An Algorithm for Labeling Stable Regions of a Class of Time-Delay Systems with Abscissa

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(Received 11 December 2017; revised 7 January 2018; accepted 20 January 2018)

Abstract: Stability is usually in the sense of Lyapunov's asymptotical stability, thus the solutions starting from points close to a stable equilibrium may have a very long transient. In the applications of time-delayed feedback controls, it is important not only to determine the stable regions in the gain plane or gain space, but also to find out the abscissa that can be used as an index of stability. Based on the D-subdivision method, this paper proposes a simple algorithm for finding and labeling the stable regions in feedback gain plane with abscissa. The labeled sub-regions with smaller abscissa are better in applications. The main results are presented for the controlled pendulum or inverted pendulum under a delayed feedback, and are illustrated with two case studies.

Key words: time delay; stability; Lyapunov's stability; abscissa; delayed feedback; optimal feedback gain

CLC number: O317 **Document code**: A **Article ID**: 1005-1120(2018)01-0094-07

0 Introduction

Time delay exists commonly in control applications, such as in digital controller where the time delay is resulted from using sampling and zero-order holder, and in human-interaction systems where the time delay is produced due to the delay of human's response. Dynamical systems with time delays are called time-delay systems, which can be classified into two categories: Retarded type and neutral type. On one hand, the presence of time delay may deteriorate the system's performance and even destabilizes the system, which may occur even when the delay is very short[1]. On the other hand, the effect of time delay on the system dynamics may be positive, such as in the study of sway reduction of cranes[2]. Thus, stability analysis of time-delay systems has been one of the major concerns in many control applications.

Usually, stability is in the sense of Lyapunov's asymptotical stability. An equilibrium of a time-delay system of retarded type is asymptotically stable if all the characteristic roots of the corresponding linearized system have negative real parts only. This is true for time-delay systems of neutral type under certain conditions. Many methods and criteria have been established for the stability analysis of time-delay systems, such as the D-subdivision method[3], the method of stability switches[4], and the stability criteria developed on the basis of Argument Principle, including the Nyquist criterion[5], Integral estimation criterion[6], Stepan-Hassard theorem[7], etc. The D-subdivision method works effectively in determining the stable regions in a parametric plane such as the feedback gain plane of a controlled system by using the critical stable conditions. The method of stability switches is preferable when only one parameter is focused. The stability criteria are used mainly for stability test of given time-delay systems, but they can also be used for

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How to cite this article: Wang Qiang, Wang Zaihua. An algorithm for labeling stable regions of a class of time-delay systems with abscissa[J]. Trans. Nanjing Univ. Aero. Astro., 2018,35(1):94-100.
http://dx.doi.org/10.16356/j.1005-1120.2018.01.094

studying the problems of stability switches[8]. In some cases, the solutions starting from points close to a stable equilibrium may have a very long transient.

Abscissa is a real number defined as the real part of the rightmost characteristic roots of a dynamical system, and it is an index for measuring stability. An equilibrium of a time-delay system is asymptotically stable in the Lyapunov's sense if the abscissa is negative, the smaller (the larger in absolute) the abscissa is, the better the stability is, and the solutions starting from points close to a stable equilibrium will decay to the stable equilibrium faster. Thus, in the control design of a delayed feedback control, it is important not only to determine the stable regions in the gain plane or gain space, but also to find out the pair of optimal feedback gains that minimizes the abscissa within a given stable region. Roughly speaking, all the above-mentioned stability criteria can be used for the calculation of the abscissa of a given time-delay system, among them the integral estimation criterion seems more effective in implementation. These criteria could be used directly for the calculation of the abscissa one-by-one at each gridding node, but seemingly it is not easy to obtain the optimal abscissa and the optimal feedback gains because manual intervention seems necessary in the calculation. Thus, more effective method or algorithms are needed to find the optimal abscissa and the optimal feedback gains within a stable region in the gain plane. This paper presents a simple algorithm for labeling the stable region in feedback gain plane with different abscissa on the basis of the D-subdivision method, and it is found that the sub-region labeled with the smallest abscissa is preferable in applications. For clarity in presentation, the main results are introduced for the controlled pendulum or inverted pendulum with delayed feedback. The proposed algorithm works also for labeling the stable regions of a pair of feedback gains of any controlled systems with delayed feedback.

