Accurate Free Vibration of Functionally Graded Skew Plates

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Abstract: The present study aims to analyze free vibration of thin skew plates made of functionally graded material (FGM) by using the weak form quadrature element method. The material properties vary continuously through the thickness according to a power-law form. A novel FGM skew plate element is formulated according to the neutral surface based plate theory and with the help of the differential quadrature rule. For verifications, Numerical results are compared with available data in literature. Results reveal that the non-dimensional frequency parameters of the FGM skew plates are independent of the power-law exponent and always proportional to those of homogeneous isotropic ones when the coupling and rotary inertias are neglected. In addition, employing the physical neutral surface based plate theory is equivalent to using the middle plane based plate theory with the reduced flexural modulus matrix.

Key words: functionally graded material (FGM); skew plate; free vibration; quadrature element method; neutral surface

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0 Introduction

The functionally graded materials (FGMs) are showing increasingly wide applications in aerospace and defense industries due to their excellent ability to mitigate the problem existing in laminated composites. The static, buckling and dynamic behavior of FGM structural members is important to structural engineers, thus has been received great attentions. It seems that no special tools are needed since FGM structural members behave like homogeneous ones[1]. However, this is only valid for certain cases, thus a vast body of literature exists[2,3]. Comprehensive reviews can be found in Refs. [2,3].

The skew plate is one of the common used structural members. The free vibration of skew plates is also a fundamental topic. Due to the strong bending moment singularities existing at the obtuse angles, it is not easy to obtain accurate fundamental frequency even for homogeneous isotropic thin skew plates with a large skew angle. A lot of existing methods may encounter serious convergence problems[4].

So far, few researchers have paid attention to the free vibration analysis of FGM skew plates. Ruan et al. [5,6] investigated the dynamic characteristics of FGM skew plates by using the differential quadrature method (DQM). Only two boundary conditions, simply supported (S) and clamped (C), are considered. If free boundary is involved, however, DQM with the widely used grid spacing may yield incorrect frequencies [7,8].

Previous research shows that the weak form quadrature element method (QEM), essentially a high-order finite element method (FEM), is highly accurate and has good computational efficiency[9]. QEM can yield accurate frequencies for homogeneous isotropic thin skew plates even with large skew angles [7]. Being FEM, the irregulari-

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ty in shape and various boundary conditions can be easily treated. Besides, QEM converges exponentially with the increase of the number of node points [8].

The objective of the present investigation is to perform free vibration analysis of thin FGM skew plates by using QEM. A novel FGM skew plate element is firstly developed according to the thin plate theory based on the physical neutral surface. Formulations are given in detail and new results are obtained. Finally, some conclusions are drawn based on the results reported herein.

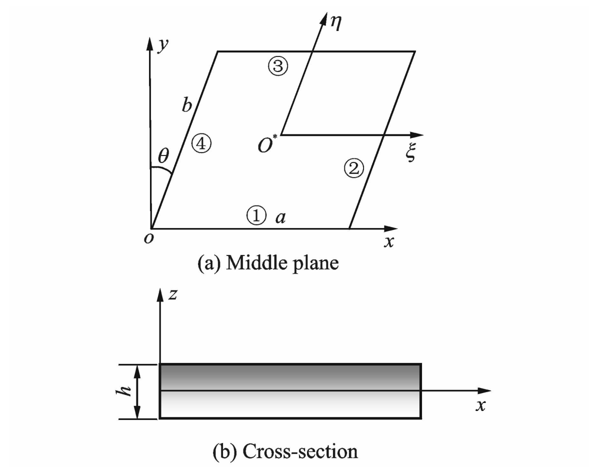

Fig. 1 Sketch of skew plate

1 Formulations of Weak Form Quadrature FGM Skew Plate Element

1.1 Expressions of strain energy and kinetic energy

An FGM skew thin plate with a skew angle θ is schematically shown in Fig. 1. The side lengths are denoted by a and b. A uniform thickness h is considered. Both Cartesian coordinate system (x, y, z) and oblique coordinate system (ξ, η, z) are set at the middle plane of the plate, thus $-h/2 \leqslant z \leqslant h/2$. Assume that Poisson's ratio μ is a constant throughout the plate, but the elastic modulus and mass density vary along the thickness direction according to the power-law form defined by

$$P(z) = (P_c - P_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + P_m \quad (1)$$

where subscripts c and m denote the ceramic and metal. $P(z)$ is either the elastic modulus $E(z)$ or the mass density $\rho(z)$ and the power-law exponent k is a non-negative variable.

