Distributed Cooperative Control Algorithm for Multi-UAV Mission Rendezvous

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Abstract: Multiple unmanned aerial vehicles (UAVs) cooperative operation is the main form for UAVs fighting in battlefield, and multi-UAV mission rendezvous is the premise of cooperative reconnaissance and attack missions. We propose a rendezvous control strategy, which divides the rendezvous process into two parts: The loose formation rendezvous and the close formation rendezvous. In the first stage, UAVs are supposed to reach the specific target locations simultaneously and form a loose formation. A distributed control strategy based on first-order consensus algorithm is presented to achieve this goal. Then the second stage is designed based on the second-order consensus algorithm to complete the transition from the loose formation to the close formation. This process needs the speeds and heading angles of UAVs to reach an agreement. Besides, control algorithms with a virtual leader are proposed, by which the formation states can reach a specific value. Finally, simulation results show that the control algorithms are capable of realizing the mission rendezvous of multi-UAV and the consistence of UAVs′ final states, which verify the effectiveness and feasibility of the designed control strategy.

Key words: unmanned aerial vehicles; loose formation rendezvous; close formation rendezvous; consensus algorithm; cooperative control

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0 Introduction

In the multiple unmanned aerial vehicles (multi-UAVs) cooperative operations, the rendezvous mission of UAVs means that the UAVs take off from different places, fly along the predesigned paths to reach a specified mission area [1-3]. Moreover, the final states and formation geometry of UAVs should satisfy certain requirements so as to facilitate the subsequent formation flight. Therefore, how to achieve the UAVs′ states consensus quickly and further realize rendezvous in an appointed area under the condition of limited communication is one of the research hotspots in aviation field.

The multi-UAV mission rendezvous is a typical cooperative control problem which generally consists of path planning and trajectory control problems. Thus the main rendezvous approaches can be divided into two categories: Trajectory-based strategies and velocity-based strategies. The trajectory-based strategy is essentially a cooperative path planning problem, whose commonly used methods include Voronoi diagram[3-4], Dubins curve[5-6] and differential geometry[7]. The speeds of UAVs can usually be determined once the path planning is completed. For example, McLain applied the cooperative control method based on coordination variables and coordination functions to the rendezvous mission and realized the simultaneous arrival successfully[3]. The velocity-based methods are to adjust the flight speed of UAVs according to path length, hence the UAVs can arrive simultaneous-

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ly. Jiang et al. proposed a rendezvous method in which velocity tuning and online trajectory adjustment are combined[8].

In recent years, multi-agent information consensus theory with the advantage of partial information interaction has been gradually applied to the formation control[9]. This method is essentially a velocity-based strategy. Olfati-Saber and Murry analyzed basic consensus algorithm and proved that the multi-agent system can guarantee the state to be consistent only through partial information exchange[10]. Ren and Beard studied the consistency of directed weighted network with switching topology, and pointed out that information consensus under dynamically changing topologies can be achieved asymptotically if the topology graph contains a spanning tree[11]. Yuan et al. designed a decentralized control strategy based on consensus algorithm for simultaneous arrival of multiple UAVs[12]. Zhu et al. proposed a sectional rendezvous strategy based on information consensus, which was decomposed of three steps: Choosing reference rendezvous and allocating target rendezvous, as well as generating loose formation and generating close formation with velocity and attitude consensus[13].

Most current studies took the simultaneous arrival as the main rendezvous objective and explored many effective solutions. However, few designed the strategy to control the final states of UAVs to reach an agreement, which was not conductive to subsequent tasks.

Therefore, we propose a novel rendezvous strategy for multi-UAV. First, a distributed control strategy based on first-order consensus theory is presented to ensure that the UAVs arrive at appointed mission area simultaneously and form a loose formation. Then, the second-order consensus algorithm is designed to complete the transition from loose formation to close formation and control the speeds and heading angles to reach an agreement. Furthermore, this control strategy is designed based on distributed architecture without central node which is capable of dealing with dynamic environment.

1 Rendezvous Problem Description

1.1 Problem description

The rendezvous problem for multi-UAV refers to the process of the UAVs with arbitrary initial states flying along different paths to arrive at the designated mission area and constitute a specific formation. We assume that the UAVs have the ability to plan path offline or online, estimate the path length in real time, and fly autonomously along the planned path.