1 Labeling of Stable Regions

Pendulum and inverted pendulum are two very popular models for many mechanical systems/structures. When a delayed acceleration-velocity-position control is used for controlling a pendulum or inverted pendulum, the controlled system is described by

$$m\ddot{x}(t)+c\dot{x}(t)+kx(t)=-k_a\ddot{x}(t-\tau_1)-k_d\dot{x}(t-\tau_2)-k_p x(t-\tau_3) \tag{1}$$

where m, c, k are the system parameters, $\tau_1 \geqslant 0$, $\tau_2 \geqslant 0$, $\tau_3 \geqslant 0$ the time delays, and k_a, k_d, k_p the feedback gains. Eq. (1) with $k>0$ corresponds to a pendulum and with $k<0$ corresponds to an inverted pendulum. Eq. (1) is called retarded (or neutral) type when $k_a=0$ (or $k_a \neq 0$). The characteristic equation is

$$D(\lambda)=p_0(\lambda)+k_a\lambda^2\,\mathrm{e}^{-\lambda\tau_1}+k_d\lambda\,\mathrm{e}^{-\lambda\tau_2}+k_p\,\mathrm{e}^{-\lambda\tau_3} \tag{2}$$

where $p_0(\lambda)=m\lambda^2+c\lambda+k$. The abscissa is defined by

$$\alpha=\max\{\mathscr{R}(\lambda):D(\lambda)=0\}$$

where $\mathscr{R}(\lambda)$ represents the real part of complex number λ. $D(\lambda)$ has infinite many roots, but the number of roots with positive real part must be finite. Thus the abscissa α must be finite[3]. For the case of $|k_a|<1$, the time-delay system is asymptotically stable if $\alpha<0$. Without loss of generality, assume that $\alpha<0$ when $\tau_1=0, \tau_2=0, \tau_3=0$. Then conditions on k_a, k_d, k_p can be obtained by using the Routh-Hurwitz criterion. Due to the continuous-dependence on the delays, the time-delay system keeps asymptotically stable if the delays are small enough.

1.1 Introduction of D-subdivision method

For given small delays, the stable regions with respect to the gains k_a, k_d, k_p can be obtained by using the D-subdivision method, where the boundaries of the stable regions are plotted by using the critical stable curves determined from $D(\mathrm{i}\omega)=0\,(\mathrm{i}^2=-1)$. For a delayed proportional-derivative (PD) feedback with equal delays, $k_a=0$ and $\tau_2=\tau_3=\tau$, for example, the boundaries of the stable regions are determined by $\mathscr{R}(D(\mathrm{i}\omega))=0$ and $\mathscr{I}(D(\mathrm{i}\omega))=0$ (where $\mathscr{I}(z)$

stands for the imaginary part of complex number z), which give

$$k_p = (m\omega^2 - k)\cos(\omega\tau) + c\omega\sin(\omega\tau)$$

$$k_d = \frac{(m\omega^2 - k)\sin(\omega\tau) - c\omega\cos(\omega\tau)}{\omega}$$

when $\omega \neq 0$, and for $\omega = 0$ one has

$$k_p = -k$$

The critical curves divide the gain plane (k_p, k_d) into many open regions, in which none, one or more are stable. For finding the stable region and labeling the stable region with abscissa, the σ-critical stable curves, determined by $D(\sigma + \mathrm{i}\omega) = 0$ or equivalently $\mathscr{R}(D(\sigma+\mathrm{i}\omega)) = 0, \mathscr{I}(D(\sigma+\mathrm{i}\omega)) = 0$, will be used, they are plotted by $\{(k_p, k_d): 0 \leqslant \omega < +\infty\}$, where

$$k_p = \frac{(m\omega^2 + m\sigma^2 - k)\omega\cos(\omega\tau)}{\omega}\,\mathrm{e}^{\sigma\tau} + \frac{(m\sigma^3 + c\sigma^2 + k\sigma + m\omega^2\sigma - c\omega^2)\sin(\omega\tau)}{\omega}\,\mathrm{e}^{\sigma\tau}$$

$$k_d = -\frac{(2m\omega\sigma + c\omega)\cos(\omega\tau)}{\omega}\,\mathrm{e}^{\sigma\tau} - \frac{(m\sigma^2 + c\sigma - m\omega^2 + k)\sin(\omega\tau)}{\omega}\,\mathrm{e}^{\sigma\tau}$$