The plate theory based on the physical neutral surface is used due to the decoupling of the stretching and bending in constitutive equations. For transverse vibration, the strain energy of the FGM skew plate is

$$U = \frac{ab}{8}\int_{-1}^{1}\int_{-1}^{1} \boldsymbol{\kappa}^{\mathrm{T}} \widetilde{\boldsymbol{D}} \boldsymbol{\kappa}\, \mathrm{d}\xi \mathrm{d}\eta \quad (2)$$

where $\widetilde{\boldsymbol{D}}$ is a 3×3 symmetric matrix and $\boldsymbol{\kappa}$ the curvature vector defined by

$$\boldsymbol{\kappa} = \left[\frac{4}{a^2}\frac{\partial^2 w}{\partial \xi^2} \quad \frac{4}{b^2}\frac{\partial^2 w}{\partial \eta^2} \quad \frac{8}{ab}\frac{\partial^2 w}{\partial \xi \partial \eta}\right]^{\mathrm{T}} = \left[\frac{4}{a^2}w_{\xi\xi} \quad \frac{4}{b^2}w_{\eta\eta} \quad \frac{8}{ab}w_{\xi\eta}\right]^{\mathrm{T}} \quad (3)$$

where $w(\xi, \eta, t)$ is the deflection. Subscripts ξ and η denote the partial derivatives with respect to the oblique coordinates ξ and η, i. e., $w_{\xi\xi} = \partial^2 w/\partial \xi^2$, $w_{\eta\eta} = \partial^2 w/\partial \eta^2$ and $w_{\xi\eta} = \partial^2 w/\partial \xi \partial \eta$.

Elements in matrix $\widetilde{\boldsymbol{D}}$ are defined by

$$\begin{aligned} &\widetilde{D}_{11} = \widetilde{D}_{22} = \widetilde{D} \\ &\widetilde{D}_{13} = \widetilde{D}_{31} = \widetilde{D}_{23} = \widetilde{D}_{32} = -\widetilde{D}\sin\theta \\ &\widetilde{D}_{12} = \widetilde{D}_{21} = \widetilde{D}(\sin^2\theta + \mu\cos^2\theta) \\ &\widetilde{D}_{33} = \widetilde{D}(1 + \sin^2\theta - \mu\cos^2\theta)/2 \end{aligned} \quad (4)$$

where

$$\widetilde{D} = \int_{-h/2}^{h/2} \frac{E(z)}{1-\mu^2}(z-e)^2 \mathrm{d}z = \frac{h^3(E_m^2 k^4 + 4E_c E_m k^3 + 4E_m^2 k^3 + 16 E_c E_m k^2 + 7E_m^2 k^2 + 28 E_c E_m k + 12 E_c^2)}{12(1-\mu^2)(k+3)(k+2)^2(E_c + E_m k)} \quad (5)$$

where e is the distance between the middle plane and the physical neutral surface which can be determined by[1,5,10]

$$\int_{-h/2}^{h/2} E(z)(z-e)\mathrm{d}z = 0 \quad (6)$$

It should be pointed out that the matrix $\widetilde{\boldsymbol{D}}$ is exactly the same as the reduced flexural modulus matrix due to the coupling of stretching and bending. The proof is given in Appendix A. It is well-known that whenever stretching-bending coupling exists, the reduced flexural modulus matrix $\widetilde{\boldsymbol{D}}$ should be used to obtain accurate results of stresses, displacements, buckling loads and natural frequencies[11].