We denote the initial information and final information of UAVs as $P_s(x_{is}, y_{is}, v_{is}, \psi_{is})$ and $P_f(x_{if}, y_{if}, v_{if}, \psi_{if})$, where x, y is the position coordinates and v, ψ are the speed and the heading angle, respectively. Besides, the formation constraint is given as C. Then, the goals of rendezvous problem can be described as: (1) UAVs arrive at specific mission area simultaneously; (2) UAVs form a stable formation; (3) the speeds and heading angles of UAVs reach an agreement when they complete rendezvous.

The research on multi-UAV rendezvous problem focuses on the design of a control strategy to realize the above goals, while reducing the influence of unfavorable factors, such as path error and sudden threat.

1.2 UAV kinematics model

UAVs are considered to be at two-dimensional area. Therefore, the model of UAV can be simplified as

$$\begin{aligned} \dot{x}_i &= v_i \cos\psi_i \\ \dot{y}_i &= v_i \sin\psi_i \\ \dot{\psi}_i &= \omega_i \end{aligned} \tag{1}$$

where $i \in I$ is the number of UAVs, (x, y) is the position coordinate, and v_i, ψ_i, ω_i represent the speed, the heading angle and the angle velocity of the i-UAV, respectively. Meanwhile, the physical performance constraints of UAVs are considered as

$$\begin{gathered} v_{\min,i} \leqslant v_i \leqslant v_{\max,i} \\ a_{\min,i} \leqslant a_i \leqslant a_{\max,i} \\ |\dot{\psi}_i| = |\omega_i| \leqslant \omega_{\max,i} \end{gathered} \tag{2}$$

where a_i is the acceleration of the i th UAV. These three constraints are the speed, the acceleration and the angle velocity limits, respectively.

We assume that the UAV autopilot has the ability to follow the given speed and heading control commands so as to hold speed and heading. The autopilot can be described as a first-order mathematical model

$$\begin{aligned}\dot{v}_i &= k_{v,i}(v_i^c - v_i)\\ \dot{\psi}_i &= k_{\psi,i}(\psi_i^c - \psi_i)\end{aligned} \tag{3}$$

where v_i^c, ψ_i^c represent the speed control command and the heading angle control command, and $k_{v,i}$, $k_{\psi,i}$ the parameters of autopilot, respectively.

1.3 Formation design and representation

Based on the premise whether the formation changes over time, the formation shapes can be divided into free formation and fixed formation. Appropriate formation can reduce fuel consumption so as to improve endurance. We take the diamond formation as research object, because it is widely used in formation flight and simulation. The common formation representation methods are l-ψ and l-l [14]. Both algorithms can obtain the position coordinates of UAVs. Compared with l-ψ which contains numerous trigonometric operations, l-l has the simpler solving process. Therefore, l-l method is adopted in following calculation.

Fig. 1 depicts the diamond formation. In the ground coordinate $x_g o y_g$, the direction of the formation movement is set as positive direction of x_b-axis, and y_b-axis is perpendicular to x_b-axis. Assume that the side length of diamond is d, the heading angle of UAV$_1$ is ψ_1. Take UAV$_1$ as vertex, the intersection angle of the side and diagonal is α. Then the position relationship of the formation can be represented as matrices $\boldsymbol{S}_x$, $\boldsymbol{S}_y$, whose elements are the projections of the distances between UAVs and reference point on x_b-axis and y_b-axis, respectively. For example, the ith row of $\boldsymbol{S}_x$ represents the projections of the distances between each UAV and the ith UAV on x_b-axis.

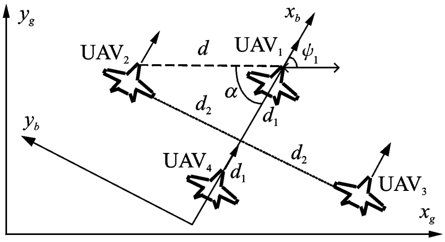


Fig. 1 Schematic diagram of diamond formation

As for the diamond formation shown in Fig. 1, $\boldsymbol{S}_x$, $\boldsymbol{S}_y$ can be calculated as

$$\boldsymbol{S}_x = \begin{bmatrix} 0 & -d_1 & -d_1 & -2d_1 \\ d_1 & 0 & 0 & -d_1 \\ d_1 & 0 & 0 & -d_1 \\ 2d_1 & d_1 & d_1 & 0 \end{bmatrix} \tag{4}$$

$$\boldsymbol{S}_y = \begin{bmatrix} 0 & d_2 & -d_2 & 0 \\ -d_2 & 0 & -2d_2 & -d_2 \\ d_2 & 2d_2 & 0 & d_2 \\ 0 & d_2 & -d_2 & 0 \end{bmatrix} \tag{5}$$