Similar results can be obtained for a delayed PD feedback with $k_a = 0$ and $\tau_2 = \tau, \tau_3 = 2\tau$, and for a delayed acceleration-derivative (AD) feedback with $k_p = 0$ and $\tau_2 = \tau_3 = \tau$, as well as the one with $k_p = 0$ and $\tau_2 = \tau, \tau_3 = 2\tau$.

For any point passed by a σ-critical stable curve, the corresponding time-delay system has a characteristic root with real part σ. The σ-critical stable curves divide the gain plane into a number of sub-regions, which can be classified into two classes: the σ-stable ones for which the corresponding $D(\lambda)$ has roots with real parts less than σ only, and the σ-unstable ones for which the corresponding $D(\lambda)$ has at least one root with real parts larger than σ. The 0-stable ones are the stable regions in the Lyapunov's sense. The stable regions as well as the σ-stable regions can be determined graphically.

The σ-critical stable curves with different values of σ may intersect with each other, or do not intersect with each other at all. For the case when intersection happens, the intersect point should be marked with the color of the σ-critical stable curve corresponding to the largest σ. Firstly, choose two real figures $\sigma_{\min} < 0, \sigma_{\max} > 0$ such that the abscissa α for all parameter combinations in a given region of the parameter plane is in the interval $[\sigma_{\min}, \sigma_{\max}]$, then the process of labeling the stable region can be completed in the following major steps.

1.2 Subdivision of given region via $\boldsymbol{\sigma}$-critical stable curves with negative $\boldsymbol{\sigma}$

Starting from $\sigma = \sigma_{\min}$ to $\sigma = 0$ by a small step $\delta\sigma$, and for each node of σ, the σ-critical stable curves are drawn, and each point on the σ-critical stable curve is marked with a designated color in the color set. Let $\sigma_2 > \sigma_1$, if the σ_1-critical stable curve intersects with the σ_2-critical stable curve, the intersect points should be marked with the color of the σ_2-critical stable curve. If $\sigma_{\min}$ is chosen small enough (or equivalently, $|\sigma_{\min}|$ is large enough), every point in the given region of the parameter plane is labeled by the color of the σ-critical stable curves with $\sigma \in [\sigma_{\min}, 0]$.

1.3 Erasure of unstable regions via $\boldsymbol{\sigma}$-critical stable curves with positive $\boldsymbol{\sigma}$

Points passed by a σ-critical stable curve with positive σ correspond to the case when the time-delay system has at least one pair of characteristic roots with positive real part, so they are not belong to the stable regions. Further effort is required to erase the unstable points from the given region of the parameter plane labeled by the color of the σ-critical stable curves with $\sigma \in [\sigma_{\min}, 0]$, by using white color of the σ-critical stable curves with $\sigma \in [0, \sigma_{\max}]$. For σ from 0 to $\sigma_{\max}$ (or from $\sigma_{\max}$ to 0) by a small step $\delta\sigma$, and for each node σ, plot the σ-critical stable curve by white color. Then, all the points in the given region of the parameter plane have been marked with different color characterizing the stable region, showing different level of the abscissa in $[\sigma_{\min}, 0)$.