In order to de-couple the stretching and bending completely in dynamics analysis, the coupling and/or rotary inertias should be discarded. Otherwise, the stretching and bending are still coupled since the coupling inertia is not zero. In present investigations, both coupling and rotary inertias are neglected for simplicity. Therefore, the kinetic energy of the FGM skew plate is given by

$$T=\frac{abI}{8}\int_{-1}^{1}\int_{-1}^{1}\cos^4\theta\left(\frac{\partial w(\xi,\eta,t)}{\partial t}\right)^2\mathrm{d}\xi\mathrm{d}\eta \quad (7)$$

where t is the time and I is given by

$$I=\int_{-h/2}^{h/2}\rho(z)\mathrm{d}z=\frac{(\rho_c+\rho_m k)h}{k+1} \quad (8)$$

where ρ_c and ρ_m are the mass densities of the ceramic and metal, respectively.

Only essential boundary conditions are required by using QEM.

Simply supported edge (S)

$$w=0 \quad \begin{cases}\xi=\mp 1\\ \eta=\mp 1\end{cases} \quad (9)$$

Clamped edge (C)

$$w=\frac{\partial w}{\partial \xi}=0 \quad \begin{cases}\xi=\mp 1\\ \eta=\mp 1\end{cases} \quad (10)$$

1.2 Weak form quadrature FGM skew plate element

Let N be the number of node in either ξ or η direction. An $N\times N$-node weak form quadrature FGM skew plate element is formulated. For simplicity in presentation, only Gauss-Lobatto-Legendre (GLL) nodes are considered.

Denote ξ_k, η_k ($k=1,2,\cdots,N$) the element node coordinates in ξ and η direction ($-1\leqslant\xi_k,\eta_k\leqslant 1$) and $\xi_k=\eta_k$. Explicit formula to calculate the GLL points does not exist. For readers' reference, the GLL integration points and corresponding weights for N varying from 3 to 21 can be found in Ref. [8].

According to the criteria for selection of displacement functions, three slightly different displacement functions can be assumed for the skew plate element, namely[7]

$$w(\xi,\eta,t)=\sum_{i=1}^{N}\sum_{j=1}^{N}l_i(\xi)l_j(\eta)w(\xi_i,\eta_j,t)=\sum_{i=1}^{N}\sum_{j=1}^{N}l_i(\xi)l_j(\eta)w_{ij}(t) \quad (11)$$

$$w(\xi,\eta,t)=\sum_{i=1}^{N+2}\sum_{j=1}^{N}h_i(\xi)l_j(\eta)\widehat{w}_{ij}(t) \quad (12)$$

$$w(\xi,\eta,t)=\sum_{i=1}^{N}\sum_{j=1}^{N+2}l_i(\xi)h_j(\eta)\widetilde{w}_{ij}(t) \quad (13)$$

where $l_i(\xi)$ and $l_j(\eta)$ are Lagrange interpolation functions, $h_i(\xi)$ and $h_j(\eta)$ Hermite interpolation functions. In Eq. (12), $\widehat{w}_{ij}(t)$ contains the nodal deflection $w_{ij}(t)(i,j=1,2,\cdots,N)$ as well as the first-order derivative with respect to ξ at nodes on edges $\xi=\mp 1$, i. e., $(w_\xi)_{i1}$ and $(w_\xi)_{iN}$ ($i=1,2,\cdots,N$). In Eq. (13), $\widetilde{w}_{ij}(t)$ contains the nodal deflection $w_{ij}(t)(i,j=1,2,\cdots,N)$ and the first-order derivative with respect to η at nodes on edges $\eta=\mp 1$, i. e., $(w_\eta)_{1j}$ and $(w_\eta)_{Nj}$ ($j=1,2,\cdots,N$). The definitions of Lagrange and Hermite interpolation functions can be found in Refs. [7, 12].

A novel way is proposed in the formulation of the plate element. Instead of using only one displacement function as commonly conducted in the formulation of a conventional finite element, three displacement functions are used in the formulation of the stiffness matrix. Eq. (11) is used to compute $w_{\xi\eta}$ as well as the mass matrix, Eq. (12) is used to compute $w_{\xi\xi}$, and Eq. (13) is used to compute $w_{\eta\eta}$. In this way, the mixed second-order derivative $w_{\xi\eta}$ is not needed as the degree-of-freedom (DOF) at the plate corner points. Therefore, the plate element contains only (N^2+4N)DOFs, one DOF at all inner nodes, two DOFs at all boundary nodes, and three DOFs at the four corner nodes.