Therefore, as long as the formation reference point is given, the position of each UAV can be obtained by position relationship matrices. However, this method researches the UAVs' position relationship in formation coordinate. Thus we should first rotate the ground coordinate to convert the UAVs' position relationship to formation coordinate by

$$\begin{bmatrix} d_{xb} \\ d_{yb} \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix} \cdot \begin{bmatrix} d_{xg} \\ d_{yg} \end{bmatrix} \tag{6}$$

where d_{xb}, d_{yb} represent the distances between UAVs along the x_b-axis and y_b-axis in formation coordinate, respectively, and d_{xg}, d_{yg} the distances in ground coordinate.

2 Multi-UAV Rendezvous Control Strategy Based on Consensus Theory

2.1 Rendezvous strategy

Few studies have applied concensus theory to the close formation, because the buffeting phenomenon of consensus theory will lead UAVs to easily collide in the process of formation. Therefore, we propose a piecewise rendezvous strategy which consists of loose formation rendezvous and close formation rendezvous.

Firstly, in the loose formation rendezvous process, Dubins algorithm is adopted to pre-plan a shortest path, and the time consensus control algorithm achieves that the UAVs distributed in wide area arrive at the appointed mission area simultaneously to build a loose formation. Then in the close formation rendezvous process, the consensus control algorithm is designed based on the speeds, heading angles and positions consensus, thus the close formation can be built and the speeds and the heading angles can reach an agreement.

2.2 Loose formation rendezvous

Assume that the UAVs take off from different places and fly along the pre-planned paths. The estimated time of arrival (ETA) is set as the coordination variable which is exchanged through communication network between UAVs. UAVs calculate the speed control command according to the state information of its own and its neighbors. Then the autopilot checks the control command to adjust the speed so as to ensure UAVs arrive at designated mission area simultaneously. Since the Dubins path has terminal constraint, the heading angles of UAVs can reach an agreement when the loose formation rendezvous is completed.

To realize the multi-UAV loose formation rendezvous, a distributed control structure base on consensus algorithm is proposed as shown in Fig. 2. The process of loose formation rendezvous can be described as follows:

(1) Dubins path planning module generates the paths for UAVs according to the initial and the final state information, and obtains the lengths of remaining paths.

(2) The consensus control module receives the information of remaining path lengths, the speed of its own and the coordination variable, then calculates the speed control command based on the time consensus algorithm.

(3) The UAV autopilot tracks the speed and heading angle control commands.

The process above is performed in real-time. Therefore, even if the path length or flight speed changes, the rendezvous time can also converge to a same value.

2.2.1 Control strategy

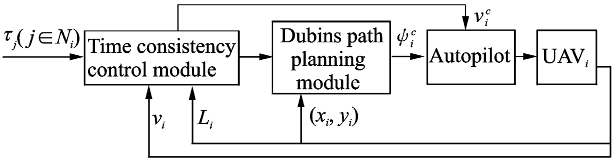


Fig. 2 Distributed control structure of loose formation rendezvous for multi-UAVs

Assume that the path planning of UAVs has been completed and the communication topology structure is invariant. The remaining path length L_i of the i-th UAV can be calculated according to information fed back by GPS. The time derivative of L_i is

$$\dot{L}_i = -v_i \tag{7}$$

where v_i is the speed of the i-th UAV at time t.