1.4 Asymptote issue

The σ-critical stable curves can be either continuous or discontinuous. For the continuous case, the stable region can be labeled simply by using the above two steps. If there are some break points on the σ-critical stable curves, just like in the applications to time-delay systems of neutral type, the plot in these points will create some asymptotes. For generality, assume that the σ-critical stable curves are plotted by $(f(\sigma,\omega), g(\sigma,\omega))$ as ω varies from 0 to $+\infty$, and they have discontinuity. The first asymptote is defined by

$$k(\sigma)(x-x_0)+y_0=0$$

where $(x_0,\ y_0)=(\lim_{\omega\to 0} f(\sigma,\omega),\ \lim_{\omega\to 0} g(\sigma,\omega))$, and $k(\sigma)$ is the derivation of the derivative of y from the implicit function $D(\sigma)$ to x, calculated by

$$k(\sigma)=-\frac{\frac{\partial D(\sigma)}{\partial x}}{\frac{\partial D(\sigma)}{\partial y}}$$

where x and y are the gain values of the feedback control.

Except the first asymptote, stability switches do not occur at the both sides of the asymptotes, which are not a part of critical stable curves, so we have to avoid these asymptotes when plotting the σ-critical stable curves. For a fixed σ, let $0<\omega_{c1},\omega_{c2},\cdots,\omega_{ck},\cdots$, be the roots of the denominator of $f(\sigma,\omega)$ and $g(\sigma,\omega)$, then all the points $(f(\sigma,\ \omega_{ci}), g(\sigma,\ \omega_{ci}))$ with $i=1,2,\cdots$ should be avoided in plotting the σ-critical stable curves with this σ.

2 Algorithm

Below is the algorithm for labeling the stable region in feedback gain plane of a time-delay system with abscissa. N,K are two integers satisfying $\sigma_{\min}=N*\delta\sigma$ and $\sigma_{\max}=K*\delta\sigma$.

INPUT: The color set C; σ-critical stable equations: $x=f(\sigma,\omega), y=g(\sigma,\omega)$; the range of considered region S; frequency length M; frequency step length $\delta\omega$; negative integer N; positive integer K; step length $\delta\sigma$.

OUTPUT: The labeled stable region S.

Step 1 For $n=N,N+1,\cdots,K$ do Steps 2—3.

Step 2 $\sigma\leftarrow n*\delta\sigma$. If $\sigma<0$, choose the color identified by σ in C, else choose white color.

Step 3 $m=0,1,\cdots,M$, do Steps 4—5.

Step 4 $\omega\leftarrow m*\delta\omega$.

Step 5 Plot point $(f(\sigma,\omega), g(\sigma,\omega))$ with the designated color.

Step 6 Output the plot of S.

Terminate.

If the denominators of $f(\sigma,\omega)$ and $g(\sigma,\omega)$ have nonzero real root ω_σ for ω with a fixed σ, Step 4 will be replaced with

Step 4 $\omega\leftarrow m*\delta\omega$. If $|\omega-\omega_\sigma|<\delta\omega$ turn to Step 3.

When plotting the σ-critical stable curves with the available mathematical software, we draw lines rather than isolated points. To avoid plotting the asymptotes, we calculate the break points of $f(\sigma,\omega)$ and $g(\sigma,\omega)$ with the index of ω firstly, and then draw the piecewise curves. A successful application of the proposed algorithm requires a suitable estimation of $\sigma_{\min}$ and $\sigma_{\max}$. In many applications, the delays are small, thus the estimated values of $\sigma_{\min}$ and $\sigma_{\max}$ can be chosen based on the abscissa when all the delays equal zero. When the delays are not small, the estimation of $\sigma_{\min}$ and $\sigma_{\max}$ is left for further investigation.

3 Examples

In the following two case studies, only the simply connected stable region close to the origin of the gain plane is considered.

3.1 Example 1

Consider the following controlled system in dimensionless form

$$\ddot{x}(t)+2\xi\dot{x}(t)+x(t)=-px(t-r\tau)-d\dot{x}(t-\tau) \tag{3}$$

where p and q are the gain values of position and velocity respectively, and r is the ratio coefficient of position delay and velocity delay. The delay values are assumed small, and only the stable region containing the origin of the parameter plane is considered. The characteristic function of system (Eq. (3)) is

$$D(\lambda)=\lambda^2+(2\xi+d\,\mathrm{e}^{-\lambda\tau})\lambda+p\,\mathrm{e}^{-r\lambda\tau}+1$$

