XIA M. Flight control system of unmanned aerial vehicle[J]. Transactions of Nanjing University of Aeronautics and Astronautics, 2015, 32(1): 1-8.

[7] FREDERICK D K, GARG S, ADIBHATLA S. Turbofan engine control design using robust multivariable control technologies [J]. Control Systems Technology, IEEE Transactions on, 2000, 8(6): 961-970.

[8] ZHEN Z Y, PU H Z, CHEN Q, et al. Nonlinear intelligent flight control for quadrotor unmanned helicopter[J]. Transactions of Nanjing University of Aeronautics and Astronautics, 2015, 32(1): 29-34.

[9] DORNAN D, OPPENHEIMER M. Improving control allocation accuracy for nonlinear aircraft dynamics[M]. Fort Belvoir:Defense Technical Information Center, 2002.

[10] BLAQUIERE A. Nonlinear system analysis[M]. Amsterdem:Elsevier, 2012.

[11] BENELGHALI S, EL HACHEMI BENBOUZID M, CHARPENTIER J F, et al. Experimental validation of a marine current turbine simulator: Application to a permanent magnet synchronous generator-based system second-order sliding mode control[J]. Industrial Electronics, IEEE Transactions on, 2011, 58(1):118-126.

[12] PANDA S, BANDYOPADHYAY B. Sliding mode control of gas turbines using multirate-output feedback[J]. Journal of Engineering for Gas Turbines and Power, 2008,130(3):460-466.

[13] WU L, HO D W. Sliding mode control of singular stochastic hybrid systems[J]. Automatica,2010,46(4):779-783.

[14] KARABOGA D, GORKEMLI B, OZTURK C, et al. A comprehensive survey: Artificial bee colony (ABC) algorithm and applications[J]. Artificial Intelligence Review,2014, 42(1):21-57.

[15] BRUNELL B J, BITMEAD R R, CONNOLLY A J. Nonlinear model predictive control of an aircraft gas turbine engine[C]// Decision and Control, 2002, Proceedings of the 41st IEEE Conference on. [S. l.]: IEEE,2002:4649-4651.

[16] ZHANG H B, SUN J G, SUN F C. Applications of nonlinear backstepping control strategy in multivariable control of aeroengine [J]. Journal of Aerospace Power, 2007,22(7):1190-1194.

[17] YAO Y L, SUN J G. Aeroengine direct thrust control based on neural network inverse control[J]. J Propul Technol, 2008,29(2):249-252.

[18] CAI K, YU K, LVY B. Multi-variable neural network adaptive control for aeroengine[C]// Intelligent Control and Automation, 7th World Congress on (WCICA 2008). [S. l.]: IEEE,2008:8687-8692.

[19] LUO G Q, LIU B, SONG D Y. Hybrid particle swarm optimization in solving aero-engine nonlinear mathematical model[J]. Gas Turbine Exp Res, 2011,24(2):5-8.

[20] WANG J, YE Z, HU Z. Nonlinear control of aircraft engines using a generalized Gronwall-Bellman Lemma approach[J]. Journal of Engineering for Gas Turbines and Power, 2012, 134(9): 094502-3-094502-6.

[21] ZHONG S, DONG Y F, SAROSH A. Artificial bee colony algorithm for parametric optimization of spacecraft attitude tracking controller[C]//Foundations and Practical Applications of Cognitive Systems and Information Processing. [S. l.]:Springer Berlin

Heidelberg, 2014: 501-510.

[22] HU Z H, ZHAO M. Research on path planning of UAV based on ABC algorithm[J]. Transducer and Microsystem Technologies, 2010,29(3):35-38.

Mr. **Lu Binbin** received his B. S. degree in Flight Vehicle Propulsion Engineering and M. S. degree in Aerospace Propulsion Theory and Engineering from Nanjing University of Aeronautics and Astronautics. His main research focuses on aero-engine control.

Dr. **Xiao Lingfei** received his Ph. D. degree in Control Theory and Engineering from Zhejiang University. She is mainly engaged in the teaching and research work of advanced control and its application in aero-engine.

Mr. **Chen Yuhan** is currently a Ph. D. candidate in College of Energy and Power Engineering, Nanjing University of Aeronautics and Astronautics. His main research focuses on aero-engine modeling and control.

(Executive Editor: Xu Chengting)