If the UAV flies to the target rendezvous location with speed v_i, the time needed is $\tau_i = L_i/v_i$, i. e. the expected arrival time is

$$T_i = t + \tau_i = t + \frac{L_i}{v_i} \tag{8}$$

In order to ensure that UAVs arrive at target rendezvous locations at the same time, the expected arrival time of all UAVs should converge to the same, i. e. $T_i \to T_j, \forall i,j \in V$, which is also can be expressed as the remaining time to arrive at destination to converge to the same, i. e. $\tau_i \to \tau_j, \forall i,j \in V$. Taking the derivation of Eq. (8) with respect to t, the derivative of T_i can be expressed as

$$\dot{T}_i = 1 + \left(\frac{\dot{L}_i}{v_i}\right) = 1 + \frac{v_i\dot{L}_i - L_i\dot{v}_i}{v_i^2} = \frac{-\tau_i\dot{v}_i}{v_i} = \frac{-\tau_i \cdot k_{v,i}(v_i^c - v_i)}{v_i} \tag{9}$$

Choose τ_i as the coordination variable and the right-most item of Eq. (9) as the control input u_i, then we can obtain the speed control command as

$$v_i^c = v_i - \frac{v_i u_i}{k_{v,i}\tau_i} \tag{10}$$

According to the first-order consensus algorithm, the time consensus control strategy can be designed as

$$\begin{cases} u_i = -\sum\limits_{j\in \mathbf{N}_i}^{n} a_{ij}(\tau_i - \tau_j) \\ v_i^c = v_i - v_i u_i/(k_{v,i}\tau_i) \end{cases} \tag{11}$$

where N_i is the set of neighbors of the ith UAV, a_{ij} the element of adjacency matrix of communication network.

The consensus algorithm designed above can be explained as each UAV obtains the state information of neighbors and then feeds back the sum of differences of state to speed controller, thus finally UAVs simultaneously arrive by adjusting speed.

2.2.2 Control strategy with virtual leader

The concensus control strategy designed above is unable to guarantee that the formation reaches an expected state. Hence a virtual leader is introduced to formation, so that the states of all UAVs converge to those of the virtual leader. In traditional leader-follower method, all UAVs take the leader as a reference point, which results in the paralysis of entire formation once the leader fails. While in the virtual structure, virtual leader has no risk of failure. Therefore, a rendezvous control strategy with virtual leader is designed.

Assume that there is a virtual leader whose kinematics model is the same as those of other UAVs. The number of leader is set as 0, then its expected arrival time can be expressed as $\tau_0 = L_0/v_0$. The motion of virtual leader will not be affected by other UAVs, which means that its neighbor set is empty. Generally, not all UAVs can receive information about virtual leader. The motion of the virtual leader can be controlled by means of artificially setting its speed and path. The virtual leader is considered as a member of formation, same as other UAVs. Then the consensus control strategy can be designed as

$$\begin{aligned} u_i &= -\beta_i(\tau_i - \tau_0) - \sum_{j\in \mathbf{N}_i}^{n} a_{ij}(\tau_i - \tau_j) \\ v_i^c &= v_i - v_i u_i/(k_{v,i}\tau_i) \end{aligned} \tag{12}$$

where β_i is the information weight. If the i-th UAV can receive information from virtual leader, $\beta_i > 0$, otherwise $\beta_i = 0$.

2.2.3 Loop wait mode

Assume that the path length of the i-th UAV is L_i and its speed limit is $v_i \in [v_{\min}, v_{\max}]$, then the range of the time to arrive at destination along the pre-planned path is $T_i \in [L_i/v_{\max}, L_i/v_{\min}]$. All the UAVs should arrive at destination at the same time, hence the range of arrival time can be expressed as $T_a = T_1 \cap T_2 \cap \cdots \cap T_N$. During the flight, T_a is checked in real time. If $T_a \neq \varnothing$, the consensus algorithm can be adopted directly. However, when the remaining path lengths of UAVs are quite different or the range of speed is small, even if the speed of UAV with shorter path reaches the lower limit and the UAV with longer path reaches the upper limit, the simultaneous arrival of UAVs still can not be achieved. It means that T_a may be an empty set resulting in the failure of arriving simultaneously. Aiming at this problem, a loop wait mode is proposed. The pre-planned path remains unchanged and the UAV with shorter path waits for other UAVs through flying in circles to increase the path length and the arrival time.