Separating the real and imaginary parts of $D(\sigma+\mathrm{i}\omega)=0$, and solving the gains p,d from linear equations $\mathscr{R}(D(\sigma+\mathrm{i}\omega))=0, \mathscr{I}(D(\sigma+\mathrm{i}\omega))=0$ gives

$$p=\frac{((\omega^2+\sigma^2-1)\omega\cos(\omega\tau)}{\cos((r-1)\omega\tau)\omega+\sin((r-1)\omega\tau)\sigma}\mathrm{e}^{r\sigma\tau}+\frac{(\sigma^3+2\xi\sigma^2+\omega^2\sigma+\sigma+2\xi\omega^2)\sin(\omega\tau)}{\cos((r-1)\omega\tau)\omega+\sin((r-1)\omega\tau)\sigma}\mathrm{e}^{r\sigma\tau}$$

$$d=\frac{((-2\sigma-2\xi)\omega\cos(r\omega\tau)}{\cos((r-1)\omega\tau)\omega+\sin((r-1)\omega\tau)\sigma}\mathrm{e}^{\sigma\tau}-\frac{(\sigma^2+2\xi\sigma-\omega^2+1)\sin(r\omega\tau)}{\cos((r-1)\omega\tau)\omega-\sin((r-1)\omega\tau)\sigma}\mathrm{e}^{\sigma\tau}$$

Fig. 1 shows the stable regions close the origin in the gain plane of system (Eq. (3)) with $r=1$ and different parameter combinations, labeled with abscissa within $[-6,0]$ by using the proposed algorithm. The algorithm can be implemented with Matlab. Fig. 2 presents the labeled stable regions of system (Eq. (3)) with $r=2$, labeled with abscissa within $[-8,0]$. Both cases show that the increase of τ not only shrinks the stable region but also decreases the abscissa, and on the contrary, the increase of ξ not only enlarges the stable region but also increases the abscissa. In addition, Figs. 1—2 show that the stable regions of system (Eq. (3)) with $r=2$ are much larger than the corresponding ones with $r=1$. This means that from the viewpoint of stable region, a delayed PD feedback with the delay in position feedback double of that in velocity is preferable in applications.

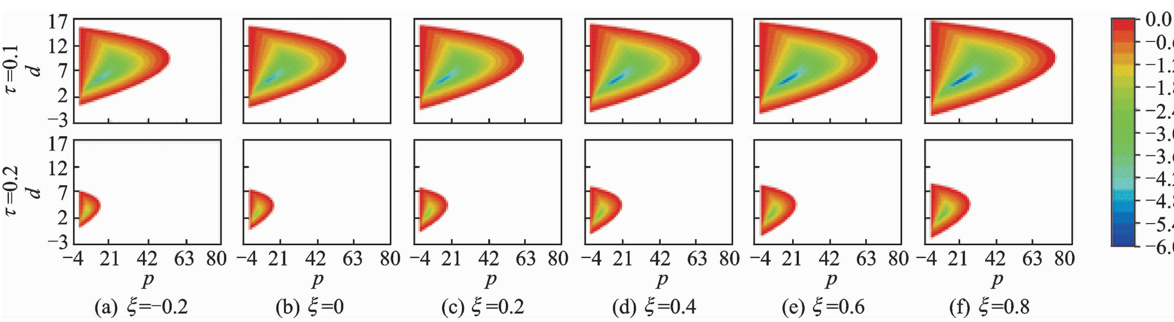

Fig. 1 Labeled stable regions of system (Eq. (3)) with $r=1$, $\sigma_{\min}=-6$, $\sigma_{\max}=15$ in $[-4,80]\times[-3,17]$