The stiffness matrix is obtained by using GLL quadrature, namely

$$\boldsymbol{k}=\frac{ab}{4}\sum_{i=1}^{N}\sum_{j=1}^{N}H_iH_j[\boldsymbol{B}(\xi_i,\eta_j)]^{\mathrm{T}}\widetilde{\boldsymbol{D}}\boldsymbol{B}(\xi_i,\eta_j) \quad (14)$$

where H_i, H_j are the weight.

The strain matrix at an integration point (ξ_i, η_j) is given by

$$\boldsymbol{B}(\xi_i,\eta_j)\bar{w}=\begin{bmatrix}\dfrac{4}{a^2}\sum_{l=1}^{N+2}\sum_{k=1}^{N}\bar{B}_{il}^{\xi}\delta_{jk}\bar{w}_{lk}\\ \dfrac{4}{b^2}\sum_{l=1}^{N}\sum_{k=1}^{N+2}\delta_{il}\bar{B}_{jk}^{\eta}\bar{w}_{lk}\\ \dfrac{8}{ab}\sum_{l=1}^{N}\sum_{k=1}^{N}A_{il}^{\xi}A_{jk}^{\eta}\bar{w}_{lk}\end{bmatrix}$$

$$i,j=1,2,\cdots,N \quad (15)$$

where δ_{il} is the Kronecker symbol, superscripts ξ and η mean that the corresponding derivatives are taken with respect to ξ and η, respectively. More precisely, A_{il}^{ξ} and A_{jk}^{η} are the weighting coefficients of the first-order derivative with respect to ξ and η, respectively, $\bar{B}_{il}^{\xi}$ and $\bar{B}_{jk}^{\eta}$ the weighting coefficients of the second-order derivative with respect to ξ and η, respectively. The degrees of freedom, $\bar{w}_{lk}$, contain the deflection at all nodes and the first-order derivative with respect to ξ at nodes on edges $\xi=\mp 1$ and the first-order derivative with respect to η at nodes on edges $\eta=\mp 1$, i. e., w_{lk}, $(w_{\xi})_{l1}$, $(w_{\xi})_{lN}$, $(w_{\eta})_{1k}$ and $(w_{\eta})_{Nk}$ $(l,k=1,2,\cdots,N)$. The weighting coefficients A_{il}^{ξ}, A_{jk}^{η}, $\bar{B}_{il}^{\xi}$ and $\bar{B}_{jk}^{\eta}$ can be explicitly calculated by using the differential quadrature rule. The explicit formulas can be found in Refs. [8, 12].

The mass matrix is also obtained by using GLL quadrature. The diagonal terms m_{II} in the mass matrix are

$$m_{II}=\frac{Iab\cos^4\theta}{4}\sum_{k=1}^{N}\sum_{m=1}^{N}H_k\delta_{ki}H_m\delta_{mj}$$
$$i,j=1,2,\cdots,N;I=N\times(i-1)+j \quad (16)$$

Note that the diagonal terms related to the derivative degrees of freedom are zero, thus they are not included in Eq. (16).

If the element nodes are not the GLL points, Gauss quadrature is usually adopted to obtain the stiffness and mass matrices. In such cases, the explicit formulas existing in DQM cannot be directly used to compute the weighting coefficients A_{il}^{ξ}, A_{jk}^{η}, $\bar{B}_{il}^{\xi}$ and $\bar{B}_{jk}^{\eta}$. The proposed method[13] should be used. Besides, the row or column summation technique should be used to obtain a diagonal mass matrix. More details can be found in Ref. [7].

For free vibration analysis of a thin FGM skew plate, $w(\xi,\eta,t)=W(\xi,\eta)\sin\omega t$, where ω is the circular frequency. If one $N\times N$-node quadrature FGM skew element is used, the equation of motion is given by

$$\boldsymbol{K}\bar{\boldsymbol{W}}=\omega^2\boldsymbol{M}\bar{\boldsymbol{W}} \quad (17)$$

where $\sin\omega t$ has been cancelled out, and the arrangement of $\bar{\boldsymbol{W}}$ is the same as the one used in DQM[8].