If the ith UAV flies in circles with the radius of $R_{\min}$, the minimum turning radius, the added flight time for one circle can be calculated by $\Delta t_i = 2\pi R_{\min}/v_i$. Assume that $t_{\min i} = L_i/v_{\max}$ and $t_{\max i} = L_i/v_{\min}$, then the minimum time needed for all UAVs to arrive can be represented by $t_{\min} = \max\{t_{\min 1}, t_{\min 2}, \cdots, t_{\min N}\}$. The circles for each UAV to fly around can be determined by

$$n_i = \begin{cases} 0 & t_{\max i} \geqslant t_{\min} \\ \mathrm{ceil}\{(t_{\min} - t_{\max i})/\Delta t_i\} & t_{\max i} < t_{\min} \end{cases} \tag{13}$$

where $i = 1, 2, \cdots, N$, $\mathrm{ceil} = \{\cdots\}$ represents the minimum integer which is equal or greater than the number in bracket; n_i is taken as an integer which means the UAV will be back to the pre-planned path so as to avoid the path replanning and reduce the calculation. According to the number of circles, the remaining path lengths can be recalculated. And then UAVs can realize simultaneous arrival based on consensus algorithm designed before.

2.3 Close formation rendezvous

After completing the loose formation rendezvous, the heading angles of UAVs are about the same due to the end constraint of Dubins curve. However, the speed is not consistent and there is a gap between the loose formation and the standard formation. Therefore, the close formation rendezvous process is proposed to further achieve the final standard formation and the consistency between speed and heading angle simultaneously.

This process is based on the loose formation, thus the path planning, threat avoidance and other issues need not be considered. The close formation building process is to compress the loose formation to reduce the distance between UAVs to a certain value so as to make the formation more compact. Therefore, the objective of close formation rendezvous process can be described as

$$\begin{cases} \lim\limits_{t\to\infty}(\psi_i-\psi_j)=0 \\ \lim\limits_{t\to\infty}(v_i-v_j)=0 \\ \lim\limits_{t\to\infty}(x_i-x_j)\in C_1 \\ \lim\limits_{t\to\infty}(y_i-y_j)\in C_2 \end{cases} \tag{14}$$

where constants C_1, C_2 are the formation geometrical constraints which represent the desired distance between the ith UAV and jth UAV. Speed and heading angle can be set as coordination variables directly. However, the positions of UAVs do not coincide because the UAVs should form certain formation. According to the formation representation method introduced above, the coordination of position can be represented by relative position relation, which means that the constant C_1, C_2 can be substituted by the corresponding elements in $\boldsymbol{S}_x, \boldsymbol{S}_y$.

2.3.1 Control strategy

In the process of building close formation, UAV should adjust position as well as adjust its speed and heading angle. Based on the consensus algorithm, the speed and the heading angle can converge to the same easily. And the position in forward direction can be adjusted by changing speed directly. However, after forming a loose formation, the heading angles of all UAVs reach a consensus, leading to the difficulty of adjusting position in lateral direction.

In order to solve this problem, we assume that the UAV has a very small deviation on its heading angle which is denoted by $\Delta\psi_i$, as shown in Fig. 3. Then there is a speed component along the y_b-axis which can be used to adjust the UAV position in lateral direction. The speed components along the x_b-axis and y_b-axis are represented by

$$\begin{aligned} v_{xi}^b &= v_i \cdot \cos(\Delta\psi_i) \\ v_{yi}^b &= v_i \cdot \sin(\Delta\psi_i) \end{aligned} \tag{15}$$

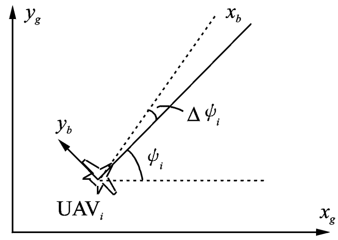


Fig. 3 Diagram of UAV motion in formation coordinate

Because $\Delta\psi_i$ is very small, Eq. (15) can be simplified as

$$\begin{aligned} v_{xi}^b &\approx v_i \\ v_{yi}^b &\approx v_i \cdot \Delta\psi_i \end{aligned} \tag{16}$$

which indicates that the position of UAV in formation can be adjusted by controlling the speed and the heading angle. Based on the second-order consensus algorithm, control strategy of close formation can be designed as

$$\begin{aligned} \dot{v}_i &= -\sum_{j\in \mathbf{N}_i} a_{ij}\left[(v_i-v_j)+\gamma_v(x_i^b-x_j^b-s_{ji}^x)\right] \\ \dot{\psi}_i &= -\sum_{j\in \mathbf{N}_i} a_{ij}\left[(\psi_i-\psi_j)+\gamma_\psi(y_i^b-y_j^b-s_{ji}^y)\right] \end{aligned} \tag{17}$$