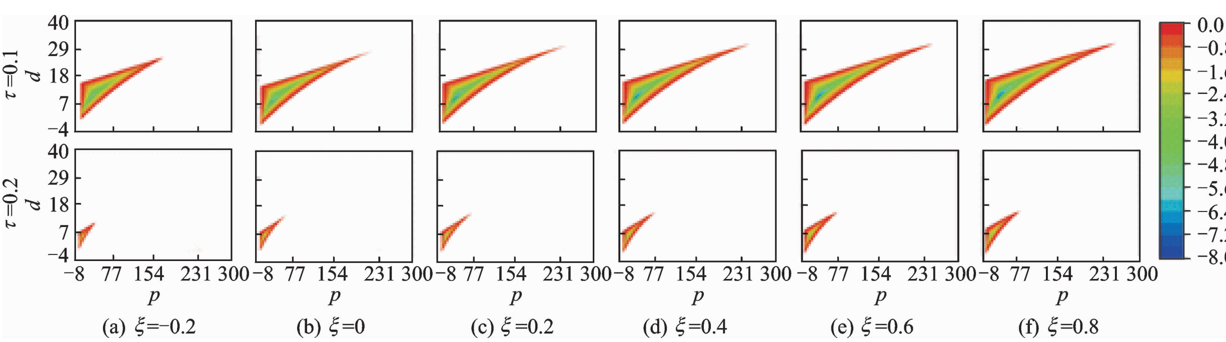

Fig. 2 Labeled stable regions of system (Eq. (3)) with $r=2$, $\sigma_{\min}=-8$, $\sigma_{\max}=15$ in $[-8,300]\times[-4,40]$

3.2 Example 2

Consider the following delayed system in dimensionless form

$$\ddot{x}(t)+2\xi\dot{x}(t)+1=-a\ddot{x}(t-r\tau)-d\dot{x}(t-\tau) \tag{4}$$

where a and p are the gain value of acceleration and velocity respectively, $|a|<1$, and r is the ratio coefficient of acceleration delay and velocity delay. The corresponding characteristic function is

$$D(\lambda)=(1+a\,\mathrm{e}^{-r\lambda\tau})\lambda^2+(2\xi+d\,\mathrm{e}^{-\lambda\tau})\lambda+1$$

By solving the linear equations $\mathscr{R}(D(\sigma+\mathrm{i}\omega))=0$, $\mathscr{I}(D(\sigma+\mathrm{i}\omega))=0$ with respect to a, d, one has

$$a=-\frac{(\omega^2+\sigma^2-1)\omega\cos(\omega\tau)}{(\omega\cos((r-1)\omega\tau)-\sigma\sin((r-1)\omega\tau))(\omega^2+\sigma^2)}\mathrm{e}^{r\sigma\tau}-\frac{(\sigma^3+2\xi\sigma^2+\sigma+\omega^2\sigma+2\xi\omega^2)\sin(\omega\tau)}{(\omega\cos((r-1)\omega\tau)-\sigma\sin((r-1)\omega\tau))(\sigma^2+\omega^2)}\mathrm{e}^{r\sigma\tau}$$

$$d=-\frac{2(\xi\sigma^2+\sigma+\xi\omega^2)\omega\cos(r\omega\tau)}{(\omega\cos((r-1)\omega\tau)-\sigma\sin((r-1)\omega\tau))(\omega^2+\sigma^2)}\mathrm{e}^{\sigma\tau}-\frac{(\omega^4+(2\sigma^2+2\xi\sigma-1)\omega^2+\sigma^4+2\xi\sigma^3+\sigma^2)\sin(r\omega\tau)}{(\omega\cos((r-1)\omega\tau)-\sigma\sin((r-1)\omega\tau))(\omega^2+\sigma^2)}\mathrm{e}^{\sigma\tau}$$

Fig. 3 shows the stable regions close the origin in the gain plane of system (Eq. (4)) with $r=1$ and different parameter combinations, labeled with abscissa within $[-19,0]$ by using the proposed algorithm. Fig. 4 presents the labeled stable regions of system (Eq. (4)) with $r=2$, labeled with abscissa within [−13,0]. Again, both cases show that the increase of τ not only shrinks the stable region but also decreases the abscissa, and on the contrary, the increase of ξ not only enlarges the stable region but also increases the abscissa. In addition, Figs. 3—4 show that the stable regions of system (Eq. (4)) with $r=2$ are much larger than the corresponding ones with $r=1$. This means that from the viewpoint of stable region, a delayed AD feedback with the delay in acceleration feedback double of that in velocity is preferable in applications, which is agreement with the result given in Ref. [9].