The dimension of stiffness and mass matrices is $(N^2+4N)\times(N^2+4N)$ and the same as the one of DQM[8]. However, the number of non-zero diagonal terms in the mass matrix is N^2 and different from the one of (N^2-4N+4) in DQM. After applying the essential boundary conditions, we can obtain

$$\tilde{\boldsymbol{K}}\tilde{\boldsymbol{W}}=\omega^2\tilde{\boldsymbol{M}}\tilde{\boldsymbol{W}} \quad (18)$$

The dimension of the matrices and vectors in Eq. (18) depends on the boundary conditions. Eq. (18) can be solved by a generalized eigen value solver.

2 Numerical Results and Discussion

For the demonstration, FGM is composed of alumina (Al_2O_3) and aluminum. The top surface of the plate is pure alumina (ceramic) and the bottom surface is pure aluminum (metal). The elastic modulus and mass density for the alumina and aluminum are $E_c=380$ GPa, $\rho_c=3\ 800$ kg/m^3, $E_m=70$ GPa, and $\rho_m=2\ 700$ kg/m^3. Poisson's ratio μ is 0.3.

Due to space limitations, rhombic plates ($a=b$) with two combinations of boundary conditions, i. e., SSSS and CFFF, are considered. The edge number denoted by ① to ④ are shown in Fig. 1. SSSS denotes all four edges are simply supported. And CFFF denotes that Edge 1 ($\eta=-1$) is clamped and the remaining three edges are free (F). The non-dimensional frequency parameter Ω is defined by

$$\Omega=\omega a^2\sqrt{PI/\tilde{D}_{11}} \quad (19)$$

The plate is modeled by one proposed $N\times N$-node quadrature skew plate element and $N=21$ for accuracy considerations. The skew angle θ takes four values, namely, 15°, 30°, 45° and 60°. Results are shown in Tables 1, 2.

It is seen that the results obtained by present method, denoted by QEM, agree well with the existing accurate upper bound solutions. Accurate non-dimensional frequency parameters are obtained by QEM although strong stress singularity

Table 1 Comparison of non-dimensional frequency parameter Ω for SSSS rhombic plates

Model number	θ 15°		30°		45°		60°	
	QEM	Ref. [14]	QEM	Ref. [14]	QEM	Ref. [14]	QEM	Ref. [14]
1	20.868	20.868	24.891	24.899	34.678	34.749	62.030	62.409
2	48.205	48.205	52.638	52.638	66.277	66.277	104.95	104.95
3	56.106	56.107	71.694	71.711	100.23	100.25	147.57	147.67
4	79.043	79.043	83.825	83.829	106.89	107.04	196.29	196.29
5	104.00	104.00	122.82	122.82	140.80	140.80	205.15	205.86

Table 2 Comparison of non-dimensional frequency parameter Ω for CFFF rhombic plates

Model number	θ 15°		30°		45°		60°	
	QEM	Ref. [15]	QEM	Ref. [15]	QEM	Ref. [15]	QEM	Ref. [15]
1	3.5831	3.5831	3.9278	3.9279	4.5051	4.5052	5.2407	5.2431
2	8.6964	8.6971	9.4095	9.4100	11.245	11.247	16.031	16.023
3	22.228	22.230	25.285	25.287	26.963	26.968	30.334	30.362
4	26.332	26.334	25.931	25.931	31.496	31.505	45.264	45.300
5	33.860	33.864	41.330	41.338	50.708	50.739	59.009	59.123

exists at the obtuse plate corners and corner functions are not included in the assumed displacements.

It is also found that Ω is independent of the power-law exponent k. Therefore, the existing non-dimensional frequency parameters of corresponding homogeneous isotropic skew plates are included for verifications. It was reported by Abrate[16] that the natural frequencies of FGM rectangular plates could be inferred from results of rectangular plates made of homogeneous materials. Similar observation exists for the FGM skew plates. In other words, if the frequencies of the corresponding homogeneous skew plate ($k=0$) are known, denoted by ω_c, the nature frequencies of the FGM skew plates can be obtained by

$$\omega=\sqrt{\frac{PI(0)\widetilde{D}_{11}(k)}{\widetilde{D}_{11}(0)PI(k)}}\,\omega_c=\alpha(k)\omega_c \qquad (20)$$

where $\alpha(k)$ is called the scaling factor and is shown in Fig. 2 for the material considered.