where s_{ji}^x, s_{ji}^y are the corresponding elements of $\boldsymbol{S}_x$ $\boldsymbol{S}_y$, respectively, γ_v, γ_ψ the weight coefficients of states. The first term on the right of Eq. (17) represents the consensus control of the speed and the heading angle, while the second term represents the consensus control of position. Therefore, the speed control command and the heading angle control command can be calculated by

$$\begin{aligned} v_i^c &= v_i-\frac{1}{k_{v,i}}\sum_{j\in \mathbf{N}_i} a_{ij}\left[(v_i-v_j)+\gamma_v(x_i^b-x_j^b-x_{ji}^x)\right] \\ \psi_i^c &= \psi_i-\frac{1}{k_{\psi,i}}\sum_{j\in N_i} a_{ij}\left[(\psi_i-\psi_j)+\gamma_\psi(y_i^b-y_j^b-s_{ji}^y)\right] \end{aligned} \tag{18}$$

This conclusion is on the premise that the initial positions of UAVs are not much different, which also illustrates the importance of the loose formation process.

2.3.2 Control strategy with virtual leader

The above consensus algorithm is unable to guarantee that the formation reaches to an expected state. Generally, in order to benefit the follow-up formation flight, it is expected that the speed and the heading angle of UAVs reach desired values when the close formation is built. Therefore, a rendezvous control strategy with virtual leader is proposed. Similarly, assume that there is a virtual leader whose kinematics model is the same as those of other UAVs and the number is 0. Then the consensus control strategy can be designed as

$$\begin{aligned}\dot{v}_i &= -\beta_i\left[(v_i - v_0) + \gamma_v(x_i^b - x_0^b - s_{1,i+1}^x)\right] - \\ &\quad \sum_{j\in N_i} a_{ij}\left[(v_i - v_j) + \gamma_v(x_i^b - x_j^b - s_{j+1,i+1}^x)\right] \\ \dot{\psi}_i &= -\beta_i\left[(\psi_i - \psi_0) + \gamma_\psi(y_i^b - y_0^b - s_{1,i+1}^y)\right] - \\ &\quad \sum_{j\in N_i} a_{ij}\left[(\psi_i - \psi_j) + \gamma_\psi(y_i^b - y_j^b - s_{j+1,i+1}^y)\right] \quad (19)\end{aligned}$$

where $s_{1,i+1}^x, s_{j+1,i+1}^x, s_{1,i+1}^y, s_{j+1,i+1}^y$ are the corresponding elements of $\boldsymbol{S}_x, \boldsymbol{S}_y$, whose first row represents the projections of the distances between each UAV and the virtual leader on x_a-axis and y_a-axis, respectively.

3 Simulation Analysis

In this section, several simulations are carried out to verify the multi-UAV rendezvous control strategy proposed above. Assume that there are 4 UAVs whose initial state and desired state are shown in Table 1. The target positions of UAVs satisfy the geometric constraint of diamond formation whose side length is 50 m. We choose the minimum turning radius of UAVs as the radius of initial circle and final circle to design the Dubins path, and set $R_{\min} = 100$ m. The path lengths of UAVs calculated by Dubins algorithm are shown in Table. 2.

Table 1 UAVs' initial state and desired state information

UAV	Initial position/m	Initial speed/ (m · s^{-1})	Initial heading/rad	Target position/m	Target heading/rad
UAV$_1$	[4 000, 200]	54	$3\pi/4$	[6 050, 4 000]	$-\pi/4$
UAV$_2$	[1 200, 1 500]	40	$\pi/2$	[6 050, 4 050]	$-\pi/4$
UAV$_3$	[2 500, 200]	48	0	[6 000, 4 000]	$-\pi/4$
UAV$_4$	[2 000, 3 200]	60	$-\pi/4$	[6 000, 4 050]	$-\pi/4$

Table 2 Initial path length of UAVs

UAV label	UAV$_1$	UAV$_2$	UAV$_3$	UAV$_4$
Path length/m	4 441	5 531	5 238	4 122

It can be seen from Table 2 that the initial path lengths are different, thus it is necessary to adjust the speed of each UAV to achieve the goal that all UAVs arrive at the designated rendezvous area simultaneously.

3.1 Simulation and analysis of loose formation rendezvous process

Assume that the UAVs are isomorphic and the characteristic parameters are shown in Table 3. The positive acceleration means UAV accelerating while the negative acceleration means UAV decelerating.