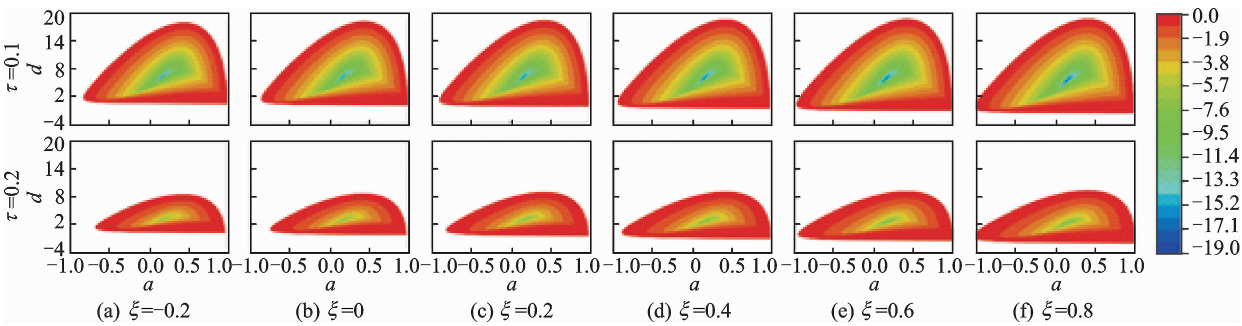

Fig. 3 Labeled stable regions of system (Eq. (4)) with $r=1$, $\sigma_{\min}=-19$, $\sigma_{\max}=15$ in $[-1,1]\times[-4,20]$

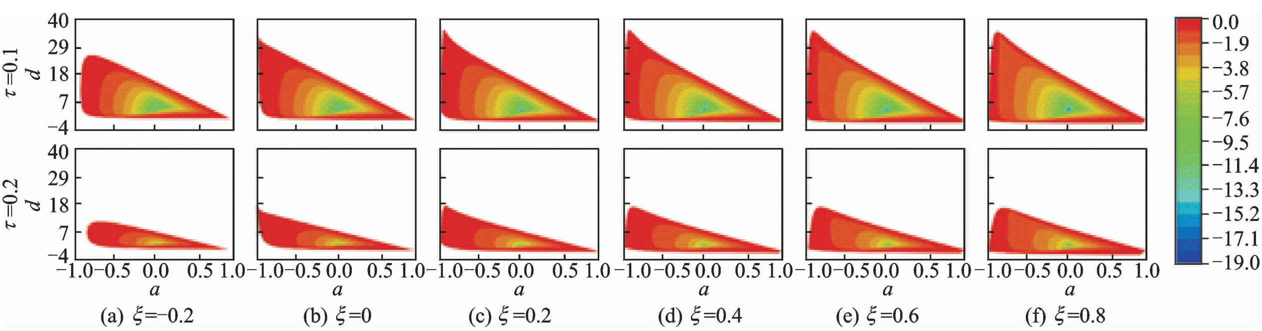

Fig. 4 Labeled stable regions of system (Eq. (4)) with $r=2$, $\sigma_{\min}=-13$, $\sigma_{\max}=15$ in $[-1,1]\times[-4,40]$

4 Conclusions

An algorithm based on the D-subdivision method is proposed for labeling the stable region in the plane of feedback gains of dynamical systems under a delayed feedback control with different abscissa. Two main steps in the labeling process are required, one is subdivision of the stable region, and the other is erasure of the unstable regions. The labeling simply uses a color in a designated color set to plot the σ-critical stable curves, and can be easily implemented with computer codes. A successful application of the proposed algorithm requires a suitable estimation of the abscissa. The two case studies show that for the controlled pendulum with a delayed feedback, the stable region can be substantially enlarged if the delays are properly chosen. The algorithm works for labeling the stable regions of a pair of feedback gains for any controlled systems with a delayed feedback.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (No. 11372354). The authors thank Dr. Zhang Li of Nanjing University of Aeronautics and Astronautics for bringing a labeling algorithm based on DDE-BIFTOOL into their attention.

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(Production Editor: Zhang Huangqun)