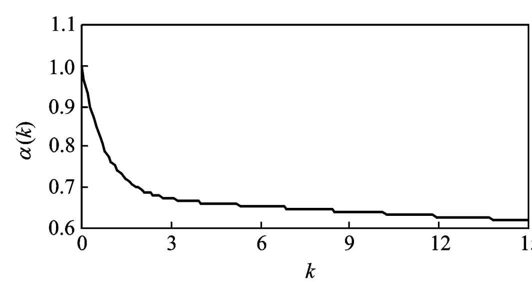

Fig. 2 Variation of scaling factor with power-law exponent

3 Conclusions

Free vibration of thin FGM skew plates is presented by using the weak form quadrature element method. The material properties vary continuously through the thickness according to a power-law form. A novel FGM skew plate element is formulated according to the neutral surface based plate theory and with the help of the differential quadrature rule. For verifications, Numerical results are compared with existing data. Numerical data show that the non-dimensional frequency parameters are independent of the power-law exponent and always proportional to those of homogeneous isotropic skew plates if the coupling and rotary inertias are discarded. It is shown that employing the physical neutral surface based plate theory is equivalent to using the middle plane based plate theory with the reduced flexural modulus matrix.

Appendix A: Discussion on the reduced flexural modulus matrix

For FGM or non-symmetric laminated plate, the stress field is given by

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{A1(a)}$$

or

$$\boldsymbol{\sigma} = \boldsymbol{Q}\boldsymbol{\varepsilon} \tag{A1(b)}$$

After integration over the plate thickness, the constitutive equation based on the middle plane is given by

$$\begin{Bmatrix} \boldsymbol{N} \\ \boldsymbol{M} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B} & \boldsymbol{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}_{0M} \\ \boldsymbol{\kappa} \end{Bmatrix} \tag{A2}$$

where

$$\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{D}\} = \int_{-h/2}^{h/2} \boldsymbol{Q}(1, z, z^2)\,\mathrm{d}z \tag{A3}$$

Let e be the distance between the middle plane and the physical neutral surface. After integration over the plate thickness, the constitutive equation based on the physical neutral surface is given by

$$\begin{Bmatrix} \widetilde{\boldsymbol{N}} \\ \widetilde{\boldsymbol{M}} \end{Bmatrix} = \begin{bmatrix} \widetilde{\boldsymbol{A}} & \widetilde{\boldsymbol{B}} \\ \widetilde{\boldsymbol{B}} & \widetilde{\boldsymbol{D}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}_{0n} \\ \boldsymbol{\kappa} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{0} \\ \boldsymbol{0} & \widetilde{\boldsymbol{D}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}_{0n} \\ \boldsymbol{\kappa} \end{Bmatrix} \tag{A4}$$

where

$$\{\widetilde{\boldsymbol{A}}, \widetilde{\boldsymbol{D}}\} = \int_{-h/2}^{h/2} \boldsymbol{Q}\{1, (z-e)^2\}\,\mathrm{d}z \tag{A5(a)}$$

and

$$\int_{-h/2}^{h/2} \boldsymbol{Q}(z-e)\,\mathrm{d}z = \boldsymbol{B} - e\boldsymbol{A} = \boldsymbol{0} \tag{A5(b)}$$

From Eq. (A5(b)), e can be determined as

$$e\boldsymbol{I} = \boldsymbol{B}\boldsymbol{A}^{-1} = \boldsymbol{A}^{-1}\boldsymbol{B} \tag{A6}$$

Therefore, one has

$$\widetilde{\boldsymbol{D}} = \int_{-h/2}^{h/2} \boldsymbol{Q}(z^2 - 2z + e^2)\,\mathrm{d}z = \boldsymbol{D} - \boldsymbol{B}\boldsymbol{A}^{-1}\boldsymbol{B} \tag{A7}$$

where $\widetilde{\boldsymbol{D}}$ is known as the reduced flexural modulus matrix[11].

The derivation of $\widetilde{\boldsymbol{D}}$ is different from the one presented in Ref. [11] and is more general than the one presented in Ref. [1] since it is also applied to anisotropic materials. It is shown that using $\widetilde{\boldsymbol{D}}$ is equivalent to employing the plate theory based on the physical neutral surface.

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