Suppose that there is a virtual leader whose kinematics model is the same as those of other UAVs and the number is 0. In this simulation, the virtual leader is assumed to communicate with UAV$_2$ and UAV$_3$ in the formation. The Communication topology of formation with virtual leader is shown in Fig. 4.

The path length and the speed of UAV$_0$ are set as 5 000 m and 50 m/s, respectively, then the ETA of UAV$_0$ is 100 s. Simulation results are shown in Fig. 5. The red curves are the variables

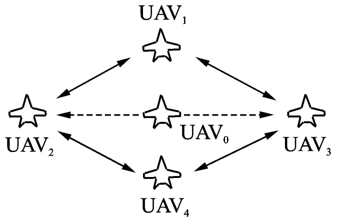


Fig. 4 Communication topology of formation with virtual leader

Table 3 Characteristic parameters of UAV

Parameter	Value
Autopilot parameter of speed $k_{v,i}/(\mathrm{s}^{-1})$	0.2
Autopilot parameter of heading angle $k_{\psi,i}/(\mathrm{s}^{-1})$	0.05
Speed $v_i/(\mathrm{m}\cdot\mathrm{s}^{-1})$	40—60
Acceleration $a_i/(\mathrm{m}\cdot\mathrm{s}^{-2})$	−2—2
Maximum angle velocity ω_i/rad	0.1

of the leader and the black curves are the variables of UAV formation. As seen from Fig. 5, the UAVs can achieve the goal of reaching target rendezvous positions at the same time and the arrival times of UAVs are consistent with that of virtual leader. After about 10 s, the speeds of UAVs are almost stable whose values are 44, 56, 52 and 40 m/s. Thus, UAVs reach at rendezvous at a specified time, and the strategy of loose formation rendezvous with virtual leader is effective.

3.2 Simulation and analysis of close formation rendezvous process

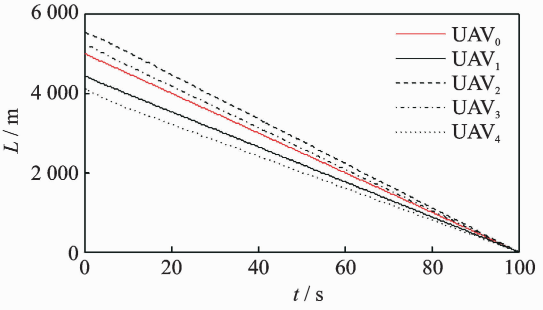
The information obtained from the loose formation rendezvous of previous simulations is used as the initial information for the close formation rendezvous which is shown in Table 4.

Table 4 UAVs′ initial state information for close formation

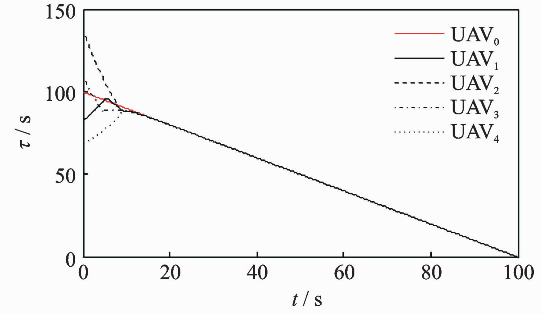
UAV	Position / m	Speed / $(\mathrm{m}\cdot\mathrm{s}^{-1})$	Heading / rad
UAV_1	[6 050, 4 000]	44	$-\pi/4$
UAV_2	[6 050, 4 050]	56	$-\pi/4$
UAV_3	[6 000, 4 000]	52	$-\pi/4$
UAV_4	[6 000, 4 050]	40	$-\pi/4$

The characteristic parameters and communication topology of UAVs remain unchanged. Suppose that the initial position and final position of UAV_0 are in the formation center, the speed is maintained at 50 m/s and heading angle is maintained at $-\pi/4$. The goal is to compress the loose formation into a close formation with side length of 5 m. Simulation results are shown in Fig. 6.

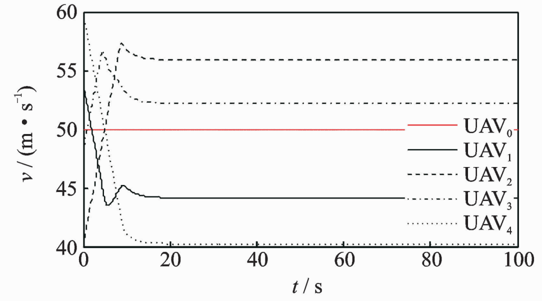
It can be seen from enlarge view of formation trajectory in Fig. 6(a) that the UAVs form the desired formation. And Fig. 6(b) shows that the distances between UAVs reach stable and satisfy the geometric constraints of close formation. In addition, Figs. 6(c, d) depict the speed and the heading angle curves of UAVs, where the red curve represents UAV_0. Obviously, the speeds and the heading angles of UAVs are consistent with those of the virtual leader eventually. Therefore, the control strategy with a virtual leader can lead the speeds and the heading angles of UAVs to reach specific values, namely, the final states of UAVs can be artificially set by presetting the speed and the heading angle of the virtual leader, which is beneficial to the subsequent formation flight.



(a) Remaining path lengths of UAVs



(b) Remaining times to arrive for UAVs

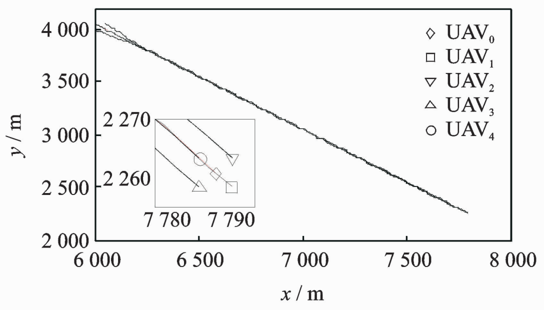


(c) Speeds of UAVs

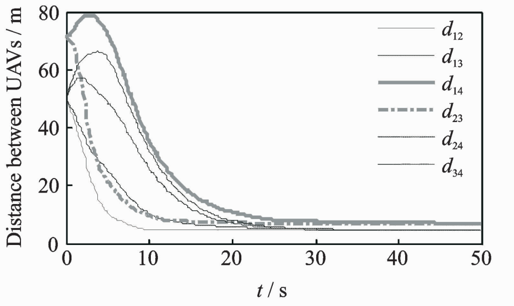
Fig. 5 Simulation results of loose formation rendezvous

4 Conclusions

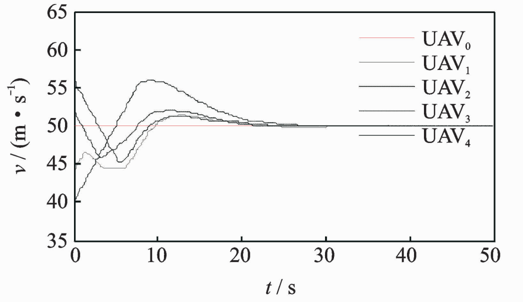
A piecewise rendezvous control strategy based on consensus algorithm for multi-UAVs is



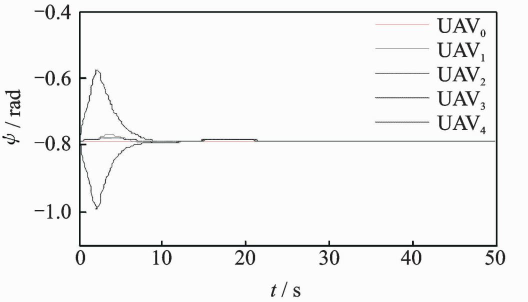
(a) Trajectory



(b) Distances between UAVs



(c) Speeds of UAVs



(d) Heading angles of UAVs

Fig. 6 Simulation results of close formation rendezvous

proposed. The rendezvous process is divided into loose formation rendezvous and close formation rendezvous. First, in the process of loose formation rendezvous, a control strategy based on the first-order consensus algorithm is designed to lead UAVs to simultaneously arrive, who are initially distributed in a wide area. Further, a virtual leader is introduced into the formation to control the UAVs to arrive at the designated rendezvous positions at a given moment. Then in the process of close formation rendezvous, a control strategy based on the second-order consensus algorithm is proposed which achieves the consistency of the final positions, the speeds and the heading angles of UAVs. Similarly, a control strategy with a virtual leader is designed to control the states of UAVs to reach specific values. Finally, simulation results illustrate the effectiveness of the proposed rendezvous strategy.

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